Macroscopic Josephson effect in superfluid ^{3}He-B

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Recently an experiment with a single vortex line of macroscopic length in superfluid ^{3}He-B has been reported. In this superfluid the periodic motion of the vortex was observed with a period of several minutes.[1] We discuss this periodic process in terms of the macroscopic ac Josephson effect and derive an expression for the period T, which is defined by the geometry and by the circulation quantum $\kappa = h/2m_3$.

The vibrating-wire technique, which is used to measure quantized circulation of superfluid velocity in superfluid ^{4}He,[2] has been recently applied to ^{3}He-B. As a result, the circulation quantum $\kappa = h/2m_3$ trapped by the wire, when the wire absorbs one vortex filament, has been measured.[3] In further experiments, the transient process of untrapping of circulation from the wire has been observed.[1] In this process a segment of quantized vortex line "unzips" from the wire. This allowed us to investigate the dynamics of a single vortex. It appears that the unzipped vortex segment performs a precessing motion around the wire with the stable period of precession, $T = 253$ s. We interpret this periodic motion as the macroscopic manifestation of the ac Josephson effect, in which the corresponding voltage is produced by the hydrodynamic energy of the trapped circulation, while the phase slip occurs due to the precession of the vortex segment. We show that the period T can be obtained from the Josephson relation and is defined by the geometry and the circulation quantum. For the given geometry of the experiment, we obtain $T = 253.2$ s.

The geometry of the experiment[1] is shown in Fig. 1: a wire of radius $R_w = 8\ \mu$m is inside a cylindrical cell of radius $R_c = 1.48$ mm at a distance $\Delta R = 0.35$ mm from the axis of the cylinder. In the transient process one quantum of circulation κ of velocity is trapped by the wire at $z < z_0$, while at $z = z_0$ a vortex filament is unzipped from the wire and terminates at the wall of the vessel. We consider here the general case of the vortex segment with $N_2 - N_1$ quanta, whose termination point z_0 on the wire separates the upper part of the wire with N_2 trapped quanta from the lower part with N_1.

First, we consider the dynamics of the vortex segment. Since the temperature is quite low, $\sim 0.2T_c$, we ignore the dissipation. To find the precession rate of the segment around the wire, one can use Newton's second law applied to the whole vortex segment:

$$dP/dt = F \quad . \tag{1}$$

Here P is the z projection of the linear momentum of the vortex line. It is a multi-

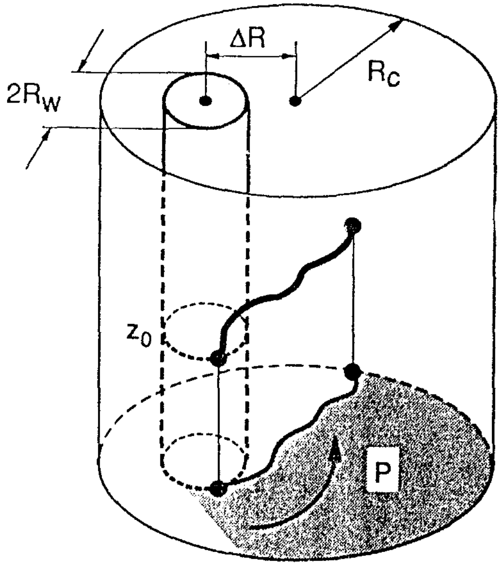

FIG. 1. Sketch of the vortex (heavy solid line) precessing around the wire. The vortex momentum P increases under the hydrodynamic force produced by the trapped flux. As a result, the projection of the vortex onto the transverse plane sweeps across the plane, producing a periodic phase slip.

valued quantity expressed through the variation of the area swept by the projection of the vortex line on the cross-sectional plane: $\delta P = (N_2 - N_1)\rho\kappa\delta S$.[4]

The external force F comes from the difference in the hydrodynamic energy of superflow around the wire, induced by the difference in trapped circulation above and below the vortex segment: $F = E(N_2) - E(N_1)$. In the case where both R_c and R_w are larger than the coherence length, the energy $E(N)$ is given by the classical expression $(1/2)\int dS\, \rho \vec{v}_S^2$. The integral is over the cross section of the vessel far above or far below the vortex segment, where the superfluid velocity field $\vec{v}_S = (\kappa/2\pi)\vec{\nabla}\Phi$ around the wire is not disturbed by the presence of the vortex line. Under these conditions the calculation of $E(N)$ corresponds to the calculation of the electric capacity of two noncoaxial cylinders, which gives:

$$E(N) = \rho N^2 \frac{\kappa^2}{4\pi} \cosh^{-1} \frac{R_w^2 + R_c^2 - (\Delta R)^2}{2R_w R_c} \tag{2}$$

