Thermodynamic description of a Josephson medium in a strong magnetic field

V. S. Gorbachev and A. V. Senatorov
Moscow Engineering-Physics Institute

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A method is proposed for generating a thermodynamic description of a ceramic high-T_c superconductor in a strong magnetic field for the case in which the length scale of the field variations is smaller than the ceramic grains. The magnetic field dependence of the transition temperature T_{cJ} is calculated in the mean-field approximation.

The thermodynamic properties of the ceramic high-T_c superconductors (and of granular superconductors in general) are usually studied in the XY model (Refs. 1–5, for example). That model is legitimate if definite values of the phase of the order parameter and of the vector $\vec{A}$ can be assigned to each grain. For coarse-grain ceramics (i.e., under the condition, $\lambda \ll L$, where λ is the London depth of a grain, and L is the size of the grain), the XY model is applicable only in weak fields,[6] $H \ll H_J = \phi_0/\lambda L$. In stronger fields, the length scale of the field variations satisfies $\phi_0/H\lambda < I$. Certain properties of a Josephson medium in this case (for $T=0$) were studied in Ref. 6. In the present letter we consider a Josephson medium at $T>0$ in fields up to the first critical field of the grain material, H^g_{c1}. We wish to stress that the relation $H^g_{c1} > H_J$ holds in the case $\lambda \ll L$. We propose a generalization of the method developed in Ref. 7, which is valid in weak fields.

We consider a 2D Josephson lattice with an edge length L. The energy of the system is

$$\epsilon = \sum_{<x,x'>} (\epsilon^J_{x,x'} + \epsilon^M_{x,x'}), \tag{1}$$

$$\epsilon^J_{x,x'} = -E_J \int_0^L \frac{du}{L} \cos\left(\chi_x - \chi_{x'} + \frac{2\pi d}{\phi_0} A_{xx'}(u)\right), \tag{2}$$

$$\epsilon^M_{x,x'} = \frac{dL^2}{8\pi} \int_0^L \frac{du}{L} \left(\frac{dA_{xx'}}{du}\right)^2. \tag{3}$$

The summation is over all the Josephson junctions; $E_J = j_c \hbar L^2/2e$ is the energy of a Josephson junction; χ_x and $\chi_{x'}$ are the phases of the wave functions in the grains; j_c is the critical current density of a Josephson junction; $d \approx 2\lambda$ is the width of a Josephson junction; and u is the coordinate along the junction.

The first term in (1) can be written

$$\epsilon^J_{x,x'} = -\frac{1}{2}E_J(\varphi_x\varphi^*_{x'}f_{xx'} + \varphi^*_x\varphi_{x'}f^*_{xx'}), \tag{4}$$

where

$$\varphi_x = \exp(i\chi_x); \quad f_{xx'} = \int_0^L \frac{du}{L}\exp(ia_{xx'}(u)); \quad a_{xx'} = \frac{2\pi d}{\phi_0}A_{xx'}. \tag{5}$$

It follows from (4) that the energy functional can be written in the symbolic form

$$H[\varphi] = -\frac{1}{2}\varphi^*\hat{\vartheta}\varphi. \tag{6}$$

Here and below, we are omitting a term with the magnetic energy; we will restore it in the final expression. Using the Hubbard–Stratonovich identity (Refs. 3, 4, and 7, for example), we find the effective Hamiltonian

$$H_{eff} = \frac{T^2}{2}\psi^*\hat{\vartheta}\psi - T\ln I_0(|\psi|), \tag{7}$$

where $I_0(s)$ is the modified Bessel function. A variation of (7) with respect to ψ^* yields the mean-field equation

$$T\hat{\vartheta}^{-1}\psi = \frac{I_1(|\psi|)}{I_0(|\psi|)}\frac{\psi}{|\psi|}. \tag{8}$$

In weak magnetic fields and in the continuous limit, the operator $\hat{\vartheta}$ has a simple form, and the inverse operator $\hat{\vartheta}^{-1}$ can be found easily.[7] This is not true in strong fields. However, an expression for the effective Hamiltonian which contains only the original operator $\hat{\vartheta}$ can be derived in the mean-field approximation. We introduce the new variable

$$\gamma[\varphi] = \frac{I_1(|\psi|)}{I_0(|\psi|)}\frac{\psi}{|\psi|}. \tag{9}$$

It is easy to see that a variation of the effective Hamiltonian

$$H_{eff} = -\frac{1}{2}\gamma^*\hat{\vartheta}\gamma + \frac{T}{2}(\psi\gamma^* + \gamma\psi^*) - T\ln I_0(|\psi|) \tag{10}$$

with respect to γ^* under condition (9) leads to Eq. (8). The meaning here is that Hamiltonians (7) and (10) lead to the identical mean-field approximation. In this approximation, γ is a global order parameter,[7] $\gamma = \langle\exp(i\chi)\rangle$, where $\langle...\rangle$ means a thermodynamic average. We wish to stress that the first term in (10) is the same as the original expression, (6), if we replace φ in it by γ. If $|\varphi| = 1$, however, we have $|\gamma(T)| \leqslant 1$.

