Resonant excitation of wakefields by a laser pulse in a plasma

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As a short laser pulse propagates through a plasma, a self-modulation of its intensity occurs. This modulation is accompanied by the resonant excitation of wakefields.

In most of the laser-plasma accelerators which are presently under development,[1,2] the particles are to be accelerated by means of a charge density wave which is propagating at a velocity close to the velocity of light. Two methods for exciting the charge density wave by laser pulses are being pursued most vigorously. In the first, the plasma beat-wave accelerator,[3] a comparatively long, two-frequency laser pulse is used. We denote the two frequencies by ω_1 and ω_2. The longitudinal dimension of the pulse satisfies $L_{\parallel} \gg c/\omega_p$, where $\omega_p = \sqrt{4\pi e^2 n_0/m}$ is the plasma frequency, and n_0 the electron density. An effective excitation of charge density waves occurs at the resonance $\omega_1 - \omega_2 \simeq \omega_p$. This approach has made it possible to generate accelerating fields of $(1–3)\times 10^9$ V/m (Ref. 4). In the second approach, the laser wakefield accelerator,[5,6] the charge density wave is to be excited by an ultrashort laser pulse ($L_{\parallel} \lesssim c/\omega_p$)

of high intensity (the excitation is of a shock nature, rather than a resonance nature, in this case).

The results of a 2D numerical calculation on the nonlinear dynamics of a short laser pulse ($L_{\parallel} \gtrsim c/\omega_p$) which we are reporting here show that an intensity modulation which arises in the course of a self-effect is accompanied by the resonant excitation of charge density waves ("wakefields") behind the pulse. The excitation efficiency is so high that one could say that new prospects arise for laser-plasma accelerators.

The calculations were based on a system of equations for the envelope of the electromagnetic pulse, $a = eE_0/m\omega_0 c$ (E_0 and ω_0 are the electric field amplitude and the frequency, respectively), and for the perturbation of the electron density, $n = \delta n/n_0$ (n_0 and δn are the unperturbed density and the perturbation, respectively) (Ref. 7, for example):

$$2i\omega_0 \frac{\partial a}{\partial \tau} + c^2 \Delta_{\perp} a + \frac{\omega_p^2}{4}|a|^2 a = \omega_p^2 n a \,, \tag{1}$$

$$v_g^2 \frac{\partial^2 n}{\partial \xi^2} + \omega_p^2 n = \frac{c^2}{4}\Delta |a|^2. \tag{2}$$

Here $\xi = z - v_g t$ is the longitudinal coordinate in a frame of reference moving at the group velocity $v_g = c^2 k_0/\omega_0$; $k_0 = c^{-1}\sqrt{\omega_0^2 - \omega_p^2}$; the variable τ characterizes the slow (at the scale $L_{\parallel}/c$) time variations in the quantities a and n; $\Delta_{\perp} = r^{-1}(\partial/\partial r)(r\partial/\partial r)$; $\Delta = \Delta_{\perp} + (\partial^2/\partial\xi^2)$; and r is the transverse coordinate.

Equations (1) and (2) were solved numerically under the condition that there are no density perturbations ahead of the pulse, under the assumption of axial symmetry, and under the assumption that a and n decay as $r \to \infty$. It was assumed that the phase front of the pulse is initially planar and that the amplitude has a Gaussian shape

$$a(\tau = 0, \xi, r) = a_0 \exp\left[-\frac{(\xi - \xi_0)^2}{L_{\parallel}^2} - \frac{r^2}{L_{\perp}^2}\right], \tag{3}$$

where $L_{\perp}$ and ξ_0 are the transverse dimension and initial coordinate of the center of the pulse.

Figure 1 is a contour map of the amplitude $|a(r=0,\xi,\tau)| = \text{const}$ of a pulse which is initially spherically symmetric ($L_{\perp} = L_{\parallel}$) and which is centered at the point $\xi_0 = 2L_{\parallel}$. We see that at the pulse axis the amplitude increases in the central region, and the leading and trailing edges steepen, over a fairly long time ($\tau c/L_{\parallel} \lesssim 400$). A modulation of the amplitude with a period $\simeq 2\pi c/\omega_p$ then arises at the trailing edge of the pulse. As time elapses, the depth of this modulation increases, goes through a maximum, and then, at $(\tau c/L_{\parallel}) > 600$, begins to decrease.

Figure 2, a–c, shows contour maps of the function $|a|$ and of n in the (ξ, r) plane at three times.

It can be seen from Figs. 1 and 2 that there are three stages in the evolution of the pulse and in the wakefields which it excites.

