Polarization states in the energy gap of superconductor-ferromagnet superlattices

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(Submitted 13 January 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **55**, No. 7, 384–387 (10 April 1992)

The spectrum of excitations of S/F and $S/I(F)$ superlattices is derived [S is a superconductor, F is a ferromagnetic metal, and $I(F)$ is a ferromagnetic insulator]. Narrow Bloch bands of quasiparticle states with one spin orientation exist in the energy gap of superlattices of both types (only under certain conditions, in the case of the S/F superlattice). The Bloch bands of an $S/I(F)$ superlattice are unusual in that their width is a rapidly oscillating function of the thickness of the superconducting layer.

Several spatially periodic structures with alternating layers of a superconducting material and a magnetic material have been fabricated in recent years. In particular, there are the direct-conductivity systems Mo/Ni (Ref. 1) and V/Fe (Ref. 2), where Mo and V are conventional, i.e., low-temperature, superconductors, and Ni and Fe are band ferromagnets; and the $Y_1Ba_2Cu_3O_{7-x}/Pr_1Ba_2Cu_3O_{7-x}$ system,[3–5] where $Y_1Ba_2Cu_3O_{7-x}$ is a high-T_c superconductor, and $Pr_1Ba_2Cu_3O_{7-x}$ is a semiconductor with antiferromagnetic order at $T \lesssim 17$ K. Since these samples have a nearly ideal periodicity, and their purity can be controlled (within limits), they are attractive for experimental and theoretical research. Nevertheless, the physical properties of superconductor–(magnetic material) superlattices have not yet been studied adequately. The theoretical work which has been carried out has been focused for the most part on calculating the superconducting transition temperature (see, for example, Ref. 6 and the literature cited there). In the present letter, in contrast, we wish to call attention to the problem of the spectrum of excitations at low temperatures, which has not previously been discussed.

In this paper we discuss pure systems of two types with ferromagnetic layers: (1) an S/F superlattice (where S is the superconductor, and F is a nonsuperconducting ferromagnetic metal) and (2) an $S/I(F)$ superlattice [where $I(F)$ is a ferromagnetic insulator]. As we know, the excitation spectra of isolated $S/F/S$ and $S/I(F)/S$ junctions have some nontrivial features. In particular, for an arbitrarily small spontaneous moment, an isolated discrete level forms in the spectrum of an $S/I(F)/S$ junction in the absence of Josephson currents and in the absence of external magnetic fields. This isolated level corresponds to a polarization quasiparticle state near a barrier[1)] (Refs. 7 and 8). The appearance of a localized state is linked with a suppression of Cooper pairing at $S/I(F)$ interfaces. Its energy is given by[8]

$$E_{0+}(t) = \Delta_\infty[1 - 2T_S(t)], \quad |t| \gg \max\{(\Delta_\infty/E_F)^{1/2}, \ [T_S(1)]^{1/4}\}, \tag{1}$$

where Δ_∞ is the gap parameter in the interior of the superconducting banks, and $T_S(t)$ is the exchange part of the tunneling probability, which depends on the cosine of the angle of incidence on the barrier $[T_S(1) \ll 1]$. As was shown in Ref. 10, in an $S/F/S$ junction there is an analog of state (1) under the conditions $h \gg \Delta_\infty$, $d \ll v_0/h$ (h is the energy parameter of the exchange field, v_0 is the Fermi velocity, d is the thickness of the S layer, and we are setting $\hbar = 1$):

$$E_{0+}(t) = \Delta_\infty \left[1 - \frac{h^2 d^2}{2v_0^2 t^2}\right], \quad |t| \gg \max\{(h/E_F)^{1/2}, \ dh/v_0\}. \tag{2}$$

It might appear that levels (1) and (2) should generate Bloch bands with similar properties in periodic $S/I(F)$ and S/F structures. We show below that this is actually not the case.