It is now a simple matter to write an expression for the Hamiltonian of a Josephson lattice in a strong field (the magnetic energy is taken into account here):

$$H_{eff}[\gamma] = \sum_{<x,x'>} \left\{ -E_J \rho_x \rho_{x'} \int_0^L \frac{du}{L} \cos(\alpha_x - \alpha_{x'} + a_{xx'}(u)) + \frac{\phi_o^2 L^2}{32\pi^3 d} \int_0^L \frac{du}{L} \left(\frac{da_{xx'}}{du} \right)^2 \right\}. \tag{11}$$

Here $\gamma = \rho \exp(i\alpha)$.

A variation of (11) with respect to a leads to a Ferrel–Prange equation which is valid for an arbitrary length of the edge of the lattice:

$$\frac{\rho_x \rho_{x'}}{\delta^2} \sin(a_{xx'} + \alpha_x - \alpha_{x'}) = \frac{d^2 a_{xx'}}{du^2}, \tag{12}$$

where $\delta^2 = c\phi_0/8\pi^2 j_c d$ is the Josephson depth.

From this point on we assume that the magnetic field is strong and that the field at each junction can be assumed uniform (this assumption is also valid in a field of arbitrary strength if the junctions are small). In this case we find

$$a_{xx'}(u) = \frac{2\pi d}{\phi_0} H u + c_{xx'}, \tag{13}$$

where $c_{xx'}$ is a constant. We also assume that the modulus of the global order parameter is the same in all grains: $\rho_x = \rho_{x'} \equiv \rho$. We find

$$H_{eff} = -2E_J \rho^2 \frac{\sin(\pi\phi/\phi_0)}{\pi\phi/\phi_0} \sum_{<x,x'>} \cos\left(\frac{\pi\phi}{\phi_0} + \frac{c_{xx'}}{2} + \frac{\alpha_x - \alpha_{x'}}{2} \right) + T\rho|\psi| - T\ln(I_0(|\psi|)) + \frac{L^2 d}{8\pi} H^2, \tag{14}$$

$$\rho = \frac{I_1(|\psi|)}{I_0(|\psi|)}, \tag{15}$$

where $\phi = dLH$ is the magnetic flux in one junction. We must now select a distribution of the phases α_x which minimizes expression (14). Because of frustration,[1–3] it is generally not possible to choose phases α_x such that we have

$$|\cos(\pi\phi/\phi_0 + c_{xx'}/2 + (\alpha_x - \alpha_{x'})/2)| = 1.$$

Some complicated numerical calculations are required for the minimization here. However, by virtue of the simplifying assumptions which we have made, we can use the results of Refs. 1 and 2 to calculate the transition temperature. In Refs. 1 and 2, the energy of a Josephson lattice was minimized at $T = 0$ (and under the condition $\lambda \gg L$) [see Fig. 2 in Ref. 1, which shows the field dependence of twice the average value $\cos(\pi\phi/\phi_0 + c_{xx'}/2 + (\alpha_x - \alpha_{x'})/2)$, which we denote by $\overline{\cos}(H)$].

We thus have an expression for the Hamiltonian of the system in the ground state:

$$H_{eff} = -2E_J\rho^2 \frac{|\sin(\pi\phi/\phi_0)|}{\pi\phi/\phi_0}\overline{\cos}(H) + T\rho|\psi| - T\ln I_0(|\psi|) + \frac{L^2 d}{8\pi}H^2. \qquad (16)$$

A variation of (16) with respect to ρ yields

$$4E_J(T)\frac{I_1(|\psi|)}{I_0(|\psi|)}\frac{|\sin(\pi\phi/\phi_0)|}{\pi\phi/\phi_0}\overline{\cos}(H) = T|\psi|. \qquad (17)$$

Equation (17) has a nontrivial solution only at $T < T_{cJ}(H)$. The transition temperature for the onset of a global coherent state, $T_{cJ}(H)$, is found from

$$2E_J(T)\frac{|\sin(\pi\phi/\phi_0)|}{\pi\phi/\phi_0}\overline{\cos}(H) = T. \qquad (18)$$

At $H = 0$ we have $\overline{\cos}(0) = 1$, and Eq. (18) becomes the customary definition of the transition temperature,[7] $2E_J(T) = T$. At $H > 0$ we have

$$\frac{|\sin(\pi\phi/\phi_0)|}{\pi\phi/\phi_0}\overline{\cos}(H) < 1, \qquad (19)$$

so $T_{cJ}(H)$ is lower than $T_{cJ}(0)$. The factor $|\sin(\pi\phi/\phi_0)|\,\overline{\cos}(H)$ in (18) is a highly complex-valued function of the magnetic field, but this field dependence should not be seen in real high-T_c ceramics, because of the averaging over grain size. We wish to stress that the lowering of the transition temperature with increasing field should occur even in this case, because of the factor $1/\phi$ in (18).

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