Current drive in a tokamak through the trapping of ions in magnetic wells of a travelling fast magnetosonic wave

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(Submitted 2 August 1991; resubmitted 23 April 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **55**, No. 11, 625–630 (10 June 1992)

If the ions formed during the injection of neutral atoms into a tokamak plasma are trapped in the magnetic wells of a fast magnetosonic wave travelling along the injection direction, there may be an accumulation of these ions, and an ion beam may form. The velocity of this beam would be the longitudinal phase velocity of the wave. This physical mechanism underlies a new inductionless method for driving a steady-state current in a tokamak. This new method has advantages over previous proposals.

In this letter we propose a new inductionless method for current drive in a tokamak. The method is based on the application of a fast magnetosonic wave, travelling along the direction of the toroidal magnetic field, to ions formed through ionization of a beam of fast neutral atoms injected into the tokamak plasma. The interaction of this wave with the ions should result in a trapping of the latter in the magnetic wells of the wave. It should also retard the collisional relaxation of the ion beam. The accumulation of ions in the wells would then make it possible to form a beam of ions, with a current velocity equal to the longitudinal phase velocity of the wave. Since the density of the beam ions turns out to depend on the amplitude of the longitudinal magnetic field of the fast magnetosonic wave, $\tilde{B}_{\parallel}$, one can establish a current of the desired strength in the tokamak by adjusting $\tilde{B}_{\parallel}$.

The accumulation of ions in magnetic wells becomes possible because injection at a longitudinal velocity equal to the longitudinal phase velocity of the fast magnetosonic wave allows the ion to remain in a trapped state for a certain time: the "lifetime" τ_t^*. If the beam velocity is above a certain critical value, $V_b \geqslant V_c \simeq V_{Te}(m_e/M_i)^{1/3}$, the longitudinal slowing of ions is caused by electrons. If the lifetime of an ion in a trapped state is comparable to the time scale (τ_s) of this slowing, i.e., if

$$\tau_t^* \geq \tau_s = \frac{3}{16\sqrt{\pi}} \frac{m_e M_b V_{Te}^3}{e^4 n_e Z_b^2 \Lambda} \tag{1}$$

(Λ is the Coulomb logarithm), then the current generated by virtue of the trapped particles turns out to be comparable to the current of passing particles.

The dynamics of the injected ions can be described in a first approximation by the equation of motion in the wave, with allowance for the friction force (in the coordinate system of the wave):

$$M_b \frac{d^2 z}{dt^2} = \mu k_{\parallel} \tilde{B}_{\parallel} \sin k_{\parallel} z - \frac{M_b}{\tau_s} \left(V_{p\parallel} + \frac{dz}{dt} \right). \tag{2}$$

The coordinate z runs along the toroidal magnetic field of the tokamak, $\vec{B}_0$; $\mu = M_b V_\perp^2/2B_0$ is the magnetic moment of an ion; and $k_\parallel$ is the longitudinal wave number of the fast magnetosonic wave. The dissipative term here leads to a trapping of passing particles, whose velocities are larger than the longitudinal phase velocity of the wave. An estimate of the probability for such trapping for Eq. (2) yields[1]

$$p_t \simeq \frac{V_\perp}{V_{p\parallel}} \left(\frac{\tilde{B}_\parallel}{B_0} \right)^{1/2} . \tag{3}$$

For parameter values approximately the same as those of the ITER tokamak reactor, this quantity would not exceed 10^{-1}; i.e., a comparatively small fraction of the originally passing particles would move into the trapping region. If this fraction is to be raised to any significant extent, the source must be directly in the resonance region, defined by the boundary

$$|u| = |V_\parallel - V_{p\parallel}| \leq \Delta V \simeq V_\perp \left(\frac{\tilde{B}_\parallel}{B_0} \right)^{1/2} . \tag{4}$$

An accumulation of particles in the resonance region opposes a diffusion among the electrons and ions of the bulk plasma. As a result, the particles which leave the trapping region become a source of passing ions with velocities ranging from the injection velocity V_b to the ion thermal velocity V_{Ti}. According to Ref. 2, the density of beam ions in this velocity interval can be estimated from

$$n_u \simeq S_0 \tau_s \ln \frac{V_b}{V_{Ti}}, \tag{5}$$

where S_0 is the source (flux density) of injected particles. The effective lifetime of the passing particles in this case is

$$\tau_u^* \simeq \tau_s \ln \frac{V_b}{V_c}, \tag{6}$$

and the characteristic current velocity is[2] $v \simeq 0.44 V_b$.

