Coherent bremsstrahlung in colliding beams

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Radiation of a new type, which arises in colliding beams with short bunches—coherent bremsstrahlung—is analyzed. This radiation can be summarized as the radiation by particles of one bunch in the collective electromagnetic field of a second. The number of photons emitted in coherent bremsstrahlung, dN_γ, is proportional to the product $N_1N_2^2$, where N_j is the number of particles in the jth bunch. For ordinary bremsstrahlung, in contrast, this number is proportional to simply N_1N_2. Since N_2 is greater than 10^{10} for a typical collider, a substantial intensification of the radiation is expected at energies $E_\gamma < 4\gamma_1^2\hbar c/l$, where l is the length of the second bunch (the target), and $\gamma_1 = E_1/m_1c^2$. The unusual properties of this radiation might be utilized for a quick check of the collisions of bunches and for measuring the properties of beams.

1. The bunches of charged particles in advanced colliders are dense. As an electron (or proton) passes through such a bunch, it is deflected through an angle θ_d, and it emits radiation.[1)] If the energy of the photon emitted is sufficiently low, the radiation which results from the interaction of the electron with the various protons undergoes a coherent intensification. The properties of such radiation vary substantially with the relation between the typical emission angle $\sim 1/\gamma_e$ and the angle θ_d.

In the case $\theta_d \gg 1/\gamma_e$ the radiation is similar to ordinary synchrotron radiation and is usually called "beamstrahlung." This case arises in linear e^+e^- colliders (Ref. 1, for example).

In the present letter we are interested instead in the case $\theta_d \ll 1/\gamma_e$ (the case of a short bunch). We call the radiation in this case "coherent bremsstrahlung." This case arises at, for example, the SSC, LHC, Sp$\bar{\text{p}}$S, TEVATRON, VÉPP-2M, VÉPP-4M, and VERS accelerators. This type of radiation at colliders has not previously been discussed. Coherent bremsstrahlung differs substantially from both ordinary (i.e., incoherent) bremsstrahlung and beamstrahlung. Let us examine these differences.

a. The number of photons in coherent bremsstrahlung satisfies $dN_\gamma \propto N_eN_p^2$ in contrast with the proportionality $dN_\gamma \propto N_eN_p$ in the case of ordinary bremsstrahlung. The reason is that, as the photon energy decreases, the coherence length scale $\sim \gamma_e^2\hbar c/E_\gamma$ becomes comparable to the length of the proton bunch, l. At $E_\gamma < E_c$, the radiation results from an interaction of the electron with the proton bunch as a whole, rather than an interaction with each of the protons individually. Under these conditions, the proton bunch is similar to a particle with a charge $N_p e$, so the emission probability is proportional to N_p^2.

b. The total number of coherent-bremsstrahlung photons is infinite (as it is for ordinary bremsstrahlung), in contrast with the case of beamstrahlung, for which the total number of photons emitted is finite (as in the case of synchrotron radiation).

In this letter we describe a calculation method, derive a general expression, and also write a convenient approximate expression. We report the results of calculations for several colliders.[2]

2. We begin by stating the conditions under which these calculations are applicable. The length of the proton bunch, l, must be short in comparison with $l_R \sim (\gamma_e m_e c^2/eB)/\gamma_e \sim l\sigma_x/r_e N_p$, which is the distance over which an electron would be deflected through an angle $\sim 1/\gamma_e$:

$$\theta_d \gamma_e \sim \frac{l}{l_R} \sim \frac{r_e N_p}{\sigma_x} \ll 1 \qquad \left(r_e = \frac{e^2}{m_e c^2}\right). \tag{1}$$

In this case the motion of the electron can be assumed to remain rectilinear over the course of the collision. This condition holds for all $\bar{p}p$ and pp colliders [r_e would of course have to be replaced by r_p in (1)] and also for several existing e^+e^- colliders.

We also assume $\chi = \gamma_e/B_s \ll 1$, where $B_s = 4.4\times 10^9$ T is a critical field. We can ignore the simultaneous interaction of the electron with several protons; i.e., we can use the QED Born approximation for the amplitude of the process. This condition is satisfied for most colliders.

3. We work from the QED formulas for the amplitude for the process. This amplitude is the sum of Born terms of the types in Fig. 1, a and b, which correspond to the interaction of one electron with the various protons. In the standard approach in which the colliding particles are described by plane waves, these terms are incoherent, so we would have $dN_\gamma \propto N_e N_p$. A point of importance to our problem is that the colliding particles are described not by plane waves but by wave packets which are localized within the bunches. If the momentum transfer is sufficiently small ($q_z \lesssim 1/l$), the states of the proton before and after the collision are not orthogonal. There is accordingly an interference of the diagrams in which the radiation by the electron results from an interaction with different protons (a coherence in terms of protons; $dN_\gamma \propto N_e N_p^2$). It is for this reason that uncoupled Feynman diagrams arise in our

