Onset of chaos during the disruption of quasiperiodic regimes and transition through intermittence in a distributed generator with retardation

V. A. Kats and D. I. Trubetskov
Saratov State University

(Submitted 17 November 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 3, 116–119 (10 February 1984)

A system discussed previously by Anisimova *et al.* has been studied experimentally. Some mechanisms for the transition to chaos which are characteristic of simple dynamic systems have been observed: the destruction of quasiperiodic regimes with two and three incommensurate frequencies and a transition through intermittence.

Anisimova *et al.*[1] have studied a multimode generator with retardation in which the active element is a nonequilibrium medium: an electron beam and a traveling electromagnetic wave. They reported that, with increasing deviation from equilibrium, this generator would exhibit a transition to chaos which would have the characteristics of the Feigenbaum mechanism.[2] In this letter we report experiments which demonstrate the existence of two other mechanisms for a transition to chaos in a distributed system of this type. These two mechanisms are characteristic of simple dynamic systems.[3]

The band of amplified frequencies of the active element of the generator, Δf, is limited by a filter resonator with a quality factor $f_0/\Delta f = 200$, which allows continuous tuning of the central frequency f_0. The number of natural modes of the generator which fall in the amplification band and which form a quasiequidistant spectrum with a mode separation $1/T$, where T is the retardation time, is $\Delta f\, T \sim 10$. In the present experiments we varied three parameters of the system: f, the frequency of the generator mode which is excited initially (the working mode); $\sigma = T(f_0 - f)$, the frequency difference between the central frequency of the filter and the frequency of the working mode; and $\lambda = 10 \log(P_{en}/P_{ex})$, where P_{en} and P_{ex} are the wave power levels at the entrance to and the exit from the active element. As the control parameter we used the feedback level; the other parameters serve as parameters of the initial conditions. To monitor the spontaneous oscillation regimes we measured the power spectrum and temporal realization of the envelope of the generated signal. We also measured a two-dimensional projection of the phase diagram of the system.

As the parameter λ is increased at a constant beam current and accelerating voltage, mode competition in the system gives rise to self-excited sinusoidal oscillations of constant amplitude at the frequency of the mode with the highest linear growth rate. With increasing deviation from equilibrium of the system, at $\lambda = \lambda_1$, this steady state gives way to a periodic regime whose amplitude increases in proportion to $\sqrt{\lambda - \lambda_1}$: A self-modulation of period $2T$ arises. The modulation spectrum for this

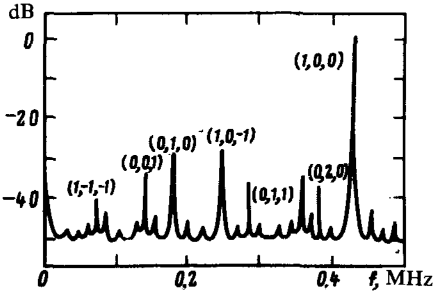

FIG. 1. Power spectrum of the envelope in the regime of a three-frequency quasiperiodic self-modulation.

process typically contains a large number of harmonics of the fundamental frequency $f_1 = 1/2T$, demonstrating that the process is very nonlinear.

A further increase in λ is accomplished by the successive "soft" appearance in the modulation spectrum of two more frequencies, f_2 and f_3, and their linear combinations $[m_1 f_1 + m_2 f_2 + m_3 f_3$; see Fig. 1, where the coefficients in the linear combination (m_1, m_2, m_3) are shown in parentheses for the statistically significant spectral peaks over the frequency range from 0 to f_1]. These perturbations of the amplitude with incommensurate frequencies f_1, f_2, f_3 may be thought of as new (modulational) modes of the nonlinear system, which are different from the system of natural linear modes. At $\lambda = \lambda_{cr}$ the discrete components in the spectrum abruptly convert into diffuse peaks with a typical "noisy" pedestal. This pedestal simultaneously rises as the peaks broaden with increasing λ. The three-frequency quasiperiodic self-modulation which results from three sequential Hopf bifurcations[4] is thus disrupted as the deviation from equilibrium increases, and self-excited stochastic oscillations with a continuous spectrum appear in the system (the mechanism of Ref. 5).

The three-frequency quasiperiodic self-modulation is stable and is detected experimentally in a narrow interval of the initial-condition parameters. As the frequency separation σ is varied at a fixed frequency of the working mode, the third Hopf bifurcation is not observed in the system. With increasing λ, the periodic self-modulation (Fig. 2a) gives way to quasiperiodic self-excited oscillations with two incommensurate frequenceis, f_1 and f_2 (Fig. 2b). As λ is increased, transition from two-frequency quasiperiodic self-excited oscillations to stochastic oscillations occurs through a regime in which two modes with incommensurate frequencies are locked. This locking regime is accompanied by (depending on the value of σ) either the disappearance or the loss of stability of the periodic self-modulation regime that has arisen. The instability results from a sequence of period-doubling bifurcations. In the former case, the quasiperiodic oscillations are disrupted as a result of a "phase capture" of two modes: the frequencies come to form a rational ratio $f_1/f_2 = 2.33 \pm 0.1 = 7/3$, which corresponds to a self-modulation of period $1/f_L$, where $f_L = f_1/7 = f_2/3$ (Fig. 2c). On the phase diagram of the system, the quasiperiodic self-excited oscillations take the form of an invariant two-dimensional torus (see Fig. 2b; the phase trajectory cuts out a ribbon of finite width on the surface of the torus, since the torus winding period, $1/f_2$, is larger than the period of the original cycle, $1/f_1$). When the two modes lock, we have a

