Contour dynamics of the convection of a magnetized plasma

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The convection which occurs in the divertor region of a tokamak can be described by contour dynamics. The results of a numerical solution of the contour equations corresponding to a hot tube which is floating up and to an instability of a stepped temperature profile are reported.

1. Hot plasma tubes were observed floating upward in the divertor region of a tokamak in the experiments of Refs. 1 and 2. In this letter we show that a model system of equations for plasma convection describing this phenomenon makes it possible to introduce contour dynamics in certain cases. We report the results of a numerical solution of contour equations corresponding to a tube which is floating up and to an instability of a stepped temperature profile [see (4)].

2. A detailed derivation of the system of equations for convection in a divertor region is given in Ref. 3, where the conditions for the applicability of these equations are analyzed. Here we simply point out the procedure for deriving the initial equations.

We assume that the magnetic field is parallel to the z axis. We use the standard method of plasma theory for deriving nonlinear equations in a magnetic field. We substitute the longitudinal current from Ohm's law $j_z = -\sigma\partial_z\phi$ into the current continuity equation

$$\partial_z j_z + \nabla \vec{j}_\perp = 0. \qquad (1)$$

We find the transverse current from the transverse MHD equation

$$\rho(\partial_t + \vec{v}\nabla)\vec{v} - \mu\Delta_\perp\vec{v} = -\nabla p + (1/c)[\vec{j}_\perp, \vec{B}]. \qquad (2)$$

We can substitute the drift velocity $\vec{v} = (c/B)[\vec{e}_z, \nabla\phi]$ into the left-hand side of this equation; on the right-hand side we need to allow for the variation of the magnetic field (this variation is responsible for the convection). For simplicity we assume that of the two factors in the pressure, $p = nT$, only the temperature is perturbed. As a result, we find

$$(\partial_t + (c/B)[\nabla\phi, \nabla]_z)\Delta_\perp\phi = -(\sigma B^2/\rho c^2)\partial_z^2\phi + (B^2/Mc) \times [\nabla T, \nabla(1/B)]_z + (\mu/\rho)\Delta_\perp^2\phi. \qquad (3)$$

We describe the temperature by the model heat-conduction equation

$$(\partial_t + (c/B)[\nabla\phi, \nabla]_z)T = \chi_\perp \Delta_\perp T + \chi_\| \partial_z^2 T. \tag{4}$$

After an average is taken over z, the terms with ∂_z^2 are replaced by $-1/L^2$, where L is the distance along the magnetic field from one divertor plate to the other, and the average of $\nabla(1/B)$ is proportional to the curvature on the outer side of the torus. System of equations (3), (4) becomes two-dimensional. If we now understand T as the temperature perturbation, and if we take the unperturbed temperature gradient into account, we find, in the corresponding normalizations,

$$(\partial_t + [\nabla\phi, \nabla]_z)\Delta\phi = \nu_1\phi + \nu_2\Delta^2\phi - \alpha T_y, \tag{5}$$

$$(\partial_t + [\nabla\phi, \nabla]_z)T = -\chi_1 T + \chi_2 \Delta T - \beta\phi_y, \tag{6}$$

where α is the effective gravitational force, which is proportional to $\nabla(1/B)$, and β is the normalized unperturbed temperature gradient.

Equations (5) and (6) are very similar to the usual equations for two-dimensional convection.[4] The potential ϕ corresponds to the stream functions, α corresponds to the thermal expansion coefficient, β is the temperature gradient in both cases, ν_2 and χ_2 are the usual viscosity and thermal diffusivity, and χ_1 characterizes the heat exchange with the ends. The quantity ν_1 apparently has no analog in the case of a liquid.

3. We now make a decisive simplification: We assume that the convection is slow, in the sense that the viscous terms are much larger than the inertial terms in (5). If we also ignore the equilibrium temperature gradient ($\beta = 0$) and the transverse thermal conductivity ($\chi_2 = 0$), we find the basic system of equations in the corresponding normalizations:

$$T_y = \phi + \Delta^2\phi, \tag{7}$$

$$(\partial_t + [\nabla\phi, \nabla]_z)T = -\nu T. \tag{8}$$

Interestingly, if we replace (7) by $T = \phi - \Delta\phi$ and ignore the right side of (8), we obtain the Hasegawa–Mima equation. Equations (7) and (8) can be written as the single equation

$$(\partial_t + [\nabla\psi_y, \nabla]_z)(\psi + \Delta^2\psi) = -\nu(\psi + \Delta^2\psi), \tag{9}$$

