Suppression of localization in quasi-1D systems by electromagnetic fluctuations

S. S. Murzin
Institute of Solid State Physics, Russian Academy of Sciences, 142432, Chernogolovka, Moscow Oblast

(Submitted 14 May 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **55**, No. 11, 665–670 (10 June 1992)

Localization effects in quasi-1D compounds are suppressed by thermal electromagnetic fluctuations at temperatures $T \gg \hbar/[\tau \cdot (\hbar V_F \epsilon_\perp / e^2)^{1/3}]$, where τ is the mean free time of an electron with respect to backscattering, $\epsilon_\perp$ is the dielectric constant of the lattice in the direction transverse with respect to the conducting chains, and V_F is the Fermi velocity. The conductivity can therefore be described by the Drude formula in this case. It is assumed that the condition $e^2/(\epsilon_\perp V_F \hbar) \ll 1$ holds, i.e., that the electron–electron interaction is weak. A situation approaching a quasi-1D situation arises in 3D semiconductors and semimetals in high magnetic fields. In this case localization effects are suppressed by thermal fluctuations under the same conditions as in quasi-1D conductors.

All the electrons in disordered 1D systems are localized, regardless of their energy, and the static conductivity at $T = 0$ is $\sigma = 0$, as has been shown by Mott and Twose[1] and as has subsequently been proved more rigorously.[2] If impurities and defects cause only a slight scattering of electrons with the Fermi energy $\mathscr{E}_F$ (if the reflection coefficient of one scattering center is much less than 1), and if the mean free time of these electrons with respect to backscattering satisfies $\tau \gg \hbar/\mathscr{E}_F$, then the localization length is four times the mean free path $l = V_F \tau$. The characteristic energy distance between spatially overlapping states is $\sim \hbar/\tau$.

In real quasi-1D compounds, consisting of a large number of 1D chains, the lifetime of an electron on a chain, $\tau_\perp$, is finite (in a perfect crystal at $T = 0$ this time would be $\tau_\perp \sim \hbar/w$, where w is the hopping integral for hopping between neighboring chains). In this case electrons are localized only in sufficiently "dirty" samples,[3,4] under the condition $\tau \ll \tau_\perp$. In the opposite case, $\tau \gg \tau_\perp$ the electrons are delocalized and the conductivity is described by the Drude formula $\sigma_0 = ne^2\tau/m$, where n and m are the density and effective mass of the electrons, provided that effects which give rise to a gap in the electron spectrum near the Fermi level are not important.[2]

A situation similar in many ways to a 1D case arises in 3D semiconductors and semimetals in a high magnetic field H ($\mathscr{E}_F \ll \hbar\Omega_c$, where Ω_c is the cyclotron frequency), if electrons are scattered primarily by ionized impurities.[5–7] Since the impurity potential is smooth over the magnetic length scale $\lambda_H = \hbar c/eH$, the displacement of an electron across the magnetic field over a time τ is far shorter than the magnetic length scale—the characteristic length scale of the electron wave function in the direction across the field. In a first approximation, in which we ignore the variation in the impurity potential over distances on the order of λ_H, we can therefore assume that the

electron is in a 1D potential and is thus localized along the magnetic field. If the transverse component of the nonuniform electric field of the impurities is taken into account, we obtain a finite lifetime in a "quasilocalized state," τ_H ($\tau_H \gg \tau$), and we obtain a nonzero conductivity along the magnetic field at $T=0$ (Ref. 6):

$$\sigma \simeq e^2(\partial n/\partial \mathscr{E})l^2/\tau_H \simeq e^2 n\tau/\tau_H \tag{1}$$

($\partial n/\partial \mathscr{E}$ is the density of states at the Fermi level). This conductivity is far smaller than follows from the theory of Ref. 8, which ignores localization effects and which leads to the Drude formula with a time τ which depends on the magnetic field.