To determine whether particles can accumulate in the trapping region, we need to solve a kinetic equation for the trapped particles and then join the resulting solution with the solution of the kinetic equation for the passing particles. The kinetic equation describing the dynamics of the ions trapped in the magnetic wells can be put in the following form in the coordinate system of the fast magnetosonic wave:

$$u \frac{\partial f_t}{\partial z} + \frac{F_\parallel}{M_b} \frac{\partial f_t}{\partial u} = St\{f_t\} + S(u, v_\perp); \tag{7}$$

$$St\{f\} = \frac{1}{\tau_s} \left[\frac{\partial}{\partial u} (V_{p\parallel} + u) f + \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp^2 f \right.$$
$$\left. + \left(\frac{T_e}{2M_b} + \alpha^3 v_\perp^2 \right) \frac{\partial^2 f}{\partial u^2} + \alpha^3 \left(-2 v_\perp V_b \frac{\partial^2 f}{\partial u \partial v_\perp} + V_b^2 \frac{\partial^2 f}{\partial v_\perp^2} \right) \right] ; \tag{8}$$

$$\alpha^3 = \frac{V_c^3}{2V_b^3}; \quad F_{\|} = -e\frac{\partial \phi_{eff}}{\partial z}; \quad e\phi_{eff} \simeq \mu \tilde{B}_{\|}(z) = \mathcal{E}_{\perp}\frac{\tilde{B}_{\|}}{B_0}\cos k_{\|}z. \tag{9}$$

The most important processes described by Eq. (7) are the slowing and scattering of beam ions by particles of the bulk plasma and the acceleration of beam ions by the travelling fast magnetosonic wave. Ions in the trapped state lose directed velocity primarily because of longitudinal slowing by electrons. This process is offset by the acceleration by the fast magnetosonic wave. Under these conditions, transverse slowing and longitudinal diffusion become the most important processes. This case differs from the problems examined by Zakharov and Karpman[3] in that there is a source and the trapping region is two-dimensional. Equation (7) can be reduced to a two-dimensional partial differential equation:

$$\frac{\partial}{\partial \tilde{u}}\left(\tilde{u}f_t + (1+\delta\tilde{\mathcal{E}}_{\perp})\frac{\partial f_t}{\partial \tilde{u}}\right) + 2\frac{\partial}{\partial \tilde{\mathcal{E}}_{\perp}}\left(\tilde{\mathcal{E}}_{\perp}f_t + \alpha^3\tilde{\mathcal{E}}_{\perp}\frac{\partial f_t}{\partial \tilde{\mathcal{E}}_{\perp}}\right) + \tilde{S}(\tilde{u},\tilde{\mathcal{E}}_{\perp}) = 0; \tag{10}$$

$$\tilde{u} = \frac{\left[\frac{u^2}{2} + e\phi_{eff}(z) + \frac{V_{p\|}}{\tau_s}z\right]^{1/2}}{V_{Ti}/\sqrt{2}}; \quad \tilde{\mathcal{E}}_{\perp} = \frac{V_{\perp}^2}{2V_{p\|}^2} \quad \delta = \frac{V_{Te}}{V_{p\|}};$$

$$\tilde{S} = \tau_s S(u, v_{\perp}); \quad V_b^2 = V_{p\|}^2 + V_{\perp m}^2. \tag{11}$$

