Theory of a 2D Luttinger liquid

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Composite particles arise on two chains in a Luttinger liquid, as in the Chern–Simons theory of the fractional Hall effect. Anderson confinement corresponding to the formation of a 1D Luttinger liquid is linked with a pairing of composite particles which occurs along with a Kosterlitz–Thouless phase transition. The spectrum of excitations in the regions with confinement consists of the spectrum of a 1D Luttinger liquid and some additional two-particle excitations with a gap in the spectrum. These additional excitations correspond to phase-coherent two-particle hops from chain to chain.

The new properties of condensed matter corresponding to the quantum Hall effect and high-T_c superconductivity cannot be described by Fermi-liquid theory. It was recently shown that anyons, corresponding to a fractional statistics, and also Laughlin and Luttinger liquids can form in strongly correlated fermion systems. The Luttinger liquid has been studied thoroughly in 1D systems, primarily because of the bosonization methods discovered by Luther.[1]

In some recent papers, Anderson raised some ideas which link high-T_c superconductivity with a 2D Luttinger liquid. He showed in Ref. 2 (see also Refs. 3 and 4) that a confinement was possible on two chains and could lead to a localization of fermions on one of the chains, with the formation of a Luttinger liquid. Anderson argued that the confinement stemmed from a separation of the spin and charge degrees of freedom. In the present letter we show that confinement generally arises even if the spin degrees of freedom are ignored. Composite solitons form, in the way that composite bosons (a fermion-boson + an odd number of vortices) form in the fractional quantum Hall effect (Ref. 5, for example).

We thus consider a Hamiltonian which describes interacting fermions which are localized on two chains (for definiteness, we will speak in terms of an upper chain and a lower one):

$$H_0=\sum_\mu \int dx[-iv_F(\Psi^+_{1\mu}\partial_x\Psi_{1\mu}-\Psi^+_{2\mu}\partial_x\Psi_{2\mu})+\pi v_F g\rho_{1\mu}\rho_{2\mu}]$$

$$+\pi v_F g'\sum_\mu \int dx\rho_{1\mu}\rho_{2,-\mu}+t_\perp\sum_\mu\int dx(\Psi^+_{1\mu}\Psi_{1,-\mu}+\Psi^+_{2\mu}\Psi_{2,-\mu}). \quad (1)$$

Here the operators $\Psi_{1(2)\mu(x)}$ represent left (right) fermions, and the density operator is defined by $\rho_{j\mu}(x)=:\Psi^+_{j\mu}(x)\Psi_{j\mu}(x):$. Here v_F is the Fermi velocity, g and g' are interaction constants, t is the interchain hopping amplitude, and the index $\mu=+1(-1)$ specifies the upper (lower) chain.

Using bosonization methods,[1] we reduce the problem to a field theory with the action

$$S=\int dtdx\left[\frac{u}{2}((\partial_t\Phi)^2)+(\partial_x\Phi)^2)+\frac{2t_\perp}{\pi\alpha}\cos\frac{1}{2}\gamma\Phi\cos\frac{1}{2}\tilde{\gamma}\tilde{\Phi}\right],$$

where $\Phi(x,\tau)$ is the boson field which corresponds to the original fermion degrees of freedom, and $\tilde{\Phi}(x,\tau)$ is the field which is the dual of $\Phi(x,\tau)$: $d_\mu\tilde{\Phi}=\epsilon_{\mu\nu}d^\nu\Phi$, where $\epsilon_{\mu\nu}$ is the Levi–Civita tensor ($\epsilon_{12}=-\epsilon_{21}=1$, $\epsilon_{11}=\epsilon_{22}=0$), $\gamma=8\pi\gamma$, $\gamma=[64\pi^2(1-g/2+g'/2)/(1+g/2-g'/2)]^{1/4}$ and $u=v_F[1-(g-g')^2/4]^{1/2}$. The partition function Z of the original problem is determined in terms of a path integral over the field $\Phi(x,\tau)$.

We break up the action into two parts: one quadratic in $\Phi(x,\tau)$ and one which corresponds to interchain tunneling. The partition function corresponding to the first part of the action can be calculated at once. We use the last part as a perturbation. When we take an average over the field $\Phi(x,\tau)$ corresponding to a quadratic action, we reduce the calculation of Z to the problem of finding the partition function of a classical 2D Coulomb gas (Refs. 6 and 9, for example):

$$Z=\sum_{n=0}^{\infty}\frac{\tau_\perp^{2n}}{(2n)!}\int\frac{d^2\vec{x}_1}{\alpha^2}\int\frac{d^2\vec{x}_{2n}}{\alpha^2}\sum_\sigma\sum_{\tilde{\sigma}}$$

$$\times\exp(-K\sum_{i<j}\sigma_i\sigma_j l_{ij}-\tilde{K}\sum_{i<j}\tilde{\sigma}_i\tilde{\sigma}_j l_{ij}-i\sum_{i\neq j}\sigma_i\tilde{\sigma}_j\phi_{ij}). \quad (2)$$

