Calculating deconfinement temperature through the scale anomaly in gluodynamics

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The deconfining temperature T_c is estimated from the minimum of the free energy of the vacuum. T_c is expressed via scale anomaly through the gluonic condensate. It is found to be about 200 MeV and does not depend on N_c as $N_c \to \infty$.

1. It is known[1] that the nonperturbative vacuum energy density ϵ is nonzero due to the scale anomaly[2]; specifically,

$$\varepsilon = \frac{\beta(\alpha_s)}{16\alpha_s} < G^a_{\mu\nu} G^a_{\mu\nu} > \cong -\frac{11}{3} N_c \frac{\alpha_s}{32\pi} < G^2 > . \quad (1)$$

One can associate with ε the zero-temperature limit of the free energy $F = E - TS$, and represent F in the form

$$F = \varepsilon V_3 + f(T), \quad (2)$$

where $f(T)$ is obtained in the usual way via statistical sum.[3] Here we calculate $f(T)$, representing gluonic field A_μ as a background field B_μ plus perturbative gluonic part a_μ. To the lowest order in a_μ, the statistical sum Z_0 can be written as follows:

$$Z_0 = < \exp\left\{ \int_0^\infty \xi(t) \frac{dt}{t} \operatorname{Tr}\left(\frac{1}{2} e^{-tW} - e^{tD^2(B)} \right) \right\} >_B = e^{-f_0(T)/T}, \quad (3)$$

where $\xi(t)$ is the regularizing factor; i.e., in the ζ-regularization it is $\xi(t) = (d/ds)[(M^2 t)^s / \Gamma(s)]$, with $s \to 0$, and M is the regulator mass. In (3) W and $D^2(B)$ are the gluon and ghost propagators in the background B_μ, and Tr is the trace in the Lorentz, coordinate and color spaces.[4] Finally, $\langle\rangle_B$ means averaging over fields B_μ.

In the limit $B \to 0$, we obtain for $f_0(T)$ the usual free-gluon-gas expression

$$f_0(T) = T(N_c^2 - 1) \int \sum_n \ln \frac{M^2}{k_0^2 + k^2} V_3 \frac{d^3k}{(2\pi)^3} = f_0(T = 0) - \frac{(N_c^2 - 1)}{45} \pi^2 V_3 T^4. \quad (4)$$

Here in $f_0(T = 0)$ we have included all infinite constant terms, which we disregard, since we normalize the free energy at $T = 0$ by the term εV_3 [see Eq. (2)]. We show below that for the confining background B_μ the free energy $f(T)$ contains contributions of two-gluon glueballs, three gluon glueballs, etc. and their interacting ensembles. From the physical point of view, $f_0(T)$ contains the most important contributions both in the confining and in the deconfined phases. In the first case, they are glueballs, and in the second case, they are the free gluon gas, which is the dominant

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contribution to the free energy immediately after the phase-transition region.[3]

In what follows we identify the main term in $f_0(T)$ and, after imposing the properties of continuity and minimality on F in (2), we calculate the deconfining temperature.

2. For the averaging procedure in (3) we can use the cluster expansion[5,6] and write

$$Z_0 = < \exp\varphi >_B = \exp\left\{ < \varphi >_B + \frac{1}{2!}[< \varphi^2 >_B - (< \varphi >_B)^2] + \dots \right\}, \tag{5}$$

where

$$\varphi \equiv \int_0^\infty \xi(t)\frac{dt}{t}\mathrm{Tr}e^{tD^2(B)} = tr\int_0^\infty \xi(t)\frac{dt}{t}Dz\frac{d^4xT}{V_3}\exp(-K)\Phi(x,x)\,. \tag{6}$$

In (6) we disregard the spin-interaction term [the difference between $-W$ and $D^2(B)$ in (3)] and use the Feynman–Schwinger (FS) representation,[7] where

$$K = \frac{1}{4}\int_0^t \dot{z}^2 d\tau,\ \Phi(x,y) = P\exp ig\int_y^x B_\mu dz_\mu\ .$$

Note that the path integral in (6) is over the trajectories $z(\tau)$ which return to the same point x. When $B^\mu = 0$, $\Phi(x,x) = 1$, and $\varphi = -f_0/T$ corresponds to the closed trajectories of a free gluon at a given temperature T, yielding (4).