Equation (10) can be solved for simple sources under the assumption that most of the ion loss results from diffusion in the longitudinal direction. We consider a source $S(u,v_{\perp}) = S_V = S_0/\Delta V_{\|}\Delta\mathcal{E}_{\perp}$, where $\Delta V_{\|}$ and $\Delta\mathcal{E}_{\perp}$ are characteristic widths of the injection region in velocity space. This source is distributed uniformly over a certain region. The distribution function f_t for this source can be written

$$f_t = \sum_i C_i X_i(x,\lambda_i)Y_i(y,\lambda_i);$$

$$x = \tilde{u}/\tilde{\mathcal{E}}_{\perp}^{1/2}; \quad y = (\tilde{\mathcal{E}}_{\perp}^2 + 2\tilde{\mathcal{E}}_{\perp}/\delta + \alpha^3\tilde{u}^2/2\delta)^{1/2}. \tag{12}$$

The functions $X(x)$ and $Y(y)$ are determined from solutions of simple differential equations. The constants C_i and the eigenvalues λ_i for the phase-space region $-x_m \leqslant x \leqslant x_m$, $0 \leqslant y \leqslant y_m$ [x_m is proportional to $(\tilde{B}_{\|}/B_0)^{1/2}$, and y_m is proportional to the transverse energy of the injected atoms, $\mathcal{E}_{\perp m}$] are found from the conditions for joining the distribution functions for the trapped and passing particles and their fluxes:

$$\lambda \simeq \frac{\xi}{x_m} = \xi\frac{V_{Ti}}{V_{p\|}}\left(\frac{B_0}{B_{\|}}\right)^{1/2}, \quad C \simeq \frac{S_V\tau_s}{3\cos\lambda x_m}, \quad \Delta y = \frac{\Delta\mathcal{E}_{\perp}}{V_{p\|}^2} \leq \frac{y_m}{3}. \tag{13}$$

The numerical factor $\xi \simeq 0.7$ is found from the boundary conditions. At $\alpha^3 \ll 1$, the function f_t can be approximated by the simple expression

$$f_t \approx AC\cos\lambda x\left(\frac{y}{y_m}\right)^q \exp(-\lambda^2/2y), \quad q = -\frac{3}{2} + \frac{\delta\lambda^2}{2}, \tag{14}$$

where the numerical factor $A(\alpha)$ is associated with the diffusion of ions in the transverse direction. It has the value $A = 1$ in the case $\alpha = 0$. The density of the ion beam trapped by the fast magnetosonic wave is given by

$$\bar{n}_t = k_{\|}\int\int\int f_t u 2\pi v_\perp dv_\perp dz \simeq S_0\tau_s H(A, q, y_m, y_s), \tag{15}$$

where H is an accumulation parameter; $y_m \simeq \widetilde{\mathscr{E}}_{\perp m}(1 + 2/\delta\widetilde{\mathscr{E}}_{\perp m})^{1/2}$; and $y_s \simeq \lambda^2/2$ is essentially the boundary of the region in which accumulation is possible. In this case we find

$$H = 2A\begin{cases} \frac{1}{|q+1|}\left(1 + \left|1 - \frac{y_s^{q+1}}{y_m^{q+1}}\right|\right), & q \neq -1; \\ 1 + \ln(y_m/y_s), & q = -1. \end{cases} \tag{16}$$

Let us look at the physical meaning of this solution. The region in velocity space in which accumulation is possible corresponds to high transverse energies, $\widetilde{\mathscr{E}}_\perp \geqslant (V_{Ti}^2/2V_{p\|}^2)(\widetilde{B}_0/B_{\|}),(y_s \leqslant y \leqslant y_m)$; the width of the region in the longitudinal direction (along the coordinate u) is large in comparison with the quantity $(T_e/M_b)^{1/2}$, and diffusion in the longitudinal direction has no bearing on the situation. At lower transverse energies $(y \leqslant y_s)$ the longitudinal diffusion of ions in phase space prevents the accumulation of the required number of particles, because the "mirror ratio" $\widetilde{B}_{\|}/B_0$ is small, and all particles which enter this region over the time $t \leqslant \tau_S$ escape from the trapping region. At a fixed value of $V_{b\perp}/V_{b\|}$, the quantity y_s actually determines the minimum amplitude of the longitudinal magnetic field of the wave, $\widetilde{B}_{\|}$, at which a beam of the required density can be formed in the magnetic wells of the wave.