1. *S/F* superlattice. We assume that the system is completely homogeneous along the y and z directions, and the x axis runs in the direction perpendicular to the boundaries of the layers. We work from the Bogolyubov–de Gennes equations of reduced order, with a piecewise-constant approximation of the real pairing potential $\Delta(x)$ (cf. Ref. 10):

$$\left[-iv_0 t\tau_3 \frac{d}{dx} + \Delta_\infty \tau_1 - (\Delta_\infty \tau_1 \pm h) \sum_{n=-\infty}^{+\infty} \Theta(x - nc)\Theta(d + nc - x)\right]\psi_\pm = E\psi_\pm. \tag{3}$$

Here t is the cosine of the angle between the direction in which the excitation is moving and the x axis; c is the period of the superlattice ($c = a + d$); τ_i are the Pauli matrices in Gor'kov–Nambu space; and we are using $\psi_+ = (u_\uparrow, v_\downarrow)$, $\psi_- = (u_\downarrow, v_\uparrow)$ (the plus and minus signs here correspond to the up and down orientations, respectively, of the quasiparticle spin). Solutions of (3) which are bounded on the entire x axis correspond to states inside the gap ($0 < E < \Delta_\infty$). These solutions satisfy the continuity conditions at the S/F boundaries and the Bloch condition $\psi(x + c) = \exp(\pm iqc)\psi(x)$, with real q. We wish to stress that excitations of two types correspond to the different signs of t in (3), and these two types of excitations do not mix, since no change occurs in the sign of the x component of the momentum in the course of Andreev reflection at the boundaries between layers. Equation (3) can be solved under the condition

$$\cos(qc) = \cos\left[\frac{(E \pm h)d}{v_0|t|}\right] \cosh\left[\frac{(\Delta_\infty^2 - E^2)^{1/2} a}{v_0|t|}\right] \tag{4}$$
$$- \frac{E}{(\Delta_\infty^2 - E^2)^{1/2}} \sin\left[\frac{(E \pm h)d}{v_0|t|}\right] \sinh\left[\frac{(\Delta_\infty^2 - E^2)^{1/2} a}{v_0|t|}\right].$$

Condition (4) actually solves the model problem of the spectrum of an S/F superlattice. We will not study it in detail for all permissible parameter values. We simply note that at $h = 0$ we find from (4) the known result of the theory of S/N superlattices (N being a normal metal),[11,12] while in the limit $a = \infty$ we obtain the spectrum of an isolated $S/F/S$ junction.[10] In the case of most practical importance, with $h \gg \Delta_\infty$ (typical values here are $h \sim 10^2$–10^3 K and $\Delta_\infty \sim 1$–10 K) and $d \ll v_0/h$ (the correlations in the phases of neighboring superconducting layers may be disrupted under the condition $d \sim v_0/h$; Refs. 1, 2, and 10), there exists a band of Bloch states with a positive spin orientation (ψ_+) in the gap for each $|t|$. In the limit $a \gg v_0^2 t^2/\Delta_\infty hd$, $|t| \gg \max\{(h/E_F)^{1/2}, dh/v_0\}$, the energy of these states is given by the simple analytic expression

$$E_{0+}(t;\ q) = \Delta_\infty \left\{ 1 - \frac{h^2 d^2}{2v_0^2 t^2} \left[1 + 4\cos(aq) \exp\left(-\frac{hda\Delta_\infty}{v_0^2 t^2} \right) \right] \right\}. \tag{5}$$

2. S/I(F) superlattice. Under the assumption that the thickness of the S layers is much greater than the thickness of the $I(F)$ layers, as in the theory of $S/I(F)/S$ junctions,[7,8] we use a δ-function approximation to describe the potential of the $I(F)$ layers:

$$\mathcal{H}_1 = (\mp J + V\tau_3) \sum_{n=-\infty}^{+\infty} \delta(x - na),$$

where J and V are the exchange and nonexchange parts, respectively ($J > 0$, $V > 0$). In contrast with the preceding case, it is not possible to lower the order of the differentiation with respect to x. The problem reduces to one of finding the eigenvalues of the singular operator

$$\mathcal{H}_0 + \mathcal{H}_1 \equiv -\left(\frac{1}{2m} \frac{d^2}{dx^2} + E_F t^2 \right) \tau_3 + \Delta_\infty \tau_1 + \mathcal{H}_1 \tag{6}$$

