Nonlinear vortex excitations (solitons) in a 2D magnetic material of the YBaCuO type

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Stable topological solitons with a finite energy are predicted in a 2D antiferromagnet such as the cuprate layer in YBaCuO.

The special role played by soliton excitations in the physics of reduced-dimensionality magnetic materials is well recognized. Solitons of the vortex type are pertinent to two-dimensional (2D) magnetic materials. These would be itinerant vortices in the case of a continuous degeneracy of the ground state and localized vortices in the case of a discrete degeneracy. The structure of these vortices is described in Ref. 1.

In easy-plane magnetic materials, itinerant vortices give rise to a (Berezinskiĭ–) Kosterlitz–Thouless transition[2,3] and are responsible for a central peak in the correlation functions (Refs. 4 and 5, for example). A central-peak component is also possible for trapped magnetic vortices,[6] which can exist in easy-axis and orthorhombic magnetic materials, but in this case there is the problem of the stability of the vortex with respect to collapse.

In this letter we show that in the cuprate layer of YBaCuO, thought of as a classical 2D antiferromagnet, there exist trapped magnetic vortices which are stable with respect to collapse and which have a finite energy and radius. We discuss their contribution to the response function.

We start from the energy of an antiferromagnet, written as a functional of the unit antiferromagnetism vector[7] $\vec{l}$:

$$W = aM_0^2 \int dxdy\{\alpha/2(\nabla\vec{l})^2 + 1/2(\beta_1 l_x^2 + \beta_2 l_z^2) + \gamma_{i,kj} l_i(\partial l_k/\partial x_j)\}. \tag{1}$$

Here a is a quantity on the order of the lattice constant, added to maintain the dimensionality. The first two terms have the usual meaning: the inhomogeneous-exchange energy and the anisotropy energy. The constants β_1 and β_2 satisfy $\beta_2 > \beta_1 > 0$; y is the easy axis and lies in the xy plane of the antiferromagnet. The last term, linear in $(\partial\vec{l}/\partial x_i)$, is specific to YBaCuO; it does not exist in the case of La_2CuO_4, for example. It exists in the case at hand because there is no inversion center. The tensor γ has the nonzero components[7]

$$\gamma_{x,zx} = -\gamma_{z,xx} = \gamma', \qquad \gamma_{y,zy} = -\gamma_{z,yy} = \gamma .$$

The corresponding constants γ and γ' are of an exchange-relativistic nature. We would thus expect $\gamma, \gamma' \gg \beta a$.

Because of the anisotropy in the xy basal plane, it is not possible to find an exact vortex solution. However, for the case of a soliton of small radius $R \ll (\alpha/\beta)^{1/2}$, which is the case of most interest (more on this below), the term $\alpha(\nabla \vec{l})^2$ makes the largest contribution to the energy of the localized magnetic vortices. In this case, we can use the Belavin–Polyakov solution as a leading approximation.[8] This solution corresponds to $\varphi = \nu\chi + \varphi_0$ and $\tan(\theta/2) = (R/r)^{|\nu|}$, where $\nu = \pm 1, \pm 2, \ldots$ is the topological charge (an invariant of the mapping of the plane of the antiferromagnet onto the sphere $\vec{l}^2 = 1$); θ, φ are angular variables for the magnetization; $l_x + il_z = \sin\theta \exp(i\varphi)$; and τ, χ are polar coordinates. The parameters R and φ_0 are adjustable for the exchange approximation. When the anisotropy and the inhomogeneous term with γ_i, k_j are taken into account, the energy of the antiferromagnet in (1), calculated from the Belavin–Polyakov solution, depends on R and φ_0. The existence of a minimum in terms of these parameters indicates that a stable vortex exists. For an anisotropic antiferromagnet and for $\nu = \pm 1$, it is a better approximation to use the substitution

$$\tan(\theta/2) = (R/l_0)K_1(r/l_0), \tag{2}$$

where $K_1(x)$ is the modified Bessel function, and $l_0 = (\alpha/\beta)^{1/2}$. The reason is that, for $r \ll R \ll l_0$, expression (2) converts into the Belavin–Polyakov solution, while with $\tau \gg l_0$ it gives the correct exponential asymptotic behavior:[9,10] $\theta \sim \exp(-r/l_0)$.

