Critical behavior of the conductivity of a medium with superconducting inclusions

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The critical behavior of the electrical conductivity near the threshold for percolation through the superconducting phase of a conducting medium with superconducting inclusions has been studied. Test samples of amorphous GaSb were used to model this medium. The critical exponent was found to be $q = 1$. This result can serve as experimental confirmation of the Shklovskiĭ–Éfros theory. Deviations from a power law are observed in a narrow region near the percolation threshold. These deviations are apparently caused by weak links between inclusions.

1. Among the various problems which have been taken up in the three-dimensional ($d = 3$) theory of percolation, that which has received the most study, both theoretical and experimental, is the conductivity σ of a medium which contains insulating ($\sigma = 0$) inclusions.[1,2] Denoting by n the volume fraction of the inclusions, we have

$$\sigma = \sigma_0\left(1 - \frac{n}{n_c}\right)^t \equiv \sigma_0\tau^t \tag{1}$$

in the critical region, where $\sigma_0 \sim \sigma(n = 0)$, and n_c is the percolation threshold (in the three-dimensional case, it is[1] $n_c \approx 0.17$). The corresponding problem for a conducting medium containing superconducting inclusions has been studied much less thoroughly. There have been suggestions[1–4] that the conductivity becomes infinite as $n \to n_c$ in accordance with

$$\sigma = \sigma_0\tau^{-q}, \tag{2}$$

where we would have $q \neq t$ in the three-dimensional case. From the theoretical standpoint, it is possible to find the critical exponent t in the $d = 3$ case if we know the critical exponent of the correlation length of an infinite cluster, ν: $t = 2\nu \approx 1.7$. The expression for q, on the other hand, contains an additional, and independent exponent.[1,3,4] In the pioneering study by Éfros and Shklovskiĭ,[1] the value $q = 1$ was suggested on the basis of numerical calculations. Somewhat later, Coniglio and Stanley[3] proposed the following formula on the basis of an alternative concept, of an "unscreened perimeter" of a superconducting cluster:

$$q = \nu(1 - (d - d_f)/2) \equiv \nu(d_f/2 - 0,5). \tag{3}$$

Here d_f is the fractal dimension of the superconducting cluster ($d_f \leqslant 3$). Using the

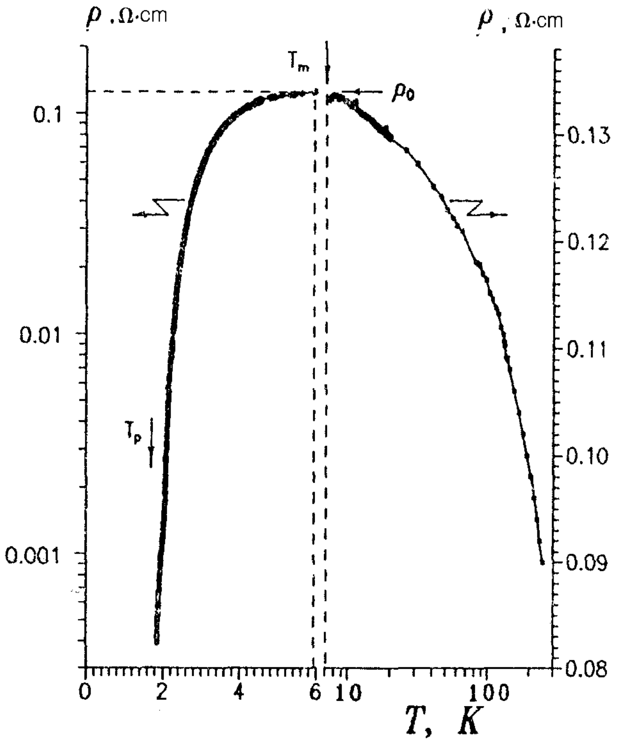

FIG. 1. Temperature dependence of the resistivity and plot of the superconducting transition of a-GaSb samples. The measurement current was $I = 0.5$ mA.

value $d_f \approx 2.54$ for a percolation cluster and also the value $\nu = 0.88$ (Ref. 5), we find $q = 0.68$.

To the best of our knowledge, no information is presently available on the value of q in real entities, and it is quite difficult to choose the better model (that of Ref. 1 or that of Ref. 3). Our purpose in the present study was to examine this question experimentally.