The discussion above ignored the electron–electron interaction. How this interaction affects the localization in quasi-1D systems has not been resolved, even in this case in which the characteristic strength of the Coulomb interaction of two electrons is small,

$$e^2/\epsilon_\perp l \ll \hbar/\tau \quad \text{or} \quad e^2/\epsilon_\perp V_F \hbar \ll 1, \tag{2}$$

and the electron–electron interaction has essentially no effect on the electron spectrum. Apel and Rice[9] took a scaling approach and reached the conclusion that a weak interaction does not give rise to a conductivity at $T=0$ in 1D systems. Even along that approach, however, it is not clear whether that assertion is valid for quasi-1D systems, in which the Debye screening length is much greater than the distance between chains, (b),

$$r_D = \{4\pi e^2(\partial n/\partial \mathscr{E})/\epsilon_\perp\}^{-1/2} \sim (\hbar V_F \epsilon_\perp/e^2)^{1/2} b \gg b, \tag{3}$$

and the electron interacts with not only electrons on its chain but also electrons on other chains. In other words, the interaction is three-dimensional.

If the localization is retained at $T=0$, we are confronted with the following question: Is an interaction capable of giving rise to a conductivity at a nonzero temperature, and if so what is the magnitude of this conductivity? The possibility of a conductivity at $T \neq 0$, as the result of an electron–electron interaction in 3D systems which are not conducting at $T=0$, has been discussed previously.[10–12] In the present letter we show that if the temperature is sufficiently high a weak electron–electron interaction ($e^2/\epsilon_\perp V_F \hbar \ll 1$) completely suppresses localization effects in quasi-1D systems, if such effects operate at $T=0$. The conductivity at $T=0$ remains an open question.

We know that localization is possible only if the phase of the electron is not changed in a random way by thermal excitations of some sort in the system. In the present letter we find the conditions under which electromagnetic fluctuations disrupt the phase of an electron, φ (i.e., change it by an amount greater than 1), over the mean free time with respect to backscattering, τ. We seek the fluctuation field and the phase change $\Delta\varphi_\tau$ over the time τ under the assumption that localization effects are unimportant. This assumption is justified if the value found as a result satisfies $\Delta\varphi_\tau > 1$. Fluctuation fields are produced by electrons, so the interaction of an electron with fluctuations is essentially an electron–electron interaction. Altshuler *et al.*[13] have studied the effect of fluctuations on the localization corrections to the metallic conduc-

tivity in the quasi-1D, quasi-2D, and 3D cases. The concept of a quasi-1D case as it was used in that previous study, however, differs from that used in the present letter. Burin and Maksimov[12] have studied a hopping conductivity resulting from fluctuations in semiconductors.

The fluctuation field can be broken up into two components: zero-point vibrations (with frequencies $\omega > T/\hbar$) and thermal vibrations ($\omega < T/\hbar$). A disruption of localization by zero-point vibrations would mean that localization would also be suppressed at absolute zero. This problem, however, has not been analyzed by the semiclassical approach, which we are taking in the present letter. The change ($\Delta\varphi_\tau$) caused in the phase of an electron by thermal vibrations (with frequencies $\omega \leqslant T/\hbar$ and with Fourier components of the fluctuation field along the direction of the chains, $k_z \sim \omega/V_F \leqslant T/\hbar V_F$) can be found by a semiclassical approach[14] if interference effects are suppressed. Taking thermal fluctuations alone into account, we find the characteristic temperature T_Γ, above which the conductivity should be described by the Drude formula. On the other hand, we do not rule out the possibility that the Drude formula may also be valid at $T < T_\Gamma$ under certain conditions.

