Light diffraction by multifrequency thick gratings

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(Submitted 11 October 1991; resubmitted 5 March 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **55**, No. 7, 380–383 (10 April 1992)

Multiphonon interactions in the course of an anisotropic diffraction of light by two acoustic waves excited in a common volume have been studied. A qualitatively new (two-state) behavior of the intensity of the diffracted electromagnetic wave as a function of the sound frequency has been observed. Multiphonon processes make the transmission of light in opposite directions a reciprocal process.

When several acoustic waves of different frequencies are excited in a common volume, multiphonon sum and difference interactions come into play. Such interactions are absent by definition in the case of Bragg diffraction of light by a single acoustic grating, and the ordinary multiphonon processes are negligible (aside from cases of degeneracy). In the case of isotropic diffraction, the sum and difference processes lead to only quantitative changes in the basic solution (Ref. 1, for example). In this letter we are reporting a theoretical and experimental study of the anisotropic diffraction of light by multiwave acoustic gratings. Since devices with such interactions are frequently employed in laser resonators, in which light is propagating in opposite directions, we supplement our study of the transmission characteristics with a study of questions pertaining to the reciprocity of the transmission of oppositely directed electromagnetic waves.

We assume that two plane acoustic waves, which have different frequencies Ω_1 and Ω_2 but are propagating in the same direction, are excited in a common volume of a uniaxial crystal. The dielectric constant in this case can be written

$$\epsilon = \epsilon_0 + \Delta\epsilon_1 \sin(\vec{K}_1\vec{r} - \Omega_1 t) + \Delta\epsilon_2 \sin(\vec{K}_2\vec{r} - \Omega_2 t), \qquad (1)$$

where ϵ_0 is the tensor of the unperturbed medium, and $\Delta\epsilon_1$ and $\Delta\epsilon_2$ are the changes caused by the acoustic waves, with wave vectors $\vec{K}_1$ and $\vec{K}_2$. For definiteness, we use the parameter values and the typical interaction geometry for paratellurite in the calculations.[2] The present experiments were also carried out on paratellurite.

We seek a solution of Maxwell's equations with dielectric constant (1) as the sum of normal wave modes of the medium:

$$\vec{E} = \sum_{p,m=0}^{\infty} \vec{e}_{pm} C_{pm}(z) \exp(\vec{k}_{pm}\vec{r} - \omega_{pm} t), \quad p, m = 0, \pm 1, \pm 2, \ldots \qquad (2)$$

(the Z axis is perpendicular to the group velocity of the sound, and the X axis is parallel to it). The frequencies ω_{pm} and the wave-vector components k_{pmz} satisfy the relations $\omega_{pm} = \omega_0 + p\Omega_1 + m\Omega_2$ and $k_{pmz} = k_{00z} + pK_{1z} + mK_{2z}$, which follow from the conservation of energy and the conservation of the Z projection of the momentum. After some transformations, we find the system of equations[3]

$$\partial C_{pm}/\partial z = q_{1p-1m}\exp(i\eta_{1p-1m}z)C_{p-1m} + q_{1p+1m}\exp(i\eta_{1pm}z)C_{p+1m}$$
$$+q_{2pm-1}\exp(i\eta_{2pm-1}z)C_{pm-1} + q_{2pm+1}\exp(i\eta_{2pm}z)C_{pm+1}, \quad (3)$$

where the coefficients $\eta_{1pm} = k_{p-1mz} + k_{pmz} + K_{1z}$ and $\eta_{2pm} = k_{pm-1z} + k_{pmz} + K_{2z}$ represent the detuning from wave matching, and

$$q_{1p\pm 1m} = \vec{e}_{pm}\Delta\epsilon_1\vec{e}_{p\pm 1m}\omega_{pm}^2/2k_{pmz}c^2,$$

$$q_{2pm\pm 1} = \vec{e}_{pm}\Delta\epsilon_2\vec{e}_{pm\pm 1}\omega_{pm}^2/2k_{pmz}c^2$$

determine the efficiency with which the incident light is coupled with each of the acoustic waves (c is the velocity of light). We restrict the calculations to the one-, two-, and three-phonon interactions, which dominate the solution (p, $m = 0, \pm 1, \pm 2$).