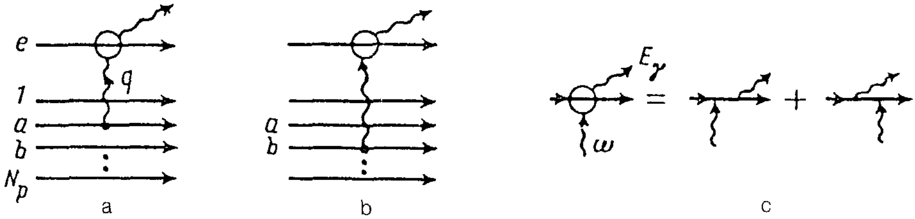

FIG. 1.

analysis. This mechanism comes into play for only those virtual photons whose wavelength is not shorter than the length of a bunch: $c/\omega \simeq 1/q_z > l$. In a collision with an oppositely directed electron, this virtual photon is scattered backward and acquires an energy greater by a factor of γ_e^2, i.e., $E_\gamma \sim \gamma_e^2 \hbar\omega < \gamma_e^2 \hbar c/l$.

The calculation technique which we are using here involves going over from wave packets to density matrices of the colliding beams or to quantum distribution functions $n(\vec{r},\vec{p},t)$. For colliders, it is a good approximation to treat the motion of the particles as classical. In this case, the quantum distribution functions become classical. Thus $n_p(\vec{r},\vec{p},t)$ is the number density of protons is phase space, and $dN_p(\vec{r},\vec{p},t) = n_p(\vec{r},\vec{p},t)d^3r d^3p$ is the number in the element $d^3r d^3p$, $dN_e(\vec{r}_e,\vec{p}_e,t)$—the same as for an electron. As a result, the number of coherent-bremsstrahlung photons emitted over a time dt is

$$dN_\gamma = \frac{M_{fi} M^*_{f'i'} d\Gamma_{\gamma e}}{4(E_e E_p E'_e E'_p)^{1/2}} dF \frac{d^3q d^3q'}{2[E_p(\vec{p}-\vec{q}/2)E_p(\vec{p}\,'-\vec{q}\,'/2)]^{1/2}(2\pi)^6}, \qquad (2)$$

$$dF = dN_e(\vec{r}_e,\vec{p}_e,t)dN_p(\vec{r},\vec{p},t)dN_p(\vec{r}\,',\vec{p}\,',t)\exp[i\vec{q}(\vec{r}_e-\vec{r}) - i\vec{q}\,'(\vec{r}_e-\vec{r}\,')]dt.$$

Here M_{fi} is the amplitude for the process $ep \to ep\gamma$ with momenta $\vec{p}_e + (\vec{q}\,'-\vec{q})/2$ and $\vec{p}+\vec{q}/2$ (and energies E_0 and E_p) for the initial electron and the initial proton. The amplitude $M_{f'i'}$ differs from M_{fi} by the substitutions $\vec{p} \to \vec{p}\,'$, $\vec{q} \leftrightarrow \vec{q}\,'$, $E_e \to E'_e$, and $E_p \to E'_p$. The quantity $d\Gamma_{\gamma e}$ is the phase volume of the final γ ray and the final electron. This formula incorporates the finite dimensions of both the electron bunch and the proton bunch. The concept of a cross section is inadequate for describing the results in this case, since the answer depends not on $|M_{fi}|^2$, as usual, but on the product of the amplitudes M_{fi} and $M^*_{f'i'}$, with different initial and final states.

4. Since the effect in which we are interested here is governed by small values of the momentum transfer, the expression for the matrix elements can be simplified substantially. When we also ignore the transverse motion of the particles in the bunches, we can derive convenient theoretical expressions for the number of coherent-bremsstrahlung photons which are emitted in one collision of bunches:

$$dN_\gamma = \frac{\alpha}{\pi}\frac{d\omega}{\omega} J(\omega) d\sigma_{\gamma e}(\omega), \qquad (3)$$

where $d\sigma_{\gamma e}(\omega)$ is the cross section for Compton scattering of an equivalent γ ray with an energy $\hbar\omega$ by the electron (Fig. 1c), and the function $J(\omega)$ is expressed in terms of the bunch form factors $F_j(\vec{q})$ [as usual, we have $F_e(0) = N_e$ and $F_p(0) = N_p$]:

$$J(\omega) = 4\pi \int \frac{\vec{q}_\perp \vec{q}\,'_\perp}{\vec{q}_\perp^{\,2} \vec{q}\,'^{2}_\perp} F_p(\vec{q}) F^*_p(\vec{q}\,') F_e(\vec{q}\,'-\vec{q}) \frac{d^2q_\perp d^2q'_\perp}{(2\pi)^4}; \quad q_z = q'_z = \omega/c. \qquad (4)$$

The energy of the equivalent photon is $\hbar\omega = E_\gamma[1+(\gamma_e\theta)^2]/(4\gamma_e^2)$, where E_γ is the energy of the final photon, and θ is the angle at which it is emitted (we have made use of the condition $E_\gamma \ll E_e$).