In the confined phase $\langle\varphi\rangle_B \sim \langle\Phi(x,x)\rangle_B$, i.e., a Wilson loop which obeys the area law:[6]

$$< \Phi(x,x) >_B = \exp(-\sigma_a S(x,x)),$$

where $S(x,x)$ is the minimal area inside the trajectory $z(\tau)$, and $\sigma_a = \frac{9}{4}\sigma$, σ is the string tension. The area law confines trajectory to the size $R \sim \sigma^{-1/2}$. One can identically rewrite the integral (6) as that corresponding to two gluons which propagate from the point x to the point y, and which interact via confining area law term

$$-\frac{f_0^{(1)}}{T} = < \varphi_B > = \int_0^\infty \xi(t)\frac{dt}{t}\int_0^t \frac{dt_1}{t}d^4y\frac{d^4x}{V_4}[Dz(\tau)Dz(\tau')\exp(-K - K' - \sigma_a S)]_n \tag{7}$$

with $K' = \frac{1}{4}\int_0^{t-t_1}\dot{z}^2(\tau')d\tau'$, and the symbol $[\,]_n$ means that the Matsubara series for the center-of-mass time is used.

Thus $\langle\varphi B\rangle$ is a two-gluon bound-state Green's function, whose zero Matsubara frequency is kept due to the integration in $d(x_4 - y_4)$. It gives no contribution to the free enegy when the relative motion of gluons is confined. When, on the other hand, $\sigma\to 0$, and the relative size of the bound two-gluon system compares with $V_3^{1/3}$, one finally recovers the limit of free gluons (4). One can visualize this picture as ensemble of pairs of gluons frozen by confinement, forming condensate for $T < T_c$, which "evaporates" at $T > T_c$ and become a free gluon gas.

3. The free glueball gas contribution is associated with the second term in (5), $\langle\varphi^2\rangle_B$, where φ is given by (6).

The confining dynamics in $\langle\varphi^2\rangle_B$ is given by the term

$$< \mathrm{tr}\Phi(x,x)\mathrm{tr}\Phi(y,y) > \sim \exp(-\sigma_a S(z(\tau), z'(\tau'))) , \tag{8}$$

where we have defined the minimal area surface S which joins the trajectories of the two gluons $z(\tau)$ and $z'(\tau')$.

We thus obtain

$$< \varphi^2 >_B = \int_0^\infty \xi(t)\frac{dt}{t}Dz\xi(u)\frac{du}{u}Dz'\frac{d^4x}{V_4}\frac{d^4y}{V_4}\exp(-K - K' - \sigma_a S) . \tag{9}$$

Introducing the full set of bound states of the two gluons (glueballs) $\langle xy|n\rangle$, and integrating over the relative coordinates, we finally find the glueball contribution to f_0

$$f_0^{(2)} = -\frac{T}{2!} < \varphi^2 >_B = -TV_3\int\frac{d^3p}{(2\pi)^3}e^{-\beta\sqrt{p^2+M_0^2}} \approx -\frac{V_3 M_0^{3/2}T^{5/2}}{(2\pi)^{3/2}}e^{-M_0/T} . \tag{10}$$

Here we kept only the lowest glueball mass M_0 and considered the case $T \ll M_0$.

In the limit $B_\mu \to 0$, the whole term $\langle\varphi^2\rangle_B - (\langle\varphi\rangle_B)^2$ tends to zero, since it is a connected contribution. In the deconfining transition when $\sigma \to 0$, $M_0 \to 0$, and $f_0^{(2)}$ vanishes.

4. Combining (2), (4), and (10), we obtain the relation

$$\frac{F}{V_3} \cong -\frac{11}{3}N_c\frac{\alpha_s < G^2 >}{32\pi} - \frac{(N_c^2-1)\pi^2}{45}T^4\tilde{\Theta}(T - T_c) - \frac{M_0^{3/2}T^{5/2}}{(2\pi)^{3/2}}e^{-M_0/T} . \tag{11}$$

Here we have introduced the transition function $\tilde{\Theta}(T - T_c)$, which is very close to the step function and which corresponds to Eq. (7). The steepness of this function is due to the fact that it is zero for $\sigma > V_3^{-2/3}$ and only for σ as small as $V_3^{-2/3}$, $f_0^{(1)}$ grows to a value given in (4). Since we know that the free energy F should be a continuous function[8] of σ near $\sigma = 0$ (or of T near T_c), the function $\tilde{\Theta}(T - T_c)$, which is nearly a step function, should be canceled by other terms in (11).

We now see that the first term on the r.h.s. in (11) indeed jumps at $\sigma = 0$ (at $T = T_c$) in the opposite direction to that of $f_0^{(1)}$. To this end, we decompose the nonlocal vacuum correlator, as in Refs. 6 and 9:

$$< \mathrm{tr}E_i(x)E_j(y) > = D^E(x-y)\delta_{ij} + 0(\partial D_1^E) , \tag{12}$$

$$< \mathrm{tr}B_i(x)B_j(y) > = D^B(x-y)\delta_{ij} + 0(\partial D_1^B) , \tag{13}$$

where $0(\partial D_1)$ means terms proportional to derivatives of another independent function D_1. At $T = 0$, we have $D^E = D^B$, $D_1^E = D_1^B$, which is not true for $T > 0$.