Accumulation at larger transverse energies, $(y_s \leqslant y \leqslant y_m)$, is opposed by the "cooling" of the ions in the transverse direction by friction with electrons. The decrease in transverse energy in the course of this slowing might offset the heating of the trapped ions in the transverse direction due to the scattering of beam ions by ions of the bulk plasma. When α is small but nonzero $(\alpha \ll 1)$, this "heating" leads to an approximate doubling of the factor A. The introduction of auxiliary transverse heating of the beam ions (by means of, for example, cyclotron heating, which—like scattering—would be described in the kinetic equation by a term with a second derivative of the distribution function with respect to $\mathscr{E}_\perp$; Ref. 4) might lead to a significant increase in the factor A and thus a significant accumulation of particles in the trapping region.

The parameter H, which determines the lifetime of the trapped particles,

$$\tau_t^* \simeq \tau_s H, \tag{17}$$

depends on q and y_m as well as the parameters A and y_s, as follows from (16). The dependence on the parameter y_m, which is proportional to the transverse energy of the injected ions, is obvious: As the ratio y_m/y_s increases, so does the parameter H, since there is an increase in the lifetime associated with the slowing of the trapped ions in the transverse direction. The dependence on the parameter q, on the other hand, stems from the diffusion of beam ions among the ions of the bulk plasma. This diffusion is important at larger transverse energies of the beam. The increase in H stems from an increase in the longitudinal injection velocity and the amplitude of the longitudinal magnetic field of the wave, $\tilde{B}_{\parallel}$. At low plasma temperatures, at which the ratio $V_{p\parallel}/V_{Ti}$ is large, the quantity λ is small; at $q \leqslant -1$, the parameter H can be increased significantly.

In order to trap the beam ions in the central part of the tokamak plasma, it is necessary to excite a fast magnetosonic wave of definite structure. Corresponding best to this structure is the $m=0$ mode: the lowest-index radial mode of the toroidal plasma resonator. This mode would be excited throughout the plasma volume by an external source at the frequency[5] $\omega_{\nu} \simeq \nu V_A/a$ (V_A is the Alfvén velocity at the plasma axis, a is the minor radius of the torus, and ν is a numerical factor which depends on the plasma density distribution along the minor radius; for example, it would be $\nu \simeq 4.67$ for a parabolic density profile). Most of the energy of a fast magnetosonic wave of this type is magnetic energy. The longitudinal wave number $k_{\parallel}$ would be chosen in order to achieve the phase resonance with the injected ions and to shift the point of the Alfvén resonance toward the plasma edge: $k_{\parallel} \ll k_{\perp}$. The longitudinal phase velocity of the wave would then have to be much greater than the Alfvén velocity: $V_{p\parallel} = \omega/K_{\parallel} \gg \omega/K_{\perp} \simeq V_A$. The magnetic field of the wave would have its maximum at the plasma axis. The region in which the ions are trapped in the magnetic wells of the wave would be a torus whose minor radius is roughly half the minor radius of the tokamak itself.

To determine the efficiency of this method, let us estimate the power which is absorbed from a fast magnetosonic wave by the ion beam. This power is determined primarily by the work performed by the wave field on replenishing the loss due to Coulomb slowing:

$$P_b \simeq \frac{n_t M_b V_{p\parallel}^2}{\tau_s} . \tag{18}$$

For a hot plasma, the primary competing mechanism is absorption by electrons. At low amplitudes of the fast magnetosonic wave, such that the electron distribution function in the resonance region remains Maxwellian as a result of collisions, the absorption by electrons occurs through Landau damping and its magnetic analog.[6] At amplitudes above a certain level, $\tilde{B}_{\parallel} \geqslant \tilde{B}_{\perp}^{*}$, the electron distribution function is greatly deformed in the resonance region ($\partial f_e/\partial V_{\parallel} \approx 0$), and the absorption is determined by collisions, according to Ref. 7:

$$P_e \simeq \frac{\mathcal{D}^c \mathcal{D}^{QL}}{\mathcal{D}^c + \mathcal{D}^{QL}} m_e n_{re} \frac{V_{p\parallel}^2}{V_{Te}^2} \simeq 0.7 \frac{n_e m_e V_{p\parallel}^3}{\tau_{ee} V_{Te}} \left(\frac{\tilde{B}_{\parallel}}{B_0} \right)^{1/2} . \tag{19}$$

Here n_{re} is the density of resonant electrons, and n_e is the density of the bulk plasma. This threshold amplitude B^* is determined by equating the collisional diffusion coefficient D^c and the quasilinear diffusion coefficient D^{QL} in velocity space; for parameter values approximating those of the ITER, we would have $B^* \simeq 25$ G.

