Stochastic dynamics of 2D electrons in a periodic lattice of antidots

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(Submitted 21 April 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **55**, No. 11, 649–652 (10 June 1992)

The motion of electrons in a periodic 2D lattice of antidots in a classically strong magnetic field is analyzed from the standpoint of dynamic-chaos theory. There exist stable trajectories which "roll" along rows of the lattice. The diffusion coefficient is calculated by numerical simulation. The results are used to explain some magnetoresistance oscillations seen experimentally.

Systems consisting of a 2D electron gas in a lattice of "antidots"—regions with a strong repulsive potential—have been developed and studied actively over the past two years.[1–5] They exhibit Shubnikov–de Haas oscillations of a new type, geometric resonances associated with a commensurate relationship between the Larmor radius and the lattice period, and Aharonov–Bohm oscillations. An exhaustive explanation of these effects has yet to be developed.

Here we are reporting a study of a system of periodically arranged circular antidots in a classically strong magnetic field. A similar problem, but with a disordered arrangement of scatterers, was studied in Ref. 6. It was shown there that, at a certain value of the magnetic field, the system undergoes a percolation transition between a diffusive motion of electrons and a localized motion. This transition occurs when the cyclotron radius r_c, the density n, and the size of the scatterers, a, satisfy the condition $\pi n(r_c+a)^2=0.68$. Below the critical magnetic field, under the condition $1/(4\pi na)\gg r_c>(0.68/\pi n)^{1/2}$, a classical negative magnetoresistance arises. We show below that a periodic arrangement of scatterers differs from a disordered arrangement in that the electrons do not become completely ergodic in the course of their diffusion, and the motion remains partially ordered. This system is actually a version of Sinaĭ billiards,[7] and it should be analyzed by the methods of nonlinear dynamics.

We assume that the antidots are circles which perform an ideal specular scattering. It can be shown that this assumption is justified in a real experiment. We also ignore bulk scattering by impurities. Solving the dynamic problem reduces to performing successive nonlinear mappings of a 2D torus. The cyclic variables on this torus, φ and ψ, is the velocity direction of the electron after a collision with an antidot and the angular coordinate of the point of departure from the surface:

$$iae^{i\psi'}-r_ce^{-i(\varphi'-2\psi')}=iae^{i\psi}+r_ce^{i\varphi}-d_{n'-n,m'-m}. \quad (1)$$

Here $d_{n,m}=d(n+im)$, d is the lattice period (the lattice is assumed to be square), (n,m) specify the particular antidot, and the variables with and without primes refer to two successive collisions. Figure 1 shows some typical trajectories found through a

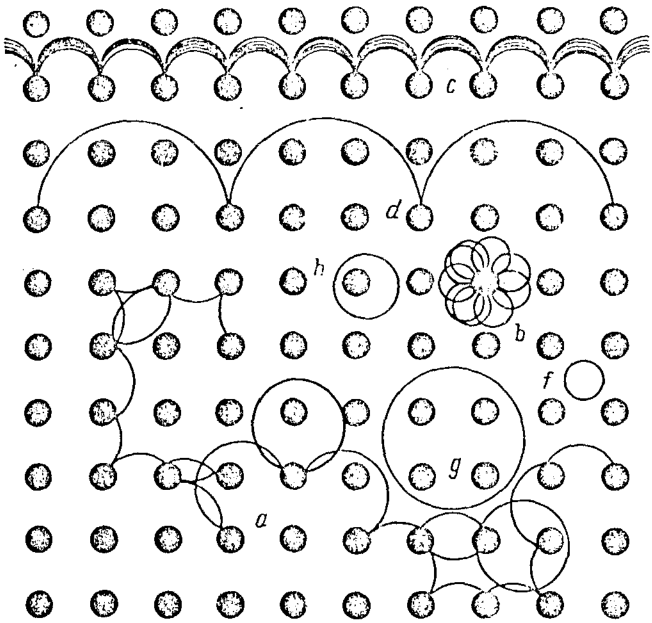

FIG. 1. Some typical trajectories traced out by an electron in a periodic lattice of scattering antidots in a magnetic field.

numerical simulation of the motion of the electrons. Among these trajectories we distinguish various types. Trajectories of types f, g, and h are trajectories of electrons which do not collide with antidots. Trajectories of type b are rosettes of radius $r + r_c$ around a single antidot, where r is the distance to the center of the orbit (this distance remains constant during the revolution), and $-a < r - r_c < a$. Trajectory a represents ordinary diffusion. Trajectories c and d correspond to motion which is directed but partically chaotic; these are "runaway" trajectories. Delocalized trajectories a, c and d appear only if $2(r_c + a) > d$.