We studied the intensities of the normal modes, $I_{pm} = |C_{pm}|^2$, as a function of the parameters of the acoustic waves for opposite light propagation directions: forward (quadrant I in Fig. 1) and backward (quadrant III). The wave vector of the incident wave, $\vec{k}_{00}$, in the second case, and the wave vector of the diffracted wave, $\vec{k}_{10}$, in the first case, were chosen in opposite directions. In this case, normal diffraction occurs in

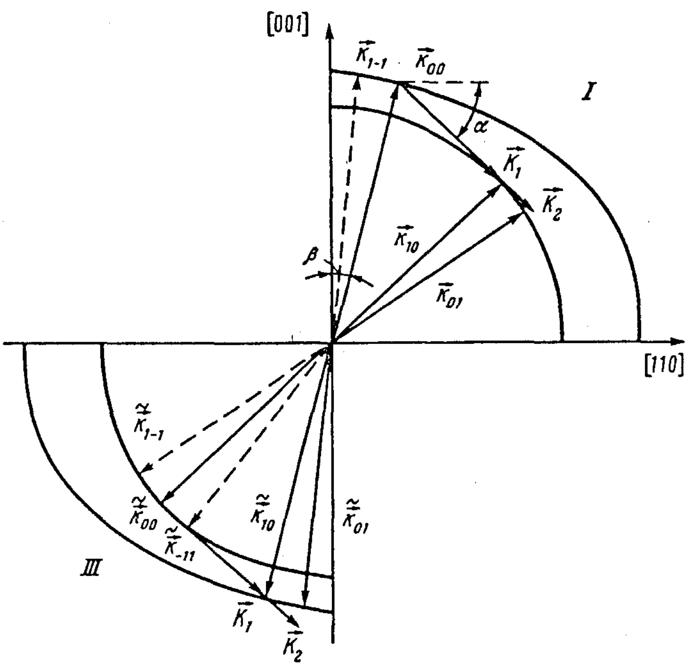

FIG. 1. Geometry of the acoustooptic interaction in an anisotropic medium. Dashed lines—The wave vectors of the natural wave modes of the most intense two-phonon processes.

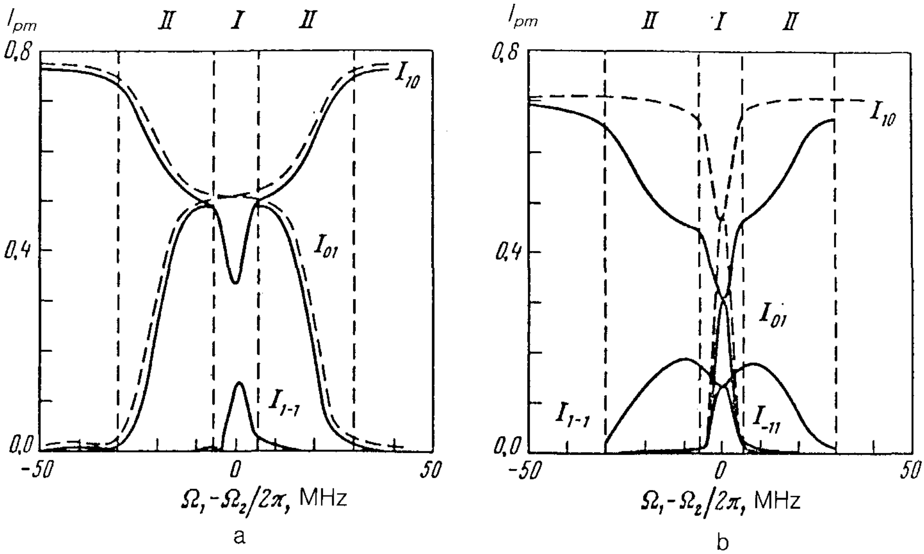

FIG. 2. Intensities of the normal modes, I_{pm}, calculated as a function of the frequency detuning $\Omega_1 - \Omega_2$ at $\Omega_1 = 60$ MHz and at identical power levels of the acoustic waves. a: For forward propagation of the light. b: For backward propagation of the light. Solid curves—With multiphonon processes; dashed curves—without such processes.

the backward direction, and anomalous diffraction (in which the angle between the vectors $\vec{K}_j$ and each of the vectors $\vec{k}_{10}$ and $\vec{k}_{01}$ is close to 90°) in the forward direction. For clarity, we have singled out regions I and II in Figs. 2 and 3, which show frequency intervals characteristic of each type of diffraction.

Ignoring small effects of higher order associated with the motion of the grating,[4] we can say that this system of equations should be reciprocal, by which we mean that the behavior of I_{10} should be the same for the opposite directions of the light. However, if we restrict the calculations to one-phonon processes, we find results which are at odds with this assertion (the dashed curves in Fig. 2): The plots of I_{10} differ substantially in width. The reason for this result, in this model, is that in the backward direction (Fig. 2b) the light in a sense does not perceive the acoustic wave with frequency Ω_2 in region II, and the diffraction efficiencies in the opposite directions are different for different detunings.

When multiphonon processes are taken into account, we obtain a fundamentally different result: Since the shape of the curves changes, the transmission coefficients for the oppositely directed waves become equal. Specifically, the efficiency of the first-order diffraction in the forward direction (Figs. 2a) decreases in region I, and it forms a narrow "beak." The changes for the backward direction (Fig. 2b) are even more substantial: The diffraction efficiency decreases in region I, and the curve of I_{10} becomes substantially broader in region II. As a result, the latter curve becomes the same as in Fig. 2a. In region II, the efficiency of the one-phonon diffraction by the second wave is zero, while I_{10} does not reach a maximum for the first wave.