on the class of functions which are bounded on the entire x axis. The simplest way to find a solution is to use the technique of retarded Green's functions. The Green's function corresponding to (6) is found from the Dyson equation by means of a Fourier transformation. Since this function may be of interest in its own right, we write it out explicitly:

$$G_E^{\pm}(x, x') = G_E^{(0)}(x, x') + \frac{m}{\pi} \int_{-\infty}^{+\infty} dq g_0(q,\ E) e^{iq(x-x')}$$

$$\times \sum_{n=-\infty}^{+\infty} \frac{2m}{a} (\mp J + V\tau_3) g_n(q,\ E) \exp(i2\pi nx/a)$$

$$\times [1 + \frac{2m}{a} (\mp J + V\tau_3) \sum_{n=-\infty}^{+\infty} g_n(q, E)]^{-1},$$

$$g_n(q, E) = \left\{ \left[\left(q + \frac{2\pi n}{a} \right)^2 - 2mE_F t^2 \right] \tau_3 + 2m\Delta_\infty \tau_1 - 2m(E + io) \right\}^{-1}, \tag{7}$$

where $G_E^{(0)}(x,x')$ is the Green's function of the operator $\mathscr{H}_0$ (corresponding to a homogeneous superconductor).[8] The limits of (7) at $\Delta_\infty = 0$ and as $a \to \infty$ are the

Green's functions of respectively a superlattice in the normal state[13] and an $SI/(F)/S$ junction.[7] For $0<E<\Delta_\infty$ we find the excitation spectrum from the condition

$$\det\left[1+\frac{2m}{a}(\mp J+V\tau_3)\sum_{n=-\infty}^{+\infty} g_n(q,E)\right]=0. \tag{8}$$

Carrying out the summation with the help of an integration in the complex plane, we see that (8) has solutions for only the upper sign on J. If the $I(F)$ layers have a low transmission ($v_0,J\ll V$), and if the conditions $a\gg v_0/\Delta_\infty [T_S(1)]^{1/2}$ ($T_S(t)=v_0^2J^2t^2/V^4$), and $|t|\gg\max\{(\Delta_\infty/E_F)^{1/2}, [T_S(1)]^{1/4}\}$ also hold, we find

$$E_{0+}(t;q)=\Delta_\infty\{1-2T_S(t)[1+4\cos(ap_0t)\cos(aq) \times\exp\left(-\frac{2a[T_S(1)]^{1/2}\Delta_\infty}{v_0}\right)]\}, \tag{9}$$

where $p_0=mv_0$ is the Fermi momentum.

The presence of the factor $\cos(ap_0t)$ in (9) is quite unusual [cf. (5)]. Because of this factor, the width of the Bloch band becomes a rapidly oscillating function of the thickness of the S layers at a given value of $|t|$, and it in fact vanishes if $ap_0|t|=\pi/2+\pi N$ (N is an integer). The physical cause of the oscillations is an interference of the $t>0$ and $t<0$ states as the result of "ordinary" reflection at $SI(F)$ boundaries. In principle, the oscillations should persist as a decreases to extremely low values, at which a superconductivity of the superlattice is still possible. With decreasing thickness of the S layers, the maximum width of the Bloch bands increases. An attempt to observe such oscillations experimentally would of course run into difficulty, since the superstructure would have to be fabricated with a precision on the order of the lattice constant of the crystal lattice. In samples with a large variability in the properties of the layers, an averaging of the cosine should occur, and the density of states should be of the same form as in an isolated $S/I(F)/S$ junction.

Finally, we note that an oscillation in the width of the Bloch bands should also occur in the case in which the moments of the $I(F)$ layers are in a paramagnetic phase. In this case the number of permissible states in the gap will double, because of a "binding" of quasiparticles with the two spin orientations (cf. the discussion of an isolated junction[8]).

We wish to thank A. N. Omel'yanchuk for a useful discussion.

This work was supported by the Scientific Council on the Problem of High Temperature Superconductivity as part of Project 90118 of the state program High Temperature Superconductivity.

[1)]There are experimental indications that states of this sort exist for $Pb/Ho(OH)_3/Pb$ structures[9] [$Ho(OH)_3$ is a ferromagnetic insulator at $T\lesssim 2.5$ K].

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Translated by D. Parsons