The phase of the electron wave function is $\varphi = S/\hbar$, where S is the action. The change caused in the phase over a time τ by the fluctuation field is

$$\Delta\varphi_\tau = \Delta S/\hbar = \hbar^{-1}\int(\Delta\mathcal{E} - e\Phi)dt \simeq \hbar^{-1}\int \Delta\mathcal{E}(t)dt$$

$$= e\hbar^{-1}\int_0^\tau dt\int_0^t E\{t', z(t')\}v(t')dt' = eV_F\hbar^{-1}\int_0^\tau dt\int_o^t eE\{t', z(t')\}dt'. \qquad (4)$$

Here ΔS is the increment in the action due to the perturbation, $\Delta\mathcal{E}$ is the change in the kinetic energy of an electron caused by the fluctuation field, Φ is the potential of this field, and E is the field along the chains (along the z axis). We show below that the relation $e\Phi \ll \Delta\mathcal{E}$ holds. The change in the electron velocity $v(t)$ can be ignored; we can assume $v(t) = V_F$.

The square of the change in the phase over a time τ is

$$\Delta\varphi_\tau^2 = e^2V_F^2\hbar^{-2}\int_0^\tau dt_1\int_0^\tau dt_2\int_0^{t_1}\int_0^{t_2} E(t', z')E(t'', z'')dt'dt''. \qquad (5)$$

Expressing $E(t',z')$ and $E(t'',z'')$ in terms of Fourier integrals; replacing z' and z'' by V_Ft' and V_Ft''; integrating over t', t'', t_1 and t_2; and taking an average, using

$$< E_{\omega_1,\bar{k}_1}E_{\omega_2,\bar{k}_2} > = (2\pi)^4 < E^2 >_{\omega_1,\bar{k}_1}\delta(\omega_1 + \omega_2)\delta(\bar{k}_1 + \bar{k}_2), \qquad (6)$$

we find

$$< \Delta\varphi_\tau^2 > = \frac{e^2V_F^2\tau^4}{(2\pi)^4\hbar^2}\int\int < E^2 >_{\omega,\bar{k}} F[(\omega - k_zV_F)\tau]d\omega d\bar{k}. \qquad (7)$$

Here $\langle E^2\rangle_{\omega,\bar{k}}$ is the spectral density of the fluctuations in the electric field E, and

$$F[x]=\frac{2(1-\cos x-x\sin x)+x^2}{x^4}=\begin{cases}1/4, & |x|\ll 1\\ x^{-2} & |x|\gg 1\end{cases}. \tag{8}$$

The spectral density of the thermal fluctuations ($\omega\leqslant T/\hbar$) of the field E is[15]

$$<E^2>_{\omega,\bar{k}}\simeq 4\pi i\frac{k_BT}{\omega}\{\Lambda_{zz}^{-1}-(\Lambda_{zz}^{-1})^*\}, \tag{9}$$

where

$$\Lambda_{zz}=\begin{pmatrix}-(k_zc/\omega)^2+\epsilon_\perp & 0 & k_\perp k_zc^2/\omega^2\\ 0 & -(k_zc/\omega)^2+\epsilon_\perp & 0\\ k_\perp k_zc^2/\omega^2 & 0 & -(k_\perp c/\omega)^2+\epsilon_{zz}\end{pmatrix}. \tag{10}$$

Here the x axis is assumed to lie in the $\bar{k}$, z plane ($k_y=0$, $k_x=k_\perp$); $\epsilon_{xx}=\epsilon_{yy}=\epsilon_\perp$; and the off-diagonal components of the dielectric tensor are zero. Using $k_z\gg\epsilon_\perp^{1/2}\omega/c$, we then find

$$\Lambda_{zz}^{-1}=\{\epsilon_{zz}-(k_\perp c/\omega)^2\epsilon_\perp/[\epsilon_\perp-(k_zc/\omega)^2]\}^{-1}\simeq k_z^2/(k_z^2\epsilon_{zz}+k_\perp^2\epsilon_\perp). \tag{11}$$

