Resonant production of mesic molecules above thermal energies

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A formula is derived for the resonance production of mesic molecules in the epithermal energy region. The "swallowing" of the spectrum of moderated mesic molecules is taken into account. Doppler broadening of the levels is also taken into account.

Calculations[2] on the production of $dd\mu$ mesic molecules based on Vesman's resonance mechanism[1] have turned out to agree well with experimental data.[3–5] In contrast, the production of $dt\mu$ mesic molecules at D_2 molecules at low temperatures in the thermal region occurs primarily because of negative subthreshold resonances (see the reviews[7,8]). In addition to being produced in the thermal region, mesic molecules are produced in a resonance fashion at above thermal energies, as mesic atoms are moderated. The spectrum of the moderated mesic atoms is not Maxwellian in this region. The spectrum is particularly distorted at the resonances, and this distortion strongly influences the production of mesic molecules.

1. The mesic molecule $dt\mu$ is produced in a resonance fashion in the reaction[9]

$$(t\mu)_F + (DX)_{\nu_i k_i} \Rightarrow [(dt\mu)_S^{vJ} dee]_{\nu_f K_f}, \tag{1}$$

where $v = J = 1$ are the vibrational and rotational quantum numbers of the weakly bound level of the mesic molecule,[10] and ν_i, K_i, ν_f, and K_f are the rotational and vibrational quantum numbers of the original DX molecule and of the mesic-molecule complex (MMC) which is produced ($X = H, D, T$). Because of the finite lifetime of the MMC, the microscopic cross section for reaction (1) is of a Breit–Wigner nature near the resonance[11] ($\nu_i = 0$; $\{F,S,\nu_f,K_iK_f\} \equiv Q$; $\hbar = c = 1$):

$$\sigma_{dt\mu-X}(E) = \frac{\pi}{p^2} \sum_Q \frac{W(K_i)\Gamma_{eQ}\Gamma_{cQ}}{(E - E_{rQ})^2 + \Gamma_{rQ}^2/4}. \tag{2}$$

Here p is the momentum in the c.m. frame, $W(K_i)$ is the population of the levels of the DX molecule at the given temperature, and E_{rQ} is the energy of the resonance, which is given by

$$E_{rQ} = -|\epsilon_{11}| + E_{\nu_f K_f}(C) - E_{\nu_i K_i}(DX) + \Delta E_S - \Delta E_F. \tag{3}$$

Here ϵ_{11} is the energy of the weakly bound level of the mesic molecule, $E_{\nu_f K_f}(C)$ and $E_{\nu_f K_i}$ are the energies of the levels of the MMC and of DX, and ΔE_F and ΔE_S are the energies of the relativistic splitting of the mesic atom and the mesic molecule. The capture width for mesic atoms, Γ_{cQ}, consists primarily of the probabilities for transitions to low-lying states of the mesic molecule (as the result of Auger processes) and of the MMC (as a result of collisions with molecules of the surrounding medium).[12,13] The entrance width Γ_{eQ} is related to the transition matrix element as follows in first-order perturbation theory:[6]

$$\Gamma_{eQ}(E) = \frac{\mu p}{\pi} |V(E)|^2_Q, \tag{4}$$

where μ is the reduced mass of $t\mu$.

The total width Γ_{rQ} incorporates all processes by which the complex decays, including the decay to the initial state $t\mu + DX$.

2. To determine the spectrum of moderated mesic atoms (MAs), we work by analogy with neutrons. In the energy region far from E_0, which is the production energy of the MAs (a matter of tens of eV), we can use the following approximate balance equation[14] for the flux of moderated MAs ($E < E_0$):

$$-\frac{\partial}{\partial E}(\Sigma\xi E\varphi) + \Sigma_a\varphi(E) + \frac{\varphi(E)}{\tau_0 v} = 0. \tag{5}$$