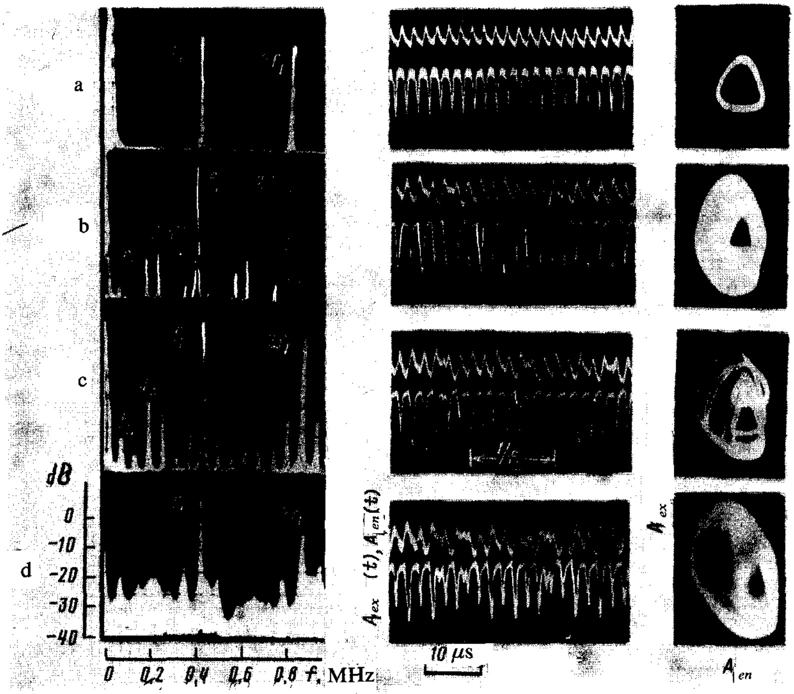

FIG. 2. Transition to chaos through a locking regime ("phase capture") of the two-frequency quasiperiodic self-excited oscillations.

degeneracy corresponding to a resonance at a 3:7 torus. These modes are accompanied experimentally by a hysteresis of $\lambda \pm 0.05$ dB, which is characteristic of the "hard" creation of a cycle on a torus (Fig. 2c). As λ is increased, stochastic self-excited oscillations arise abruptly in the system at $\lambda = \lambda_{\rm cr}$; they are seen on the phase diagram as a strange attractor (Fig. 2d). In the latter case, two adjacent peaks, f_2 and $(f_1 - f_2)$, in the quasiperiodic modulation spectrum (Fig. 2c) move closer together as λ increases (the frequency ratio f_1/f_2 decreases), and at some $\lambda = \lambda_s \pm 0.1$ dB these two peaks rapidly collapse to form a single narrow peak at the frequency $f_1/2$, which corresponds to a resonance on a 1:2 torus. The quasiperiodic self-excited oscillations are disrupted. We find a regime of a periodic modulation with $4T$, which is rendered stochastic with increasing deviation from equilibrium as a result of a sequence of period-doubling bifurcations (experimentally, two bifurcations are detected before the onset of chaos).

If the self-excited oscillations in the system retain their regular nature as the initial-condition parameters are varied, at values of λ above the critical value $\lambda_{\rm cr}$, then there is a transition to chaos in the system which is fundamentally different from the transitions described above: There is a transition through intermittence.[6] With increasing λ, the regime of determinate self-excited oscillations gives way at $\lambda = \lambda_i$ to a self-excited oscillatory regime with random and infrequent bursts of chaos, which are separated by long intervals of order. The duration of the chaos and order phases in a temporal realization of this process is random in nature. With $|\lambda - \lambda_i|$, the average duration of the chaos phase increases, and they become more frequent. The regular

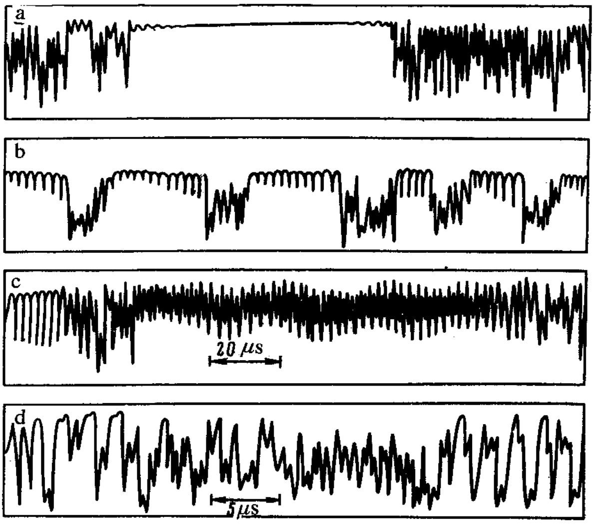

FIG. 3. Oscilloscope traces of the envelope illustrating the intermittent chaos.

regime gradually disappears, and self-excited stochastic oscillations arise in the system. Depending on the initial-condition parameters, the experiments reveal transitions to chaos through intermittence from regimes of steady-state oscillation (Fig. 3a), a periodic self-modulation (Fig. 3b), and a quasiperiodic self-modulation (Fig. 3c). Furthermore, we observe a chaos-chaos intermittence between two regimes of stochastic self-excited oscillations which are produced as a result of the loss of stability by the cycles of different periods (Fig. 3d).[7]

As in Ref. 1, we observe a transition to chaos through a sequence of Feigenbaum period-double bifurcations (we find four bifurcations). We found experimental values for Feigenbaum's constants[2]: $\delta = 4.23 \pm 0.1$ and $\alpha = 2.51 \pm 0.1$.

We wish to emphasize that the system which we have studied demonstates a large variety of transitions to chaos, including all those which have been identified for hydrodynamic systems.[8]

We wish to thank Yu. A. Zyuryukin for assistance in the preparation for the experiments.

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Translated by Dave Parsons
Edited by S. J. Amoretty