Under the force F the momentum P increases with time, which means that the vortex segment is forced to precess, sweeping the whole cross-sectional area, $S = \pi(R_c^2 - R_w^2)$ in one period T. The change in the momentum during the precession, $P(t+T) - P(t) = (N_2 - N_1)\rho\kappa S$, should be equal to FT, which gives for T the equation

$$T = \frac{(N_2 - N_1)\rho\kappa S}{E(N_2) - E(N_1)} \tag{3}$$

The period is thus determined by the circulation quantum κ, by the numbers N_1 and N_2 of the trapped flux, and by the geometry. It does not depend on the particular features of the vortex precession, i.e., on the shape of the vortex line or on its core structure. Inserting the values of R_c, R_w, and ΔR, we obtain $T = |N_1 + N_2|^{-1} \times 253.2$ s, which coincides with the experimental value $T = 253 \pm 1$ s, obtained for the case $N_1 = 1$, $N_2 = 0$. The theoretical result presented in Ref. 1 is slightly different: It was assumed in Ref. 1 that the vortex segment moves as a solid body, which is valid only if the wire is not shifted from the axis of the vessel, i.e., if $\Delta R = 0$. We used here the less restrictive assumption that the motion is periodic.

This process bears all the features of the ac Josephson effect. The constant force F is applied to the liquid, which results in the time-dependent (periodic) motion of the vortex line. One can represent this force in the familiar form of the difference in the chemical potentials, which play the role of voltage in the electrically neutral liquid. The force per unit area of the vessel F/S plays the part of the pressure difference applied between the top and the bottom walls of the vessel. Divided by the particle density ρ/m_3, it gives the effective difference in the chemical potentials,

$$\mu_2 - \mu_1 = \frac{Fm_3}{\rho S} = \frac{m_3(E(N_2) - E(N_1))}{\rho S} . \tag{4}$$

According to Eq. (3), the frequency of the periodic process, $w = 2\pi/T$, is related to this voltage by the Josephson relation

$$\omega = \frac{2}{\hbar}(\mu_2 - \mu_1) \quad . \tag{5}$$

This is not surprising since the vortex, which sweeps the cross section of the vessel, realizes the periodic phase-slip process: After each period T, the phase difference $\Phi_2 - \Phi_1$ between the top and the bottom walls changes by 2π due to the vortex motion, which compensates for the change caused by the difference in the chemical potentials. The kinematic phase slip equation is[5]

$$\partial_t \vec{v}_S + \nabla \mu = \kappa(N_2 - N_1) \oint d\sigma \;\; \vec{r}_\sigma \times \dot{\vec{r}} \;\; \delta^3(\vec{r} - \vec{r}(\sigma, t)) \quad ,$$

$$\mu = \frac{1}{2}\vec{v}_S^2 + \frac{\partial \varepsilon(\rho)}{\partial \rho} . \tag{6}$$

The integration of this equation over the volume of the vessel and over period T under condition that there is no difference in the mass density ρ between the upper and the lower parts of the container gives

$$\int_0^T dt \int d^3 r \partial_t \vec{v}_S = 0 \quad , \tag{7}$$

$$\int_0^T dt \int d^3 r \nabla \mu = T\hat{z} \frac{E(N_2) - E(N_1)}{\rho} \quad , \tag{8}$$

$$\int_0^T dt \int d^3r\kappa(N_2 - N_1) \oint d\sigma \ \vec{r}_\sigma \times \dot{\vec{r}} \ \delta^3(\vec{r} - \vec{r}(\sigma, t))$$

$$= \kappa(N_2 - N_1) \int d\sigma \ dt \ \vec{r}_\sigma \times \dot{\vec{r}} = \kappa(N_2 - N_1)S\hat{z} \ . \tag{9}$$

Here $\vec{r}(\sigma,t)$ denotes the position of the vortex line, and σ is the coordinate along the line with

$$\vec{r}_\sigma = \partial_\sigma \vec{r}(\sigma, t) \quad , \quad \dot{\vec{r}} = \partial_t \vec{r}(\sigma, t) \quad ,$$

S is the cross-sectional area of the vessel, which is swept by the vortex during one period. From Eq. (6) it follows that Eq. (8) is the same as Eq. (9). As a result, we obtain Eq. (3) for the precession period.

Hence the precession frequency can be derived directly from the Josephson equations (5) and (4), without using the motion equation for the vortex [Eq. (1)]. The Josephson relation, of the kinematic origin, does not depend on the particular features of the vortex dynamics. In particular, it is not important if the vortex is frozen into the liquid, i.e., if it moves with the local velocity, or if it has its own mass. The Josephson equation is so general that it can be applied even when the hydrodynamic equations, which define the vortex motion, do not hold, for example, in the case where R_w is of the order of, or even less than, the coherence length. Even in this case Eqs. (5) and (4) hold, but the energy $E(N)$ of the trapped flux is not given solely by the hydrodynamic energy of the superflow: one should also add the energy of the distortion of the superfluid state in the layer of the coherence length near the wire. Note that the precession frequency ω changes if an external pressure or temperature difference is applied, in addition to the internal "voltage" caused by the trapped circulation.

In summary, we have shown that the experiment reported in Ref. 1 represents the observation of the quantum Josephson effect on the macroscopic scale, where the phase slip is realized by the extremely slow precessing motion of a single vortex line.

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