If the electron lifetime on a chain satisfies $\tau_\perp\gg\tau$, the electron dielectric constant in the direction across the chains for frequencies $\omega>1/\tau$ is much smaller than the lattice component $\epsilon_{\perp 0}$, so we have $\epsilon_\perp=\epsilon_{\perp 0}=\text{const}$. Substituting in the value of $\langle E^2\rangle_{\omega,k}$ with Λ^{-1} from (11), we find

$$<\Delta\varphi_\tau^2>=\frac{Te^2V_F^2\tau^4}{2\pi^3\hbar^2}\int\frac{F\omega^{-1}\epsilon_{zz}''k_z^2dk_zdk_\perp^2d\omega}{k_z^4\epsilon_{zz}''^2+\{k_z^2\epsilon_{zz}'+k_\perp^2\epsilon_\perp\}^2}. \tag{12}$$

The integration over ω is carried out up to $T/\hbar$. The real part (ϵ_{zz}') and the imaginary part (ϵ_{zz}'') of ϵ_{zz} are independent of $k_\perp$ (see the Appendix) so an integration over $k_\perp$ from 0 to ∞ yields

$$<\Delta\varphi_\tau^2>=\frac{Te^2V_F^2\tau^4}{2\pi^3\hbar^2\epsilon_\perp}\int\frac{Fk_z^2dk_zd\omega}{\omega}\{\pi/2-\arctan(\epsilon_{zz}'/\epsilon_{zz}'')\}. \tag{13}$$

The integral of the first term is dominated by values of ω and k_z which satisfy the condition $|\omega-k_zV_F|\leqslant 1/\tau$. In this region we have $\epsilon_{zz}'/\epsilon_{zz}''\leqslant 1$, and this ratio is odd with respect to $\omega-k_zV_F$ if ϵ_{zz}'' is greater than the lattice dielectric constant along the chains, $\epsilon_\parallel$ (see the Appendix). We can therefore discard $\arctan(\epsilon_{zz}'/\epsilon_{zz}'')$. As a result, we find

$$<\Delta\varphi_\tau^2>\sim(e^2/\epsilon_\perp\hbar V_F)(T\tau/\hbar)^3. \tag{14}$$

Localization effects are suppressed, and the conductivity is given by the Drude formula, under the condition $\Delta\varphi_\tau^2\gg 1$, i.e., at

$$T\gg T_\Phi=\hbar/\tau(\hbar V_F\epsilon_\perp/e^2)^{1/3}. \tag{15}$$

The ratio of the change in the electron kinetic energy over a time τ to the potential energy is $\Delta\mathscr{E}/e\Phi \sim k_z l \sim T\tau/\hbar \gg 1$ for $k_z \sim T/\hbar V_F$ at $T \geqslant T_\Phi$, as mentioned above. Limitations on our analysis follow from the condition $\epsilon_\parallel \ll \epsilon''_{zz}$ with $\omega = T_\Phi/\hbar$ and $k_z = T_\Phi/\hbar V_F$. We thus find that condition (15) holds if either

$$\hbar/\tau \ll (e^2/\epsilon_\perp \hbar V_F)^{2/3}\mathcal{E}_F, \quad (e^2/\epsilon_\perp \hbar V_F)^{2/3}(\mathcal{E}_F \hbar^2 b^{-2}/m)^{1/2}(\epsilon_\perp/\epsilon_\parallel)^{1/2} \tag{16}$$

or

$$(e^2/\epsilon_\perp \hbar V_F)^{2/3}\mathcal{E}_F \ll \hbar/\tau \ll (\mathcal{E}_F e^4/\epsilon_\parallel \epsilon_\perp b^2)^{1/3}. \tag{17}$$

Semiconductors and semimetals in an ultraquantum magnetic field. The analysis above is also valid for semiconductors and semimetals in an ultraquantum magnetic field. Since the displacement of an electron across the magnetic field over a time τ is much shorter than the magnetic length scale[6] λ_H, at frequencies $\omega \geqslant 1/\tau$ the electron part of the dielectric constant across the magnetic field is small in comparison with the lattice component ϵ_0 (we are assuming that the lattice dielectric constant is isotropic):

$$\epsilon_{xx} = \epsilon_{yy} \ll 4\pi \partial n/\partial \mathcal{E} e^2 \lambda_H^2/(\tau\omega) \sim \epsilon_0(\lambda_H^2/r_D^2)/(\tau\omega) \ll \epsilon_0. \tag{18}$$

