Magnetoresistance and Hall effect in tunneling-coupled quantum wells with asymmetric scattering

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A semiclassical description of the energy spectrum is not valid for tunneling-coupled quantum wells whose relaxation times τ_l and τ_r are different, because the 2×2 Green's function cannot be diagonalized in the case $\tau_l \neq \tau_r$. Quantum-mechanical expressions are accordingly derived for the magnetoresistance and the Hall effect even in weak magnetic fields ($\omega_c \tau_{l,r} < 1$, where ω_c is the cyclotron frequency).

Tunneling-coupled electron states in double quantum wells have been studied by optical methods and also on the basis of the "resistance resonance" which arises when the scattering in the left well (l) and that in the right one (r) are asymmetric.[1–4] This resonance arises at $\Delta \simeq 0$, where Δ is the splitting of the levels of the double quantum wells at $T=0$, where T is the tunneling matrix element. This effect occurs because the electron densities in the l and r wells are independent of Δ (a transport in real space) and also because the scattering probability is changed by a tunneling-induced mixing of states. If the tunneling frequencies $T/\hbar$ are smaller than the difference between the relaxation frequencies of the l and r wells, $\nu \equiv (\tau_l^{-1} - \tau_r^{-1})$ (we are assuming $\tau_l < \tau_r$), however, scattering processes suppress the tunneling-induced superposition of the states of the wells, and a semiclassical description of the energy spectrum is incorrect. The peak representing the resistance resonance changes in shape. In addition, galvanomagnetic effects in weak magnetic fields ($\omega_c \tau_{l,r} < 1$) are described by the quantum-mechanical expressions derived below.

In describing the electron states in the double-quantum-well structure, we use the basis of orbitals of the l and r quantum wells. The overlap of these orbitals determines the tunneling matrix element T, which decays exponentially with the barrier thickness.[5] Considering a double-well structure in a perpendicular magnetic field, and considering scattering by statistically independent random potentials of the l and r wells, $U_l(\mathrm{x})$ and $U_r(\mathrm{x})$, we find the following 2×2-matrix Hamiltonian in this representation:

$$\frac{\pi^2}{2m} + \frac{\Delta}{2}\hat{\sigma}_z + T\hat{\sigma}_x + U_l(\mathrm{x})\hat{P}_+ + U_r(\mathrm{x})\hat{P}_-. \tag{1}$$

Here $\vec{\pi}$ is the kinematic momentum, $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are the Pauli matrices, and $\hat{P}_\pm = (1 \pm \hat{\sigma}_z)/2$ are projection operators which project onto the states of the l and r wells, with equal effective masses m. A diagram expansion of the one-particle Green's function for this model contains only scattering contributions which are proportional to $\hat{P}_\pm$. In the Born approximation, in the region of classical magnetic fields, $\hbar\omega_c < \bar{\epsilon}$ ($\bar{\epsilon}$ is the average electron energy[1)]), the retarded Green's function in the translationally invariant representation is determined by the 2×2 matrix

$$\left[\epsilon_p + \frac{\Delta}{2}\hat{\sigma}_z + T\hat{\sigma}_x - E - \frac{i\hbar}{2}(\hat{P}_+/\tau_l + \hat{P}_-/\tau_r)\right]^{-1}, \tag{2}$$

in which $\epsilon_p = p^2/2m$, $\mathbf{p}$ is the 2D momentum, and the relaxation times τ_l and τ_r are independent of $\mathbf{p}$ and the energy E for δ-correlated random potentials. We have omitted from (2) a small renormalization of ϵ_p and Δ, and we have also ignored some higher-order contributions (terms containing $\bar{\sigma}_{x,y}$). Incorporating those contributions complicates the analysis of the diagram expansion for the conductivity without changing the results.

The matrix with $\tau_l \neq \tau_r$, obtained in the denominator of (2), does not commute with its Hermitian conjugate matrix and the Green's function cannot be diagonalized by any unitary transformation.[6] The usual picture of the energy spectrum of slowly decaying quasiparticles therefore cannot be introduced in the double-quantum-well structure with asymmetric scattering at $T \simeq \Delta, \hbar\nu$, and a quantum analysis of the kinetic phenomena is necessary even at large Fermi electron energies, $\epsilon_F \gg T, \Delta$.