Under the condition $(2r_c - d) \ll a$, mapping (1) simplifies near angles $\varphi = \psi = 0$, which correspond to trajectories to type c:

$$\psi' = \psi - (2r_c - d)/a + r_c/a\varphi^2, \quad \varphi' = \varphi + 2\psi' - a/2r_c(\psi^2 - \psi'^2). \tag{2}$$

Figure 2 is a phase portrait of these trajectories for the values $a/d = 0.25$ and $(2r_c - d)/d = 0.02$. The white region represents initial angles for which the given particle is still in its row after 127 collisions. The various shades of gray represent trajectories which run away after K steps, where $2^N < K < 2^{N+1}$ $(N < 7)$. Black represents trajectories which run away from the given row after fewer than eight collisions. The area of the white region, which gives us the fraction of runaway trajectories in the phase-space square $|\varphi| < \pi/4$, $|\psi| < \pi/4$, is $S = 0.05845$. The structure of the region of stable trajectories is a typical fractal, as can be seen, in particular in the $5\times$ fragment in the inset in Fig. 2. Stable trajectories are possible for r_c in the interval $0 < (2r_c - d/$

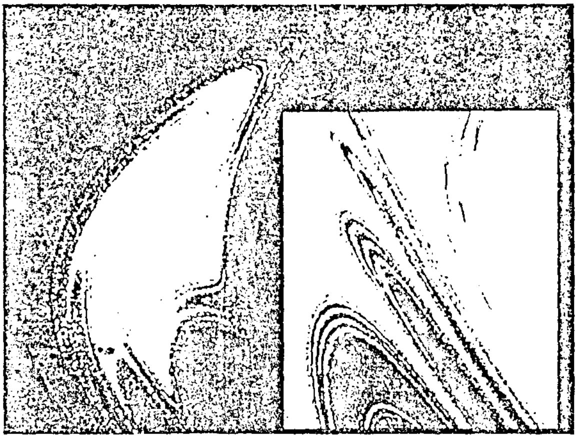

FIG. 2. Phase portrait of stable runaway trajectories (the white region) and of nearby trajectories (various shades of gray) in the coordinates (ψ,φ). ($-25° \leqslant \psi \leqslant 35°$, $-12.5° \leqslant \varphi \leqslant 32.5°$, $a/d = 0.25$, $r_c/d = 0.51$. The inset shows a fragment of the portrait in larger scale $(5\times)$.

$a)(r_c/a) < 0.5$. At the edges of this interval, the boundary of the region becomes less fractal.

The conductivity σ_{xx} is determined by the diffusion coefficient of the electrons which are colliding with the antidots. This coefficient was calculated for the non-runaway trajectories through a numerical simulation based on the expression $D_{xx} = \langle \vec{z}^2(t) \rangle / 2t$, where $\vec{z}(t)$ is the displacement of an electron over a time t in the course of the diffusion. The starting point and the departure angle of the electron were fixed in the region of unstable (diffusion) trajectories. An averaging over the initial conditions occurs automatically, because the motion along these trajectories is ergodic. The results are shown in Fig. 3b. The peaks on the plot of D_{xx} correspond to regions in which runaway trajectories arise. Such trajectories are possible not only in the (1,0) direction but also in other singular directions, provided that they are not prevented by a shadowing by antidots in other rows. This result is evidence of the appearance of nearly runaway trajectories in corresponding magnetic fields. The resultant conductivity is the sum of the two contributions, multiplied by f_s, which is the fraction of the electrons which collide with antidots (Fig. 3c).

This fraction is determined by the conditions for the positions of the centers of the cyclotron orbits, $\rho = x + iy$: $d - (r_c + a) < |\rho - d_{n,m}| < r_c + a$ for $r_c + a < d - |r_c - a|$ and $|r_c - a| < |\rho - d_{n,m}| < r_c + a$ for $r_c + a > d - |r_c - a|$. The fraction f_s is the ratio of the combined area of these rings to the total area. According to Fig. 3b, f_s oscillates as a function of r_c. Its average value is equal to the function $1 - \exp(-4\pi r_c a/d^2)$, which corresponds to the fraction of electrons which collide with randomly arranged antidots. The oscillation amplitude is less than 20% and considerably smaller than the oscillations in D_{xx}.