Substituting the Hall components $\Lambda_{xy} = -\Lambda_{yx} = i4\pi\sigma_{xy}/\omega = i4\pi nec/H\omega$ into tensor (10), we see that these components are also inconsequential if $\hbar\sigma_{xy}/T\epsilon_0^{1/2} \ll c/V_F$ ($\sigma_{xy} = nec/H$ is the Hall conductivity). This inequality always holds under real experimental conditions. In a field of 1 T, at $T = 100$ K, with $n = 10^{16}$ cm^{-3} and $\epsilon_0 = 10$, for example, the left side is 0.3, while the right side is much greater than 300. Accordingly, localization effects should also be suppressed in semiconductors with a degenerate electron gas in a high magnetic field at $T \gg T_f = \hbar/\tau(\hbar V_F \epsilon_0/e^2)^{1/3}$.

Comparison with experimental data. Although the experimental information available on quasi-1D compounds is extensive, we have been unable to find evidence which would let us draw any conclusions about the validity of the assertion of this paper. The reason is that in most cases the lowering of the temperature causes phase transitions, which are accompanied by the opening of a gap in the electron spectrum. Furthermore, the condition $\tau \ll \tau_\perp$ apparently does not always hold.[2]

With regard to semiconductors in a high magnetic field, we note that an increase in the conductivity along the magnetic field has been observed[17] with increasing temperature in n-InSb and n-InAs under the condition $\mathscr{E}_F > \hbar/\tau$. This behavior is evidence that localization effects have an important influence on the conductivity. These results cannot be explained in terms of an interaction of electrons with phonons, since the time scale of the electron–phonon interaction, τ_{e-ph}, found from that interpretation turns out to be two orders of magnitude shorter than the time τ_{e-ph} found from other measurements.[17] It may be that the increase in the conductivity along the magnetic field with increasing temperature stems from a suppression of localization effects by fluctuations. Under these experimental conditions the interaction parameter is $e^2/\epsilon_0 \hbar V_F \simeq 0.4$–$0.8$. Here we have $\hbar/\tau(\hbar V_F \epsilon_0/e^2)^{1/3} \simeq 15$–$30$ K. The increase in the conductivity with increasing temperature occurs in specifically that temperature interval (5–20 K) in which the relation $T \sim T_f$ holds.

I wish to thank D. N. Shovkin for useful discussions.

APPENDIX. THE DIELECTRIC CONSTANT ϵ_{zz}

The dielectric constant ϵ_{zz} for $\omega < T/\hbar$ and $k_z < T/\hbar V_F$, in the absence of localization effects, can be found by solving the Boltzmann kinetic equation. When there is a spatial dispersion, the solution of the kinetic equation with the collision integral in the τ approximation does not satisfy the continuity equation $\partial\rho/\partial t + \mathrm{div}\, j = 0$ (ρ is the charge density, and j is the current density). The simplest collision integral which can be used is therefore the Bhatnagar–Gross–Krook collision integral[16]

$$-(\partial f/\partial t)_{sct} = -(f - f_0')/\tau. \tag{A1}$$

Here f is the electron distribution function, and f_0' is a Fermi distribution function with a Fermi energy $\mathscr{E}_F' = \pi^2 b^4 \hbar^2 n'^2/8m$. At small values of $\delta n = n' - n$ this function is $f_0' = f_0 + (2\mathscr{E}_F/n)(\partial f_0/\partial \mathscr{E})\delta n$, where f_0 is the equilibrium distribution function. Solving the kinetic equation as in the 3D case,[16] we easily find the imaginary part and the real part of ϵ_{zz}.