Comparing P_b with P_e, we find

$$\frac{P_b}{P_e} \simeq 1.8 \frac{n_t}{n_0} \frac{V_{Te}}{V_{p\parallel}} \left(\frac{B_0}{\tilde{B}_\parallel} \right)^{1/2} . \qquad (20)$$

For plasma parameter values typical of the ITER tokamak reactor, this ratio is close to 2 in the case of hydrogen injection; it would double in the case of deuterium. Much of the power deposited is thus absorbed by beam ions.

It is important to note that electron currents associated with absorption of the wave by electrons or with the trapping of electrons would not flow in the case in which a standing fast magnetosonic wave was excited in the interior of a tokamak, since a standing wave can be written as the sum of two oppositely directed waves. The two would interact identically with the electrons, but only one would interact with the ion beam—the one travelling in the direction of the injected ions.

We can estimtate the efficiency of this method in terms of the local efficiency η, defined as the ratio of the current density generated to the power density deposited:

$$\eta = \frac{j}{P_{RF} + P_s} = \frac{j_t + j_u}{P_b + P_e + P_s} \simeq \eta_{Nb} \frac{1.6(H + 0.44)}{\left(1 + \frac{P_e}{P_b}\right) H + 1}, \qquad (21)$$

where P_{RF} is the density of rf power deposited, P_s is the power density of the beam of neutral atoms, j_t and J_u are the current densities of the trapped and passing particles, and H is the accumulation parameter.

For plasma parameter values typical of the planned ITER tokamak reactor ($n_e \simeq 10^{20}$ m^{-3}, $T_e \simeq T_j \simeq 20$ keV, $V_{Te} \simeq 8.4 \times 10^7$ m/s, $V_c \simeq 3 \times 10^6$ m/s), achieving a value $H \geqslant 2$ at a beam current $I_b \simeq 13$ MA through the injection of hydrogen atoms with $V_{b\parallel} \simeq V_{b\perp} \simeq V_{p\parallel} \simeq 1.4 \times 10^7$ m/s and $\mathscr{E}_b \geqslant 2$ MeV would require

$$\frac{n_t}{n_e} \leq 2 \cdot 10^{-2}; \quad \frac{\tilde{B}_\parallel}{B_0} \geq 10^{-2}; \quad \tilde{B}_\parallel \geqslant 500\ \text{G}. \qquad (22)$$

In a tokamak reactor, in which the absorption of rf power by electrons would be strong, the efficiency of this method at $H \geqslant 1$ would be close to the efficiency of current drive by "pure" neutral injection. However, at values as low as $H \simeq 2$ this new method would reduce the power required of the source of neutral atoms by a factor of nearly 5, because much of the current would be carried by trapped ions, which have a current velocity 2.3 times that of the passing ions. This change is advantageous, since the rf power source, which excites the fast magnetosonic wave in the interior of the tokamak plasma, is vastly more efficient than the source of neutral atoms.

In small devices, e.g., the T-11M, in which this new method could be tested experimentally, the absorption of the fast magnetosonic wave by electrons would be slight, and the efficiency of this method might be nearly 1.5 times that of pure neutral injection.

Furthermore, as we mentioned earlier, there are ways to increase the accumulation parameter H. For example, one might use auxiliary cyclotron heating of the trapped ions.

We wish to thank V. V. Parail, A. R. Polevoĭ, V. P. Patrukhov, and A. I. Neĭshtadt for useful comments offered in the course of this study. We also thank Yu. V. Petrov for assistance.

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Translated by D. Parsons