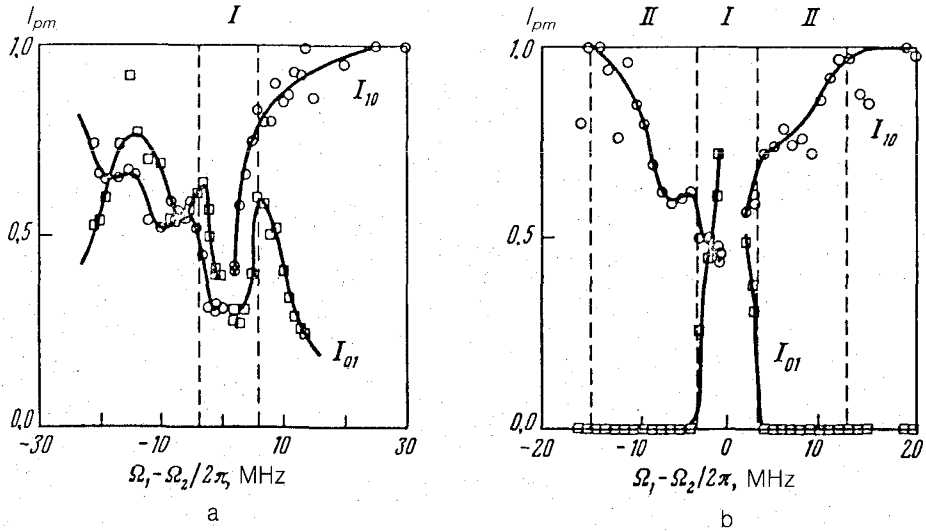

FIG. 3. Experimental results on the intensities of the normal modes, I_{pm}, as a function of the detuning of the acoustic frequencies for (a) forward and (b) backward propagation of the light. ○—I_{10}; □—I_{01}.

These changes in the transmission characteristics arise because of the simultaneous manifestation of multiphonon processes of two types. For the processes of one type, there is a normal diffraction (I_{-11} in Fig. 2b and I_{1-1} in Fig. 2a). For the processes of the second type, there is an anomalous diffraction (I_{1-1} in Fig. 2b). These circumstances are responsible for the two-stage shape of the curves. We might also point out that specifically these multiphonon processes impart reciprocity to the system, by "canceling" the asymmetry of the interaction configurations for the opposite directions.

Experiments were carried out with a paratellurite crystal for both forward and backward propagation of the light in the interaction geometry corresponding to Fig. 1, with $\alpha = 6°$ and $\beta \approx 4.2°–4.4°$. The light sources were a He–Ne laser ($\lambda = 0.63\ \mu$m) and an Nd:YAG laser ($\lambda = 1.06\ \mu$m). The intensities of the various normal modes were detected simultaneously by a line array of charge-coupled devices. This approach made it possible to determine the relations between these intensities. Figure 3 shows some experimental results, which confirm the two most important predictions of the calculations regarding the behavior (Fig. 2). Let us discuss these two aspects of the behavior.

1. That the curves of I_{pm} are of a two-stage nature can be seen in Fig. 3a, where we find a narrow dip (in which the diffraction efficiency falls off by $\approx 50\%$) against the background of the broad curves in region I. The width of this dip is characteristic of normal diffraction. Although the magnitude of the decrease is difficult to measure at $\Omega_1 – \Omega_2 \approx 0$, because the beams overlap (the regions where the curves are cut off at $\Omega_1 \approx 70$ MHz in Fig. 3), this decrease nonetheless exceeds the calculated value in Fig.

2a. The reason is that multiphonon processes of orders higher than those considered in the calculations play a larger role at $\Omega_1-\Omega_2\approx 0$.

2. Figure 3b shows the broadening of the I_{10} curve for the backward direction [the broadening is larger by a factor $\approx(5-6)$ than in the single-frequency case]. As predicted by the calculations (Fig. 2b), the shapes of the I_{01} and I_{10} curves are quite different and are determined by normal and anomalous diffraction, respectively. We see yet another important aspect of this system in Fig. 3: If one of the acoustic waves is outside the region of the anomalous acoustooptic interaction, then the presence of this wave has no effect on the diffraction (the values of I_{10} outside region II are the same as in the case in which only a single acoustic wave is excited), despite the disruption of the periodicity of this wave by the resultant acoustic grating.

We note in conclusion that the behavior established here is a general behavior, independent of the nature of the gratings and of the interacting waves. Corresponding processes will be seen in interactions of light with charge waves, with thick grating holograms, etc.

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Translated by D. Parsons