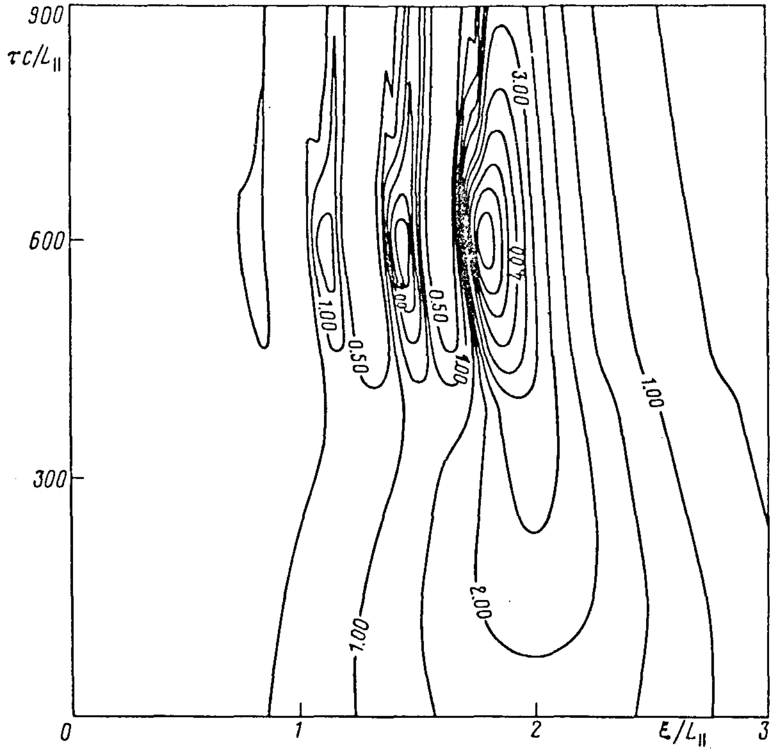

FIG. 1. Time evolution of the fixed values of the amplitude at the axis of the pulse ($r=0$). The parameter values $L_\parallel = L_\perp$, $(\omega_0/\omega_p)=22.5$, $a_0=0.27$, and $k_0L_\parallel=450$ were used here. The curve labels are the values of $E=|a|(10/\sqrt{2})(k_0L_\parallel)(\omega_p/\omega_0)^2=6.66|a|$.

In the first stage ($\tau \lesssim 400L_\parallel/c$), the ponderomotive nonlinearity associated with the density perturbation n makes only a small contribution to Eq. (1). In the central part of the pulse, where the amplitude is high, a relativistic self-focusing occurs. At the leading and trailing edges of the pulses, where the amplitude is low, a diffractive spreading occurs (Fig. 2a). The pulse in a sense contracts, and its leading and trailing edges steepen.

In this stage the amplitude of the wakefield is expressed in terms of $\Delta|a|^2$ in accordance with Eq. (2), and it depends on the relation between the length scale of the variations in $|a|^2$ and the length of the wakefield,[5] c/ω_p. The wakefield amplitude is initially proportionally to $\exp[-\frac{1}{2}(\omega_pL_\parallel/c)^2]$ and is very small [$\exp(-200)$]. At the end of the first stage, the length scale reaches a value on the order of c/ω_p, and the wakefield amplitude approaches $|a_{\max}|^2$.

In the second stage, the wakefield which has arisen triggers a distinctive self-focusing instability. In regions with an elevated electron density ($n>0$) the light amplitude decreases, while it increases in regions with a depressed density ($n<0$)

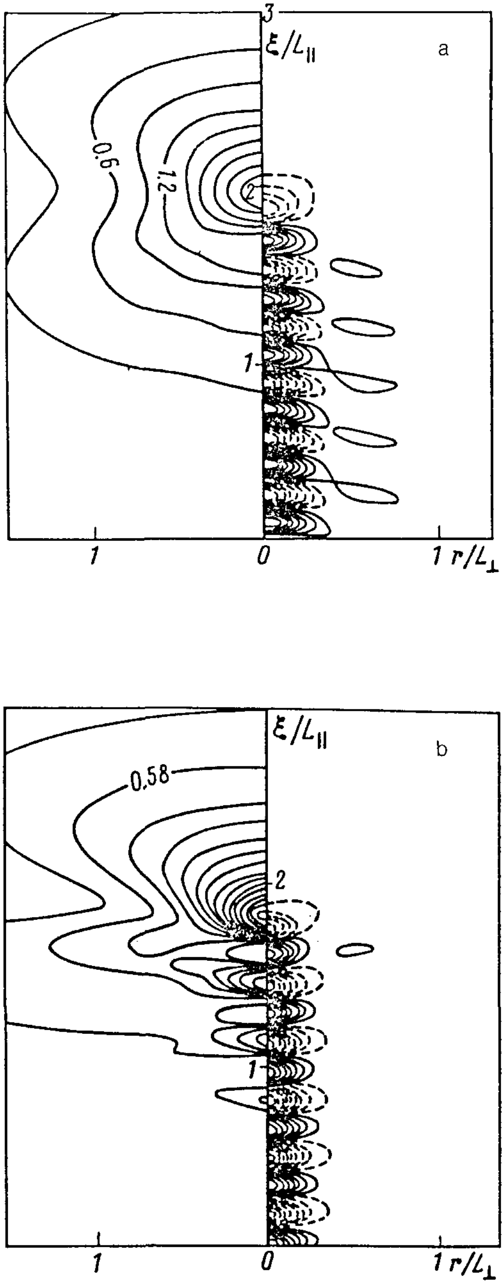

FIG. 2. (*Continued*)