We calculate the conductivity tensor in the ladder approximation (ignoring localization corrections). We take into account the matrix structure of (2) and of the scattering potentials. For the model of scattering by δ-correlated potentials, the transport relaxation times are the same as the outgoing times $\tau_{l,r}$, and the integral equations for the Green's function describing the linear response convert into algebraic equations. Such equations can be summed over $\mathbf{p}$. As a result, we find a closed equation for the 2×2 matrix $\hat{\Sigma}_{\alpha\beta}$. The trace of this matrix determines the conductivity tensor $\sigma_{\alpha\beta}$:

$$\sigma_{\alpha\beta} = \sigma_R \mathrm{tr} \hat{\Sigma}_{\alpha\beta}, \qquad \sigma_R \equiv \frac{e^2 n}{m} \tau, \tag{3}$$

Here n is the total electron density in the double-well structure, $\tau^{-1} = (\tau_l^{-1} + \tau_r^{-1})/2$ is the average relaxation frequency, and σ_R determines the conductivity in the case of a pronounced tunneling-induced mixing of l and r states. In this case the probabilities for scattering by inhomogeneities of the l and r wells add together. It is convenient to work with the 2×2 flux matrix $\hat{\mathbf{W}} = \hat{\Sigma}\mathbf{e}$ (where $\mathbf{e}$ is the unit vector along the electric field in the plane of the double-well structure) in place of $\hat{\Sigma}_{\alpha\beta}$. For this flux matrix we find a quantum generalization of the momentum balance equation:

$$[\hat{\mathbf{W}} x \vec{\omega}_c] - \frac{i}{\hbar}\left[\hat{\mathbf{W}}, \frac{\Delta}{2}\hat{\sigma}_z + T\hat{\sigma}_x\right]_- + \frac{\hat{\mathbf{W}}}{\tau} + \frac{\nu}{4}\left[\hat{\mathbf{W}}, \hat{\sigma}_z\right]_+ = \frac{\mathbf{e}}{2\tau}, \tag{4}$$

where $\vec{\omega}_c = (0,0,\omega_c)$, $[...]_-$ and $[...]_+$ are the commutator and anticommutator of the 2×2 matrices. The usual semiclassical approximation[3,4] is found in the case $\sqrt{(\Delta/2)^2 + T^2} \gg \hbar\nu/2$, after a diagonalization of $(\Delta/2)\hat{\sigma}_z + T\hat{\sigma}_x$. In this case, Eq. (4) becomes the momentum balance equation for the symmetric and asymmetric states of a double-quantum-well structure, and the nondiagonal part of the flux matrix $\hat{\mathbf{W}}$ can be ignored.

We can write a general solution of (4) for values of $\hbar\nu$ comparable to the distance between levels by transforming to circular coordinates. For the components of the conductivity tensor $\sigma_d = \sigma_{xx} = \sigma_{yy}$ and $\sigma_\perp = \sigma_{xy} = -\sigma_{yx}$ we find

$$\begin{pmatrix} \sigma_d \\ \sigma_\perp \end{pmatrix} = \frac{\sigma_R}{2\tau} \int_{-\infty}^{0} dt e^{t/\tau} \begin{pmatrix} \cos\omega_c t \\ \sin\omega_c t \end{pmatrix} \mathrm{tr}\exp\left[t\left(\frac{\nu}{4}\hat{\sigma}_z + i\frac{\Delta}{2}\hat{\sigma}_z + iT\hat{\sigma}_x\right)\right]$$

$$\times \exp\left[t\left(\frac{\nu}{4}\hat{\sigma}_z - i\frac{\Delta}{2}\hat{\sigma}_z - iT\hat{\sigma}_x\right)\right]. \tag{5}$$

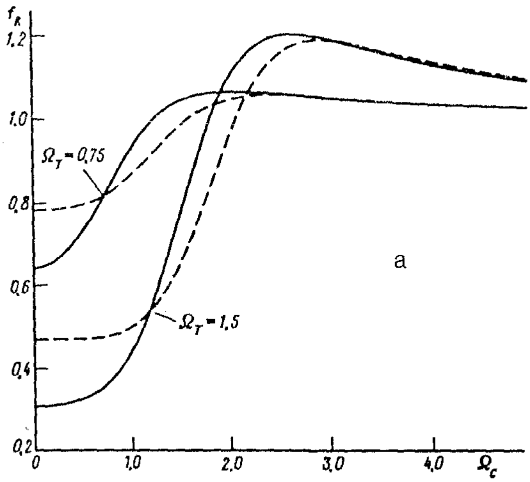
We can then immediately calculate the trace and carry out the integration in (5). As a result, we find