Out of for independent nonperturbative correlators D^E, D^B, D_1^E, and D_1^B, only the first one is responsible for the confinement,[6,7] providing nonzero string tension

$$\sigma = \frac{1}{2}\int_{-\infty}^{\infty} D^E(\sqrt{x^2+t^2})dx\,dt + \dots , \tag{14}$$

where the dots stand for the contribution of higher-order cumulants.

On the other hand, all four correlators enter as a sum in the vacuum condensate in (1):

$$< G^2 >= D^E(0) + D_1^E(0) + D^B(0) + D_1^B(0) . \tag{15}$$

At the transition point, $T = T_c$, D^E vanishes according to (14), causing a jump in $\langle G^2\rangle$ due to (15). This jump should be matched (and canceled) in the total expression for the free energy (11) by the second term, $f_0^{(1)}$, thus making F continuous.

In addition to this continuity argument, one can also use the principle of minimality of F[8] to argue that three other quantities $D_1^E(0)$, $D^B(0)$, and $D_1^B(0)$ should stay unchanged at $T > 0$.[9] This statement has been confirmed in Monte Carlo computations in two ways. It was shown that only a part of the condensate $\langle G^2\rangle$ has a jump across $T = T_c$, but the rest stays nonzero at $T > T_c$[11]. In spacelike Wilson loops[12] the string tension σ^B was measured for $T > T_c$ [connected to D^B as in (14)] and found nonzero and close to σ at $T = 0$.

Therefore, we can define $D^E(0) = \eta\langle G^2\rangle[1 - \tilde{\Theta}(T - T_c)$, where η is weakly dependent on the temperature and

i) $\eta(T=0) = 1/2$ if $D_1^E(0) = D_1^B(0)$ is small as compared to $D^E(0)$.

ii) $\eta(T=0) = 1/4$ if $D_1^E(0) = D^E(0)$.

We note that η is a part of $\langle G^2\rangle$, which disappears during the deconfinement transition.

The correlator D_1 defines the nonperturbative tensor force in heavy quarkonia,[13] and computations[14] show that $D_1(0)$ is at least of the same order as $D(0)$. Moreover, recent lattice calculations[15] of $D(0)$ and $D_1(0)$ support this conclusion. We can now put the continuity condition near $T = T_c$ for the free energy F in (11) in the form

$$\frac{11}{3}N_c\frac{\alpha_s\eta < G^2 >}{32\pi} + \frac{M_0^{3/2}T_c^{5/2}}{(2\pi)^{3/2}}e^{-M_0/T_c} = \frac{(N_c^2-1)\pi^2}{45}T_c^4 . \tag{16}$$

Solving for T_c, we obtain

$$T_c = \left(\frac{165N_c\eta\alpha_s < G^2 >}{32\pi^3(N_c^2-1)}\right)^{1/4}\left(1 + \frac{C(M_0)}{N_c^2}\right) , \tag{17}$$

where $C(M_0)$ is taken from the second term on the l.h.s. of (16),

$C(M_0 = 1\,\text{GeV}) \cong 7.65\times10^{-4}$.

5. We see in (17) that T_c is $0(1)$ at $N_c\to\infty$, since $\alpha_s\langle G^2\rangle$ is $0(N_c)$.[16] For $N_c = 3$ we obtain

$$T_c = 220\ \text{MeV}(\eta \cdot \lambda)^{1/4}, \tag{18}$$

where λ measures the gluonic condensate in units of the standard value,[1] $\alpha_s/\pi\langle G^2\rangle_{st} = 0.012\ \text{GeV}^4, \lambda = \langle G^2\rangle/\langle G^2\rangle_{st}$. For the options $\lambda = 1, \eta = \frac{1}{2} - \frac{1}{4}$ we obtain

$$T_c = 185 \div 156\ \text{MeV}. \tag{19}$$

This value should be compared with the lattice value for gluodynamics, $T_c = 197$–254 MeV (Ref. 17) and implies that $\lambda \sim 2$–4.

The fact that T_c is $O(1)$ for large N_c is crucial for our mechanism of deconfinement. Note that the small glueball term $C(M_0)$ is suppressed at large N_c, which means that the Hagedorn-type mechanism[18] is not operative in our case.

In summary, we have suggested a mechanism of deconfinement, in which a part of gluonic condensate evaporates into gluons in the deconfining transition. Requirements of minimality and continuity of the free energy make it possible to estimate the deconfinement temperature within the uncertainty region (19), which is reasonably close to the lattice results.

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