Fractional quantum Hall effect from the viewpoint of magnetic symmetry

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The plateau in the Hall conductivity at rational Landau-level filling factors can be qualitatively understood from the point of view of the symmetry of the electron states in an external magnetic field.

The fractional and quantized Hall effect, which was discovered by Stormer *et al.*,[1] was immediately thereafter explained under the assumption that for rational Landau-level filling factors (see below), the two-dimensional system of interacting electrons forms a "stable liquid." The stability is guaranteed by two factors: the gap in the spectrum of electronic excitations and incompressibility. Furthermore, in the series of papers by Laughlin,[2] Haldane,[3] and Halperin,[4,5] an uniquely beautiful scheme was proposed for constructing the specific wave functions of such a stable liquid, which permits describing a large number of properties of the spectrum of excited states. In this paper I want to show that the gross property of the liquid—its stability—can be understood without any calculations based on the symmetry of the states of an electron in an external magnetic field, an idea which was developed in 1964 by Zak and Brown (see, for example, the review by Brown[6]).

According to Zak and Brown, the states of both the free and the interacting two-dimensional system of electrons, which are placed in a strong, perpendicular magnetic field $\mathbf{H}$ and which belong to a single Landau level, can be classified according to the representations of the symmetry of the two-dimensional magnetic crystal with periods $\mathbf{a}_1$ and $\mathbf{a}_2$, if the volume of its unit cell $v = [\mathbf{a}_1\mathbf{a}_2]$ satisfies the rationality relation

$$\frac{eH}{2\pi\hbar c}\, v = \frac{p}{r}, \tag{1}$$

where p and r are prime integers. The states are described by the usual quasimomenta k_1 and k_2, and the size of the Brillouin zone depends on the parity of the number p: it is $2\pi^2/vr^2$ for odd p and $4\pi^2/vr^2$ for even p. It is extremely interesting that all r states are multiply degenerate. This is a result of the infinite degeneracy of a Landau level for free electrons.

If the actual state of the system of interacting electrons is indeed a Wigner crystal with periods $\mathbf{a}_1,\mathbf{a}_2$ and electron density $n = 1/v$, then from what was said above it follows that such a crystal will be a dielectric: depending on the parity of p, either the first $2r$ or the first r bands are completely filled, while the electronic excitations reveal a gap. This type of Wigner crystal has been examined by many investigators, beginning with Lozovik and Yudson.[7] However, the fractional Hall effect is apparently missing in this state (see, for example, Ref. 8).

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The Wigner crystal, however, may turn out to be energetically unfavorable. Its energy in this case can be lowered without changing its density (!) or, equivalently the filling factor of Landau levels

$$\nu = n \Big/ \frac{eH}{2\pi \hbar c} = \frac{r}{p}, \tag{2}$$

by adding a pair of dislocations (and, perhaps, disclinations as well) until the crystal no longer melts.[1)] After melting, the classification of states with respect to quasimomenta k_1 and k_2 is no longer meaningful. However, it may be expected that the gaps in the electronic densities of states will not close.

If the density of the system does not satisfy quantization condition (2), then $1/n \neq v = [\mathbf{a}_1 \mathbf{a}_2]$ in the starting Wigner crystal and the substance remains a "metal" with a gapless spectrum. The quantization condition (2) indicates essentially that the dependence of the density and ν on the chemical potential is a devel's staircase.

We note finally that the theory of symmetry makes one non-gross prediction: the electronic excitations must be r-fold degenerate with r from (2). It is also interesting to what extent the above-mentioned r (or $2r$) gaps (bands) in the spectrum remain after melting. An argument supporting the last assertion is that it does not depend on the specific choice of periods $\mathbf{a}_1, \mathbf{a}_2$ of the parent crystal and is determined solely by the density, i.e., again by Eq. (2).

[1)]This mechanism of melting of a two-dimensional crystal was examined by Nelson and Halperin.[9]

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