$$\begin{pmatrix} \sigma_d \\ \sigma_\perp \end{pmatrix} = \frac{\sigma_R}{2} \begin{cases} \Psi(1 + i\Omega_c) + \Psi(1 - i\Omega_c) \\ [\Psi(1 + i\Omega_c) - \Psi(1 - i\Omega_c)]/i \end{cases},$$

$$\Psi(s) = s^{-1}\left[1 + \mu^2 \frac{s^2 + \delta^2}{(s^2 - \mu^2)(s^2 + \delta^2) + s^2\Omega_T^2}\right]. \tag{6}$$

The dimensionless cyclotron frequency Ω_c, the dimensionless tunneling frequency Ω_T, the level splitting δ, and the extent of asymmetry in the scattering, μ (the value $\mu = 0$ corresponds to identical scattering in the wells; the value $|\mu| = 1$ corresponds to scattering in only one of the wells), are introduced in (6) by means of

$$\Omega_c = \omega_c \tau, \quad \Omega_T = 2T\tau/\hbar, \quad \delta = \Delta\tau/\hbar, \quad \mu = \frac{\tau_r - \tau_l}{\tau_r + \tau_l}. \tag{7}$$



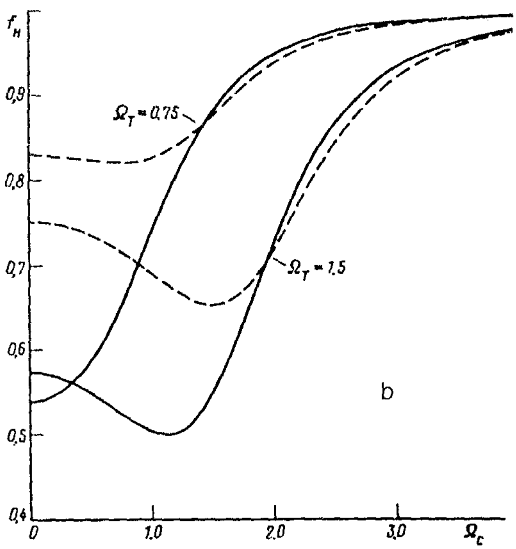


FIG. 1. Magnetic-field dependence modulating (a) the classical magnetoresistance and (b) the classical Hall constant (the functions f_R and f_H). Solid lines—resonance, $\delta = 0$; dashed lines—nonresonant case, $\delta = 1$.

From (6) we find the following results for the dimensionless resistance $\rho\sigma_R$ and the dimensionless Hall constant R/R_0 ($= -1/|e|nc$):

$$\rho\sigma_R = 1 - \frac{\mu^2}{1+\Omega_c^2} f_R(\Omega_c|\delta, \Omega_T), \quad R/R_0 = 1 + \frac{\mu^2}{1+\Omega_c^2} f_H(\Omega_c|\delta, \Omega_T). \tag{8}$$

The functions f_R and f_H, shown in Fig. 1, are equal to one in the classical limit. For the double-quantum-well structures studied in Ref. 2, with $\Omega_T \simeq 1.6$, these modulating factors are quite different from one, but the magnetic-field dependence in (8) would be weak because of the value $\mu \simeq 0.13$. For the double-quantum-well structures which were studied in Ref. 4, with a greater asymmetry in the scattering, the plots of (8) are not monotonic. The relation $R/R_0 > 0$ holds. The sign of the magnetoresistance $(\rho - \rho_{|\omega_c = 0})$ becomes negative with increasing δ, at $\Omega_T > 1.37$, even if $\Omega_c \ll i$.

The qualitative distinction between (8) and the classical dependence for $\Omega_c < 1$ demonstrates the onset of macroscopic quantum-mechanical effects in a double-quantum-well structure with asymmetric scattering (the frequency dispersion of the conducitivity, the nature of the relaxation of the populations in the wells, and other properties also change in such a structure).

[1)]Under the assumption that $\bar{\epsilon}$ is on the order of the spreading of the Fermi distribution, we are ignoring here not only the quantum Hall effect but also magnetooscillation phenomena.

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