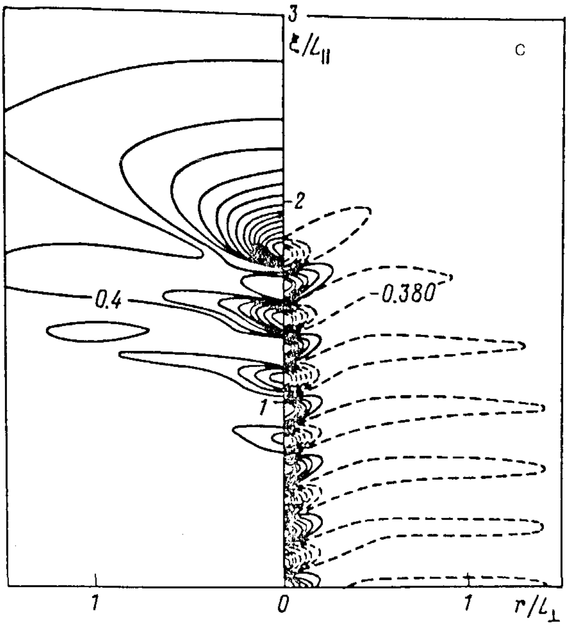

FIG. 2. Contour maps of the amplitude (on the left) and of the density perturbations (on the right) in the ξ, r plane. The parameter values are the same as in Fig. 1. Dashed lines—Negative values of n; solid lines—positive. a) $\tau c/L_{\parallel} = 450$, $E_{\max} = 2.937$, $|n|_{\max} = 0.026$; b) $\tau c/L_{\parallel} = 540$, $E_{\max} = 4.44$, $|n|_{\max} = 0.184$; c) $\tau c/L_{\parallel} = 630$, $E_{\max} = 5.94$, $|n|_{\max} = 0.452$.

(Fig. 2, b and c). Ponderomotive forces amplify initial density perturbations, and the amplitude of the wakefield increases. The density perturbations in turn increase the modulation depth of the pulse.

In the third stage, the zone of diffractive spreading of the leading edge of the pulse approaches the region of maximum amplitude, and the size of this maximum begins to decrease (Fig. 2c).

In the case of a neodymium-glass laser ($\omega_0 \simeq 2\times10^{15}$ s^{-1}), the parameter values used in the calculation correspond to a pulse length of 0.25 ps, to an energy of 5 J, to $L_\perp = L_\parallel = 75$ μm, to a maximum initial intensity $\sim 10^{17}$ W/cm^2, and to a diffraction length of 6.5 cm. At $n_0 = 2\times10^{18}$ cm^{-3}, the maximum amplitude of the electron-density perturbations is $|n|_{\max} = 0.45$. This figure corresponds to an accelerating field of 6×10^{10} V/m (the rate of electron acceleration is 60 GeV/m).

According to Eq. (1), the integral $\int_0^\infty dr r|a|^2$ does not depend on the time, and its value is determined by initial condition (3). The meaning here is that the change in the light intensity in the pulse results exclusively from a redistribution of the amplitude in the transverse direction, and the two-dimensional nature of the problem is of fundamental importance.

Estimates show that the total energy which is transferred to the wakefield over the diffraction length is roughly 1% of the pulse energy.

The system of equations which we have used, (1), (2), has several limitations. The derivation of these equations was based on the assumptions $|n| < 1$ and $|a|^2 < 2$. In our calculations these quantities reached values of 0.45 and 0.8, respectively; a further increase in the initial value a_0 requires a revision of the initial equations. Furthermore, this system of equations ignores the stimulated Raman scattering at various angles. We are presently studying this process. However, estimates based on Ref. 8 raise the hope that this process will not be important under the conditions discussed here.

In summary, the conditions for resonant excitation of wakefields which we have been discussing here arise because of a self-modulation of the pulse amplitude. In this regard, this case is fundamentally different from the method of exciting charge density waves in a beat-wave accelerator.[3] In that method of excitation the amplitude modulation arises because of the two-frequency nature of the laser light. The self-modulation precedes the stage in which the leading and trailing edges of the pulse steepen; that stage becomes shorter as the pulse becomes smaller. Consequently, pulses for which $L_{\parallel}$ is a few times c/ω_p would be the best choice for observing the predicted effect. In practice, this means pulses in the picosecond range, which happens to be just where the most intense pulses are presently being produced.

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