For $k_z \ll k_F/(\tau T)$ they are

$$\epsilon_{zz}' = \epsilon_{\|} - \frac{4\pi n e^2}{m}\frac{\omega^2 - k_z^2 V_F^2}{(\omega^2 - k_z^2 V_F^2)^2 + \omega^2/\tau^2}, \quad \epsilon_{zz}'' = \frac{4\pi n e^2}{m}\frac{\omega/\tau}{(\omega^2 - k_z^2 V_F^2)^2 + \omega^2/\tau^2}. \tag{A2}$$

For values of ω and k_z which satisfy $|\omega - k_z V_F| \leqslant 1/\tau$ we have $\epsilon_{zz}'/\epsilon_{zz}'' = (\omega^2 - k_z^2 V_F^2)\tau/\omega \leqslant 1$ if $\epsilon_{\|} \ll \epsilon_{zz}''$. Since the largest values of interest here are $k_z \sim T/\hbar V_F$, the condition $k_z \ll k_F/(\tau T)$ becomes

$$T \ll (\mathscr{E}_F \hbar/\tau)^{1/2}. \tag{A3}$$

For $k_z \gg k_F/(\tau T)$, under the condition $|\omega - k_z V_F| \leqslant 1/\tau$, the imaginary part of the dielectric constant is

$$\epsilon_{zz}'' \simeq 2\pi^2 n e^2 V_F/(\omega k_z T) = 4\pi^2 (n e^2/m\omega^2)\mathscr{E}_F/T. \tag{A4}$$

The real part is $\epsilon_{zz}' \ll \epsilon_{zz}''$, if $\epsilon_{\|} \ll \epsilon_{zz}''$.

[1]N. F. Mott and W. D. Twose, Adv. Phys. **10**, 107 (1961).
[2]L. P. Gor'kov, in *Electron–Electron Interaction in Disordered Systems* (ed. A. L. Efros and M. Pollak), North-Holland, Amsterdam, 1985.
[3]V. N. Prigodin and Yu. A. Firsov, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 241 (1983) [JETP Lett. **38**, 284 (1983)].
[4]O. N. Dorokhov, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 94 (1986) [JETP Lett. **38**, 284 (1983)].
[5]A. A. Abrikosov and I. A. Ryzhkin, Adv. Phys. **27**, 254 (1978).
[6]S. S. Murzin, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 228 (1987) [JETP Lett. **45**, 283 (1987)].
[7]D. G. Polyakov, in *Twentieth International Conference on the Physics of Semiconductors* (ed. E. M. Anastassakis and Joannopoulos), Vol. 3, World Scientific, Singapore, 1990, p. 2321.
[8]P. N. Argyres and E. N. Adams, Phys. Rev. **104**, 900 (1956).
[9]W. Apel and T. M. Rice, Phys. Rev. B **26**, 7063 (1983).
[10]L. Fleishman, D. C. Licciardello, and P. W. Anderson, Phys. Rev. Lett. **40**, 1340 (1978).
[11]L. Fleishman and P. W. Anderson, Phys. Rev. **21**, 2366 (1980).
[12]A. L. Burin and L. A. Maksimov, Zh. Eksp. Teor. Fiz. **95**, 1345 (1989) [Sov. Phys. JETP **68**, 776 (1989)].

[13]B. L. Altshuler, A. G. Aronov, D. E. Khmel'nitskii, Solid State Commun. **39**, 619 (1981); J. Phys. C **15**, 7367 (1982).
[14]E. M. Lifshitz and L. P. Pitaevskiĭ, *Physical Kinetics*, Pergamon Press, Oxford, 1981.
[15]A. I. Akhiezer *et al.* (editors), *Plasma Electrodynamics*, Nauka, Moscow, 1974.
[16]A. F. Aleksandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics*, Springer, New York, 1984.
[17]F. A. Egorov and S. S. Murzin, Zh. Eksp. Teor. Fiz. **94**(5), 315 (1988). [Sov. Phys. JETP **67**, 1045 (1988)].