5. Let us consider the case of Gaussian beams in more detail. For this case we find the following spectrum of coherent-bremsstrahlung γ rays from (4):

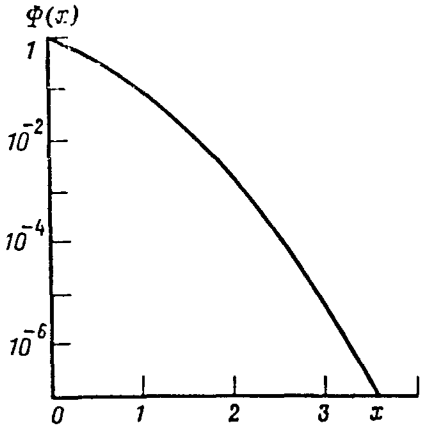

FIG. 2.

$$dN_\gamma = N_0\Phi(E_\gamma/E_c)dE_\gamma/E_\gamma, \quad \Phi(x) = \frac{3}{2}\int_0^\infty \frac{1+z^2}{(1+z)^4}\exp[-x^2(1+z)^2]dz,$$

$$N_0 = \frac{8}{3}\alpha r_e^2 J(0). \tag{5}$$

The function $\Phi(x)$, normalized by $\Phi(0) = 1$, is plotted in Fig. 2. The strong dependence of the photon energy distribution on the length of the oppositely directed bunch, l, means that we could in principle determine this length from the photon spectrum.

Table I lists values of E_c and N_0 for several colliders; in the case of the HERA, we considered the version in which protons are radiating.

6. If the axis of the electron beam is shifted a vertical distance R_y away from the axis of the proton beam, the luminosity of the *ep* collisions, $L(R_y)$, falls off exponentially rapidly (as does the number of events of ordinary reactions). The number of coherent-bremsstrahlung γ rays, in contrast, at first increases, and then it begins to fall off, but only very slowly, $dN_\gamma(R_y) \propto 1/R_y^2$. As an example, we show corresponding curves for the VÉPP-4M accelerator in Fig. 3.

TABLE I.

	SSC	LHC	HERA $<p>$	VÉPP-2M	VÉPP-4M	BEPC
E_1, GeV	20000	8000	820	0.5	6	2.8
E_c, eV	6000	770	70	20	2000	500
N_0	50	$2\cdot10^4$	14	$1.6\cdot10^5$	10^8	$5\cdot10^7$

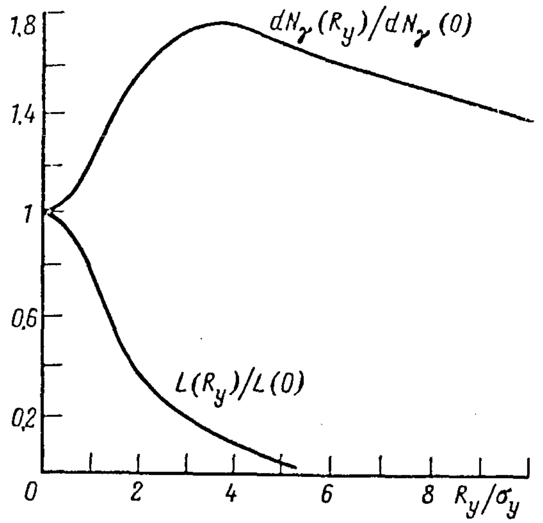

FIG. 3. Upper curve—The ratio of the number of coherent-bremsstrahlung photons in the case in which there is a relative vertical displacement R_p of the axes of the beams, $dN_\gamma(R_y)$ (on the one hand), to the number of photons in the case $R_y = 0$, $dN_\gamma(0)$ (on the other), as a function of R_y/σ_y, where σ_y is the vertical size of a bunch; lower case—ratio of luminosities, $L(R_y)/L(0)$ (for the VÉPP-4M accelerator).

When there is a relative displacement of the beam axes, there is also an appreciable azimuthal asymmetry in the coherent bremsstrahlung.[2]

The properties of coherent bremsstrahlung which we have been discussing in this section of the paper might be utilized to make a quick check for a relative displacement of beams and to determine the transverse dimensions of the bunches. (An experiment of this type has been carried out successfully at SLAC for the case of long bunches.[3])

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[1]For definiteness, we discuss the coherent bremsstrahlung which occurs in *ep* collisions. We denote by N_e and N_p the numbers of particles in the bunches, by $l = \sigma_z$ the longitudinal dimension of a proton bunch, by σ_x and σ_y the horizontal and vertical transverse dimensions of a proton bunch, by $\gamma_e = E_e/m_e c^2$ the Lorentz factor of the electron, and by $E_c = 4\gamma_e^2 \hbar c/l$ a characteristic (critical) energy for emission of photons in coherent bremsstrahlung.

[1]P. Chen, *An Introduction to Beamstrahlung and Disruption. Lecture Notes in Physics*, Springer-Verlag, 1988.

[2]I. F. Ginzburg, G. L. Kotkin, S. I. Polityko, and V. G. Serbo, Preprints TP 28(192), 29(193), Inst. Math., Novosibirsk, 1991; Preprints TPI-MINN-91/50-T, 91/51-T, Univ. Minnesota, 1991; in *Proceedings of the First All-Union Conference on Physics at VLÉPP, Vol. II*, Protvino, 1991, p. 151.

[3]G. Bonvicini *et al.*, Preprint SLAC-PUB-4856, Stanford, 1989.

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