Josephson effect for a nonideal Bose gas

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A Josephson junction for a slightly nonideal Bose gas can be described well by an ideal sinusoidal dependence of the current on the phase difference. Some possibilities for experimentally observing a Josephson effect are discussed.

There is no theoretical reason to doubt that a Josephson effect, i.e., a periodic dependence of a current on a phase difference, would occur in a slightly nonideal Bose gas. The detailed shape and numerical values of this dependence, however, should be determined by the structure and geometry of the Josephson junction. The simplest imaginable construction would be a short, narrow channel connecting two vessels. In practice, however, it would be exceedingly difficult to find a single-valued dependence of the current on the phase difference in such a system. The reason is that the correlation length ξ in a slightly nonideal Bose gas at $T=0$ is given by

$$\xi = \hbar/mu.$$

Evaluating this expression for ^{4}He, we find $\xi \sim (Na)^{-1/2} \sim a$, where a is the scattering length, which is the same as the interatomic distance in ^{4}He. A necessary condition for a single-valued dependence of the current on the phase difference is that the size of this small channel, l, be smaller than the correlation length: $l < \xi$. Since $\xi \sim a$, we conclude that it would be impossible in practice to observe a clearly expressed Josephson effect in such a structure at low temperature. However, even this structure has possibilities, because ξ increases toward the λ point, becoming infinite right at this point.

In the present letter we take a more detailed look at another possible structure: a tunnel junction. In practice, such a structure could be devised in the following way for ^{4}He. A droplet of ^{3}He or a bubble with gaseous ^{4}He is placed between two volumes containing ^{4}He. There are yet other ways to suppress the order parameter of ^{4}He. Clearly, a correct description of this system would use the method of a tunneling

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Hamiltonian. This approach is widely used in describing superfluid Fermi systems. It reduces to the Hamiltonian[1]

$$H = H_L(a, a^+) + H_R(b, b^+) + H_t\,,$$

$$H_t = \sum_{k,q}[T_{kq}a_k^+ b_q + T_{kq}^* b_q^+ a_k], \tag{1}$$

where the Hamiltonians $H_L(a, a^+)$ and $H_R(b, b^+)$ describe the slightly nonideal Bose gases in the vessels on the right and left, respectively, H_t is the tunneling Hamiltonian, and T_{kq} is the tunneling amplitude. The current of the condensate particles, I, is found from the change in the number of particles in one of the vessels:

$$I = \dot{N}_L = \frac{i}{\hbar}[H, N_L] = \frac{i}{\hbar}T_{00}(a_0^+ b_0 - b_0^+ a_0). \tag{2}$$

Taking the expectation value of I over the ground state of the unperturbed Hamiltonian, we immediately find

$$I = I_c \sin(\varphi_L - \varphi_R) \qquad I_c = \frac{2}{\hbar}T_{00}\sqrt{N_L N_R}. \tag{3}$$

In addition to the condensate, there are particles above the condensate in a slightly nonideal Bose gas at absolute zero. These particles above the condensate also contribute to the superfluid current. The reason is that the ground state of a slightly nonideal Bose gas contains actual gas particles $\phi_k = v_k a_k^+ |0\rangle$, where v_k are the coefficients of a u–v transformation. We know that the coefficients of a u–v transformation depend on the phase: $v_k \sim v_k e^{i\varphi}$. Taking the expectation value of I over the ground state ϕ_k, we find

$$I_2 = \frac{2}{\hbar}\sum_k T_{kk} v_k^{L*} v_k^R \sin(\varphi_L - \varphi_R). \tag{4}$$

If the vessels on the left and the right are filled with the same gas, and if the tunneling amplitude T_{kk} is essentially independent of k, we can add (3) to (4), obtaining as a result a simple expression for the amplitude of the Josephson current:

$$I = I_c \sin(\varphi_L - \varphi_R); \qquad I_c = \frac{2}{\hbar}TN, \tag{5}$$

where N is the total number of actual gas particles. At a nonzero temperature, we would add to the dissipationless Josephson current a dissipative current which stems from thermal quasiparticles and which is proportional to the difference between the chemical potentials of the vessels.

In the tunneling-Hamiltonian method for a Bose gas, as for a Fermi gas, we obtain an ideal sinusoidal dependence of the current on the phase difference. Interestingly, however, a nonzero value of the superfluid current is found in the tunneling-Hamiltonian method for a Bose gas even in the zeroth order of a perturbation theory in T, in contrast with the case of a superfluid Fermi gas. The dissipative quasiparticle current at a nonzero temperature is of second order in the tunneling amplitude. Consequently, to obtain a well-expressed Josephson effect in a tunneling structure we should use a reasonably low temperature or a reasonably small value of the tunneling amplitude.

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[1]A Barone and G. Paternò, *Physics and Applications of the Josephson Effect*, Wiley-Interscience, New York, 1981.

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