A calculation of the energy shows that the terms with $\gamma_{i,kl}$ are zero except for $|\nu| = 1$. The value φ_0 corresponds to the minimum energy. For the energy $E = E(R)$ in the case $|\nu| = 1$ we find

$$E = 4\pi a M_0^2\{\alpha - 0.5\gamma R + \beta R^2 \ln(2.42 l_0/R)\}, \tag{3}$$

where the first term is the usual energy of the Belavin–Polyakov solution, and the second and third terms are determined by the inhomogeneous exchange-relativistic interaction and the anisotropy, respectively. Here $\beta = (\beta_1 + \beta_2)/2$. The sign of the second term in (3) is determined by the product of the constans γ and ν; it can always be made negative by choosing $\nu = \pm 1$. If we instead have $|\nu| \neq 1$, then the second term is absent, and we have $E = 4\pi\alpha M_0^2 |\nu| a + \beta A R^2$ and $A = \text{const} \sim 1$.

Clearly, with $|\nu| \neq 1$ a minimum is reached only in the case $R = 0$; i.e., the soliton collapses. If we instead have $|\nu| = 1$, $\gamma\nu < 0$, the energy in (3) has a minimum at a finite $R = R_0$, and the radius of the soliton is, at logarithmic accuracy,

$$R_0 = (\gamma/4\beta)/\ln[4\beta l_0/\gamma]. \tag{4}$$

Since we have $\gamma \gg \beta a$, the value of R_0 is large in comparison with the lattice constant; i.e., a macroscopic description of the soliton is adequate. On the other hand, even at the maximum possible $\gamma \leqslant (\beta\delta)^{1/2} a$, where δ is the exchange constant, the relation $R_0 \ll l_0$ holds. Consequently, as was assumed in the analysis, the soliton is a small-radius localized magnetic vortex, with $a \ll R_0$, l_0, and its energy is close to the Belavin–Polyakov value:

$$E = 4\pi a \alpha M_0^2 (1 - (\gamma^2/16\beta\alpha)(\ln(2l_0/R_0))^{-1}) \simeq 4\pi a \alpha M_0^2. \tag{5}$$

The mass of the vortex is $m = E/c^2$, where c is the magnon velocity in the xy plane.

Since the energy of a localized magnetic vortex is finite, there is a finite density of such vortices in a 2D antiferromagnet at any temperature $T \neq 0$. In a real quasi-2D antiferromagnet, the same is true at $T > T_{3D}$, where $T_{3D} \simeq 200$ K is the temperature of the three-dimensional ordering. In this case the localized magnetic vortex gives rise to a central peak in the dynamic structure factor for neutron scattering, $S(q,\omega)$, or in the imaginary part of the magnetic susceptibility, $\chi''(q,w)$. The calculation is carried out in the same way as for easy-axis magnetic materials.[6] The shape of the peak is determined by the mean free path l_r, where $l_r = mV_T/\eta$, and η is the viscosity of the vortex. At $q > 1/l_r$, the peak has a Gaussian shape and a width $\Gamma_\omega \sim qV_T$, where V_T is the thermal velocity of a vortex. If we instead have $q < 1/l_r$, the peak is Lorentzian, with a width $\Gamma_\omega = Dq^2$, where $D = T/\eta$ is the diffusion coefficient for a localized magnetic vortex.

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[1]A. M. Kosevich, B. A. Ivanov, and A. S. Kovalev, *Nonlinear Magnetization Waves: Dynamic and Topological Solitons*, Nauk. Dumka, Kiev, 1983.
[2]V. L. Berezinskiĭ, Zh. Eksp. Toer. Fiz. **61**, 1144 (1971) [Sov. Phys. JETP **61**, 610 (1971)].
[3]J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[4]M. E. Gouvea, G. M. Wisin, A. R. Bishop, and F. G. Mertens, Phys. Rev. B **39**, 11840 (1989).
[5]V. L. Pokrovskiĭ and D. V. Khveshchenko, Fiz. Nizk. Temp. **14**, 385 (1988) [Sov. J. Low Temp. Phys. **14**, 213 (1988)].
[6]B. A. Ivanov and A. K. Kolezhuk, Fiz. Nizk. Temp. **16**, 335 (1990) [Sov. J. Low Temp. Phys. **16**, 184 (1990)].
[7]V. G. Bar'yakhtar, V. M. Loktev, and D. A. Yablonskiĭ, Physica C (1988).
[8]A. A. Belavin and A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 503 (1975) [JETP Lett. **22**, 245 (1975)].
[9]V. P. Voronov, B. A. Ivanov, and A. M. Kosevich, Zh. Eksp. Teor. Fiz. **84**, 2235 (1983) [Sov. Phys. JETP **84**, 1303 (1983)].
[10]B. A. Ivanov, V. A. Stephanovich, and A. A. Zhmudskii, J. Magn. Magn. Mater. **88**, 116 (1990).

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