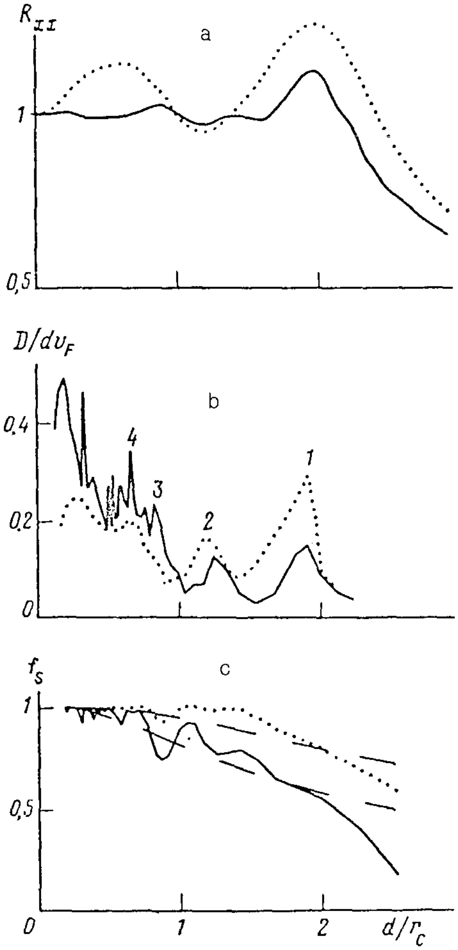

FIG. 3. Several properties versus the reciprocal of the Larmor radius divided by d. a: Experimental resistance. b: Diffusion coefficient calculated by numerical simulation. c: Fraction of trajectories in the diffusion category. Peaks 1, 2, 3, and 4 in frame b correspond to trajectories of type c, which run away along the directions (1,0), (1,1), (1,2), and (1,3). Solid lines—Sample with $a/d = 0.1346$; dotted lines—sample with $a/d = 0.25$; dashed lines in frame c—approximation of the fraction of trajectories in the diffusion category by the expression $f_s + 1 - \exp(-4\pi r_c a/d^2)$.

It is not difficult to see that incorporating the regions of runaway electrons c and d in the initial conditions would have led to a divergence of the diffusion coefficient, since the relation $\vec{z}(t) \propto t$ holds for these electrons. In bounded regions, the contribution of these regions to σ_{xx} is finite and not particularly large, because the phase volume of regions c and d is small. It can be found from the theory for the resistance of point contacts, by treating the entire sample as a point contact. The conductance G of an $L \times L$ sample is made up of a parallel connection of the contributions of the diffusion and runaway trajectories. The second contribution is equal to the product of the quantum of conductance ($2e^2/h$), the total number of channels in the sample $L/d(d-2a)/\pi\hbar p_f$), and the fraction of the phase space which is filled by runaway trajectories: $G = 2e^2/hL/d(d-2a)/\pi\hbar p_f 2S/\pi^2$.

The ideas developed above yield an explanation of the oscillations which have been seen in several studies[1–4] in the magnetoresistance as the result of a commensurate relationship between the Larmor radius and the lattice period. Figure 3c shows the results of measurements of a diagonal component of the resistance tensor versus the reciprocal of the Larmor radius divided by d. Comparison of Figs. 3a and 3c shows a satisfactory agreement between the theoretical and experimental positions of the singularities. Not all the singularities predicted by the theory are seen in this experiment. This result is not surprising, since the theory ignores the diffuse aspect of the surfaces of the antidots and also the finite mean free path in the interior.

Oscillations of this type have previously been attributed[2] to structural features in f_s. That explanation runs into some difficulties. First (as we mentioned earlier), these features are much smaller than the oscillations in D_{xx}. Second, the positions of some of them are completely at odds with the experimental results (there is a corresponding discrepancy in Ref. 2).

We note in conclusion that a 2D electron gas in a lattice of antidots is a good model for studying the stochastic dynamics of electrons. In addition, technical advances have now made it possible to fabricate a wide variety of billiards using 2D ballistic electrons. There are accordingly wide opportunities for experimental research on classical and quantum dynamic chaos in solid-state electron systems.

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Translated by D. Parsons