Here $\Sigma_a = \Phi N_0 \Sigma_i c_i \sigma_{dt\mu,i}$ is the macroscopic cross section for the production of an MMC; $N_0 = \Phi N_0$ is the density of the medium with respect to the density of liquid hydrogen, $N_0 = 4.25 \times 10^{22}$ cm^{-3}; c_i is the relative concentration of molecules of species i; $\sigma_{dt\mu,i}$ is their cross section for the production of mesic molecules; $\tau_0 = 2.2 \times 10^{-6}$ s is the lifetime of the muon; v is the rate at which the MAs are moderated; Σ is the total macroscopic cross section, which includes the scattering cross section Σ_s; and ξ is the average logarithmic energy loss, given by

$$\xi = \frac{\Sigma_i c_i \xi_i \Sigma_{si}}{\Sigma_s}; \quad \xi_i = 1 + \frac{\epsilon_0 \ln \epsilon_0}{1 - \epsilon_0}; \quad \epsilon_0 = \left(\frac{A_i - 1}{A_i + 1}\right)^2. \tag{6}$$

Here $1 - \epsilon_0$ is the maximum possible energy loss in a single elastic collision, and $A_i = M_i/m_a$ is the ratio of the mass of molecule i and that of the MA.

In the absence of absorption and decay, according to (5), the density of collisions is conserved in each energy interval: $Q = \Sigma\xi E\varphi = \text{const} = Q_0$, where Q_0 is a given density of the production of MAs at the energy E_0. It follows that the spectrum of moderated MAs is described by the classical Fermi spectrum

$$\varphi(E) = \frac{Q_0}{\xi \Sigma_s E}. \tag{7}$$

In a dense medium ($\Sigma_s \tau_0 v \gg 1$) the fraction of MAs which have been moderated to an energy E and which have avoided resonant absorption and the fraction of the MMCs which have formed are, respectively,

$$p(E) \equiv Q(E)/Q(E_0) = \exp\left[-\int_E^{E_0} \frac{\Sigma_a(E')}{[\Sigma_s + \Sigma_a(E)]} \frac{dE'}{\xi E'}\right]; \quad \delta = 1 - p. \tag{8}$$

3. Let us examine the case of a strong resonance, in which the value of the cross

section at the resonance satisfies $c_i\sigma_{rQ_i} = c_i W(K_i)4\pi\Gamma_{eQ_i}/p_r^2\Gamma_{rQ_i} \gg \sigma_s = \Sigma_i c_i \sigma_{si}$. We will evaluate (8) in the approximation of a narrow resonance, in which the width of the zone in which the production of MMCs is important satisfies $\Delta E \ll \xi E$, where the right side is the average energy loss per collision. We find the width ΔE from the condition that at $E = E_r + \Delta E/2$ the function in the integrand falls to half its value at the maximum:[15] $\Delta E = \Gamma_{Q_i}\sqrt{1 + c_i\sigma_{rQ_i}/\sigma_s}$. Under the condition that the resonances do not overlap, and under the further condition that the distance between levels satisfies $D_Q \gg \Delta E$, we find the following result from (2) for resonance integral (8) above the lower boundary of the Fermi spectrum, E_c:

$$p = \exp\left\{-\frac{1}{\xi}\sum_i c_i \sum_{Q_i} \frac{\pi}{2}\frac{\Gamma_{rQ_i}}{E_{Q_i}} \frac{\sigma_{rQ,l}}{\sigma_s\sqrt{1 + c_i\sigma_{rQ_i}/\sigma_s}}\right\}. \qquad (9)$$

[For simplicity we have omitted from (9) a correction for the interference between potential scattering and resonance scattering.]

Expression (9) contains a factor in addition to the weak-absorption integral, $c_i\sigma_{i0} \ll \sigma_s$. This additional factor, which is small, is $(1 + c_i\sigma_{rQ_i}/\sigma_s)^{-1/2} = \Gamma_{Qi}/\Delta E$. It results from the "swallowing" of the spectrum of mesic atoms at the resonance which was discovered for neutrons by Zel'dovich and Khariton.[16] This effect can be summarized by saying that all the mesic atoms which reach a resonance as a result of a previous scattering at a higher energy may be absorbed, but no more. As a result, the flux of mesic atoms, $\varphi = Q/\xi\,\Sigma(E)$, drops sharply near a resonance.