where $\psi_y = \phi$. The substitutions $\psi \to e^{-t}\psi$, $-e^{-t} \to t$ put (9) in the form

$$(\partial_t + [\nabla\psi_y, \nabla]_z)(\psi + \Delta^2\psi) = 0. \tag{10}$$

Equation (10) allows us to introduce contour dynamics (see Ref. 5 and the papers cited there). Specifically, we assume that the temperature $T = \psi + \Delta^2\psi$ initially takes on the value 1 in a certain region Ω, while it has the value 0 outside Ω. Then for any point $\vec{R}$ outside Ω we have

$$\psi(\vec{R}) = \int_\Omega d^2r G(|\vec{R} - \vec{r}|), \tag{11}$$

where G is the Green's function of the operator $1 + \Delta^2$, given by

$$G(r) = \int dkk(1 + k^4)^{-1} J_0(kr), \qquad (12)$$

and J_0 is the Bessel function. From (11) we find

$$\phi(\vec{R}) = \int_\Omega d^2 r \partial_Y G = \int_\Omega d^2 r \partial_y G = \int_\Omega ds(\partial x/\partial s) G(|\vec{R} - \vec{r}(s)|), \qquad (13)$$

where $\vec{r}(s)$ is the boundary of Ω, and s is an arbitrary parameter. Evaluating the gradient in (13) from $d\vec{r}/dt = \vec{v} \equiv [\vec{e}_z, \nabla\phi]$, we find the equation of the contour motion:

$$\partial_t \vec{r}(s) = \int ds' (\partial x/\partial s') F(|\vec{r}(s) - \vec{r}(s')|)[\vec{e}_z, \vec{r}(s) - \vec{r}(s')], \qquad (14)$$

where the function F is found from the Green's function G:

$$F(r) = (1/r) G'(r) = (1/r) \int dkk^2 (1 + k^4)^{-1} J_1(\vec{k}\vec{r}). \qquad (15)$$

Here J_1 is again a Bessel function.

4. An unexpected result is that the numerical simulation of (14) leads to structures which are very similar to those that are found in a direct simulation of the complete system of equations, i.e., (5), (6) (Ref. 3). For example, for an initial condition corresponding to a stepped temperature profile—a slightly curved line running nearly parallel to the x axis—some "mushrooms" (Fig. 1) characteristic of convection form. Similar formations are found in the convection of a liquid.[6,7] An initial condition in the form of a closed contour leads to characteristic drop-shaped forma-

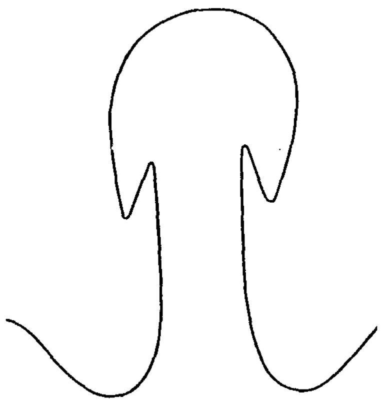

FIG. 1. "Mushrooms" which form from a nearly straight initial contour as a result of the instability.

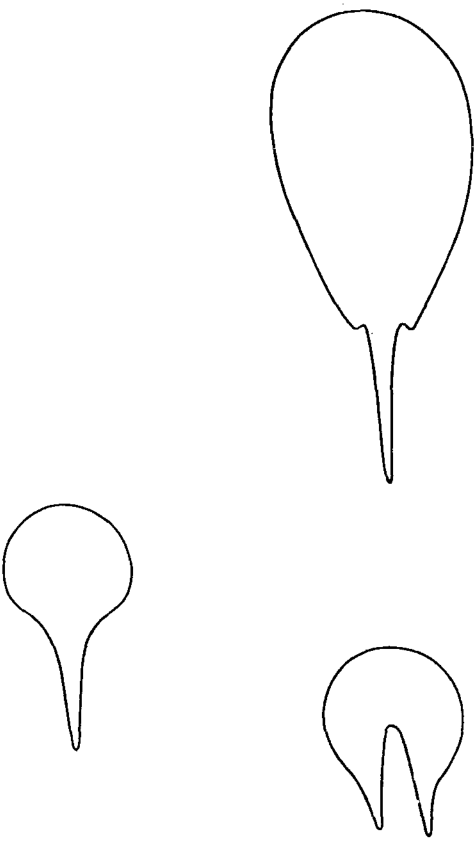

FIG. 2. Hot tubes which are floating up. The initial contours are ellipses.

tions which float up (Fig. 2). These formations have been observed in the simulation of the complete system of equations, (5), (6) (Ref. 3).

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