Here $l_{ij} = \ln[|\vec{r}_i - \vec{r}_j|/\alpha]$ is the correlation of the field $\Phi(x,t)$, $\vec{r}_i = (x_i, t_i)$ is the vector specifying the position of the particle in the 2D space-time, $\tau_\perp$ is the effective amplitude for hopping from one chain to another, and α is a cutoff associated with the introduction of the boson picture. The constant $\widetilde{K}$ is $\widetilde{K} = 1/K = \gamma^2/8\pi$, and ϕ_{ij} is the Aharonov–Bohm phase. This phase plays the role played by the Chern–Simons term in the Ginzburg–Landau theory of the quantum Hall effect:[5]

$$\phi_{ij} = \tan^{-1}[(x_i - x_j)/u(\tau_i - \tau_j)].$$

Each particle has an electric charge σ_i and a magnetic charge $\tilde{\sigma}_i$. The total number of positive electric (magnetic) charges is equal to the total number of negative electric (magnetic) charges:

$$\sum_{i=1}^{2n} \sigma_i = \sum_{i=1}^{2n} \tilde{\sigma}_i = 0. \quad (3)$$

In other words, the plasma is on the whole neutral. A 2D Coulomb gas is known to undergo a (Berezinskiĭ–) Kosterlitz–Thouless transition, in the course of which the charges combine into pairs.[10,11]

In a completely analogous way, two Kosterlitz–Thouless transitions occur in our plasma. At the first, the electric charges of opposite signs pair up, forming a single-component plasma of magnetic charges ($+2, -2, 0$). Finally, at the second transition, the magnetic charges pair up, forming a single-component, electrically charged plasma with charges $+2$, -2, 0. The scaling equation for the hopping amplitude τ derived in first-order perturbation theory is

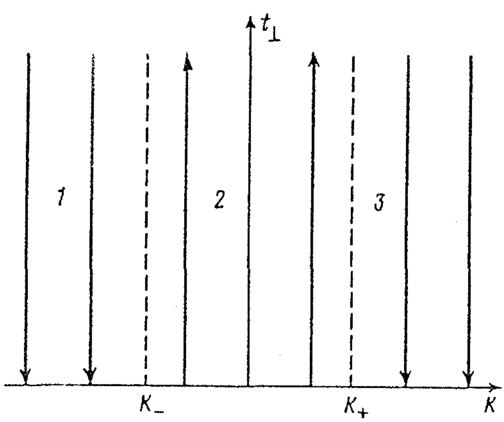

FIG. 1. (t, K) phase diagram. Regions *1* and *3* correspond to confinement, in which case the amplitude for one-particle hopping, t, is renormalized to zero. The arrows show the renormalization flux. The vertical lines corresponding to $K_+ = 2 + 3^{1/2}$ and $K_- = 2 - 3^{1/2}$ represent Kosterlitz–Thouless transitions. Region *2* corresponds to a situation in which a one-particle hopping exists. In this region, the Fermi surface becomes split in a manner reminiscent of the splitting of energy levels in a two-well potential. Region *1* corresponds to a pairing of magnetic charges and to a two-particle exchange between chains. Region *2* corresponds to a Coulomb plasma of unpaired particles. In region *3*, electric charges undergo a pairing, and there is a two-particle hopping.

$$d\tau/dl = (2 - 0.5K - 0.5/K)\tau, \tag{4}$$

where l is a scaling parameter. Figure 1 shows a phase diagram found from Eq. (4).

Let us establish the correspondence between the composite particles of the Coulomb gas and the original fermion problem. It turns out that the elementary particle of the Coulomb gas, which has an electric charge and a magnetic charge, corresponds to the hopping of a fermion from chain to chain. There are a total of four possible types of hops: of left fermions (1) from the upper chain to the lower one and (2) from the lower one to the upper one, and of right fermions (3) from the upper chain to the lower one and (4) from the lower one to the upper one. Each of these transitions corresponds to one combination of the electric charge σ and the magnetic charge $\tilde{\sigma}$: (1) $(+1, -1)$; (2) $(-1, +1)$; (3) $(+1, +1)$; (4) $(-1, -1)$.

We now consider the pairing of electric charges, i.e., a plasma of magnetic charges $+2$, -2, and 0. Neutral composites corresponding to a pairing of particles with the charge combinations $(1, -1)$ and $(-1, 1)$ correspond to a localization of the charge in the case in which left fermions are localized on the lower (upper) chain. A pairing of charges $(1, 1)$ and $(-1, -1)$ corresponds to a localization of right fermions on the upper (lower) chain.

In a completely similar way, upon the pairing of magnetic charges, with the formation of neutral composites, left fermions localize on the upper (lower) chain, and right fermions do the same on the lower (upper) chain. The formation of neutral composites thus corresponds to Anderson confinement.

The pairing of electric charges, with the formation of pairs of magnetic charges $+2$, -2, corresponds to a two-particle tunneling from the lower chain to the upper one and from the upper one to the lower one. The pairing of magnetic charges, with the formation of electrically charged pairs with charges of $+2$ and -2, corresponds to the hop of a left fermion from the lower chain to the upper one and the simultaneous hop of a right fermion from the upper chain to the lower one (or of a right fermion from the lower chain to the upper one and of a left fermion from the upper chain to the lower one).

We have thus shown that a Luttinger liquid on two chains exists and that it forms when carriers are confined on one of the chains. Composite solitons arise here, as in the formation of composite bosons in fractional statistics and in the fractional Hall effect.

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