The individual terms in (9) fall off as $E_{rQ_i}^{-3/2}$, so the first term of the sum, with the smallest ν_f, is predominant. As was shown in Ref. 17, the value of σ_{rQ_i} is particularly large in a collision of $t\mu$ with DH molecules, in which case we have $\nu_i = 0$ and $\nu_f = 2$ (Ref. 16). As an example we consider the transition $\{F = 1, K_i = 0 \to S = 2, K_f = 3\}$. A calculation yields[8,17] $E_{rQ} = 0.172$ eV, $|V|_Q^2 = 1.55\times10^{15}$ eV·cm^3/s, $W(0) = 5/9$, and $\sigma_{rQ} = 0.74\times10^{-20}$ eV·cm$^2/\Gamma_{rQ}$. With $\Gamma_{rQ} \simeq 2\times10^{-3}$ eV, the cross section at the resonance peak, $\sigma_{rQ} \simeq 37\times10^{-19}$ cm^2, is considerably larger than the scattering cross section $\sigma_S(E_{rQ}) = 0.9\times10^{-19}$ cm^2 (Ref. 18) for an equilibrium mixture with $c_p = c_d = c_{HD} = 0.5$ ($c_t \ll 1$). The resonance capture due to the "swallowing" effect decreases by a factor of $\sqrt{1 + c_{HD}\sigma_{rQ}/\sigma_s} = 4.6$. The width of the zone in which the MMCs form is $\Delta E = 9.2$ meV, and only about 4% of the mesic atoms are captured at this resonance.

4. The production of mesic molecules accelerates if Dopper broadening of the levels is taken into account. Taking account of the relationship between the energy of the MA in the c.m. frame, E', and the projection of the molecule's velocity V_z in the linear approximation, $E' \simeq E - \sqrt{2\mu E}\cdot V_z$, and taking an average over a Maxwellian distribution of V_z, we find in place of (2) the results $z = 2(E' - E)/\Gamma_{rQ}$, $\Delta_D = \sqrt{4TE_{rQ}/A}$; $\zeta_0 = \Gamma_{rQ}/\Delta_D$):

$$\bar{\sigma}_{dt\mu-x}(z) = \sigma_{rQ}\psi(x,\zeta_0); \quad \psi(x,\xi_0) = \frac{\zeta_0}{2\sqrt{\pi}}\int_{-\infty}^{\infty} \frac{\exp[-\zeta_0^2(x-y)^2/4]}{1+y^2}dy. \qquad (10)$$

With $\zeta \ll 1$ and $x < 4/\zeta_0^2$, expression (2) is replaced by a purely Doppler distribution, and the argument of the exponential function, ω_i, in (8) is replaced near the resonance by

$$\psi_D(x,\xi) = \frac{\sqrt{\pi}}{2}\zeta_0 \exp\left[-\frac{\zeta_0^2 x^2}{4}\right]; \quad \omega_i = \frac{\Gamma_{Q_i}}{\xi E_{Q_i}} \int_0^\infty \frac{\psi_D(x,\zeta_0)dx}{[\psi_D(x,\zeta_0)+\beta_i]}, \tag{11}$$

where $\beta_i = \sigma_S / c_i \sigma_{rQ_i}$. With $\beta \ll 1$ and $\sqrt{\pi}\zeta_0/2\beta \gg 1$ we have

$$\omega_i \simeq \frac{2\Delta_D}{\xi E_{rQ}} \sqrt{\ln\left[\frac{\sqrt{\pi}\zeta_0}{2\beta}\right]}. \tag{12}$$

Under the opposite inequality,[20] $\sqrt{\pi}\zeta_0/2\beta_i < 1$, we have

$$\omega_i = \frac{\pi}{2}\frac{\Gamma_{Q_i}}{\xi E_{Q_i}} \frac{c_i \sigma_{rQ_i}}{\sigma_S} \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}} \left(\frac{\sqrt{\pi}\zeta_0}{2\beta}\right)^n. \tag{13}$$

It can be seen from (13) that at small values of β the value of ω_i approaches resonance absorption at infinite dilution. For the resonance discussed above at $T = 25$ meV ($\Delta = 0.13$ eV, $\sqrt{\pi}\zeta_0/2\beta = 0.26$), we find $\omega_i = 0.15$ from (13). This result is considerably larger than the earlier result. Since the production of mesic molecules in the epithermal region also occurs during other transitions $K = 0 \rightarrow K_f$, the total contribution of epithermal production of mesic molecules is significant.

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