2. As test samples we selected bulk samples of amorphous gallium antimonide, a-GaSb, which were synthesized by quenching at high pressure. We have shown elsewhere[6] that inclusions of a nonstoichiometric amorphous phase, Ga_xSb_{1-x} with $x > 0.5$, arise in the interior of a-GaSb samples under certain conditions (for certain synthesis temperatures and pressures). These inclusions exhibit a superconductivity. The typical size of these inclusions is[6] ~ 250 Å. Variations in the value of x lead to variations in the superconducting transition temperature T_c, so the superconducting transition of a-GaSb is stretched out over a wide range (Fig. 1). First, at $T = T_m \sim 7$ K, the resistance of the inclusions with the highest transition temperature $T = T_c$ vanishes. At $T = T_p \sim 1.8$ K, an infinite cluster forms from the superconducting regions, and a state with $\rho = \sigma^{-1} = 0$ is reached. In the case of a-GaSb, the temperature is thus a convenient parameter, allowing us to smoothly vary the relative amount (n) of superconducting phase in the sample. Amorphous gallium antimonide can be used as a model medium for studying percolation along superconducting inclusions.

We specify $n(T)$ to be of the form

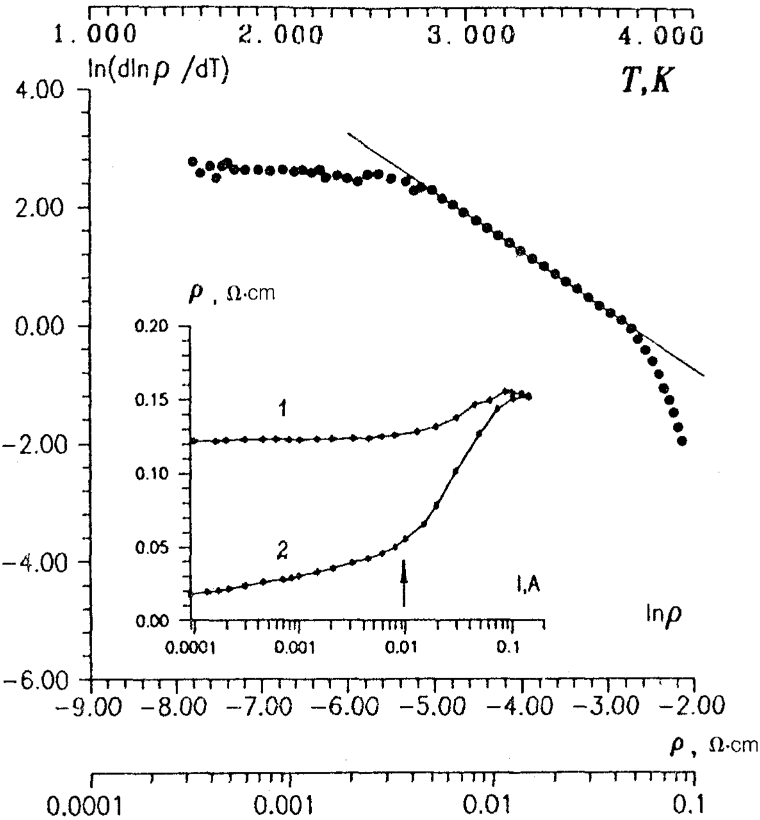

FIG. 2. The results on $\rho(T)$ in the region $T < T_m$, plotted in special coordinates (see the text proper) for a determination of the critical exponent q. The inset shows the current dependence of the resistivity for (1) $T = 4.2$ K and (2) $T = 2.5$ K. The arrow shows the typical value of the current which separates the region in which the current-induced disruption of weak links between inclusions is predominant from the region in which the superconductivity of the inclusions is destroyed.

$$n(T) = \int_T^{T_m} \phi(T_c) dT_c, \tag{4}$$

where $\phi(T_c)$ is the distribution of clusters with respect to T_c From (2) and (4) we find the following equation for the resistivity $\rho(T) = \sigma^{-1}$:

$$\frac{1}{\rho}\frac{\partial \rho}{\partial T} = q\frac{\phi(T_c)}{n_c} c \left(\frac{\rho}{\rho_0}\right)^{-1/q} \tag{5}$$

Here $\rho_0 = \sigma_0^{-1} \sim \rho(T \gg T_m)$ is the resistivity of the medium in the absence of inclusions (Fig. 1). As a result, if the function $\phi(T)$ does not have any sharp structural features [this conclusion is suggested by the smooth shape of the $\rho(T)$ curve at $T < T_m$], we can find the exponent q by replotting the experimental data as $\ln[(1/\rho)(\partial\rho/\partial T)] = f(\ln \rho)$ and finding the slope of the straight section. This technique has the advantage that we can determine q without knowledge of the percolation threshold

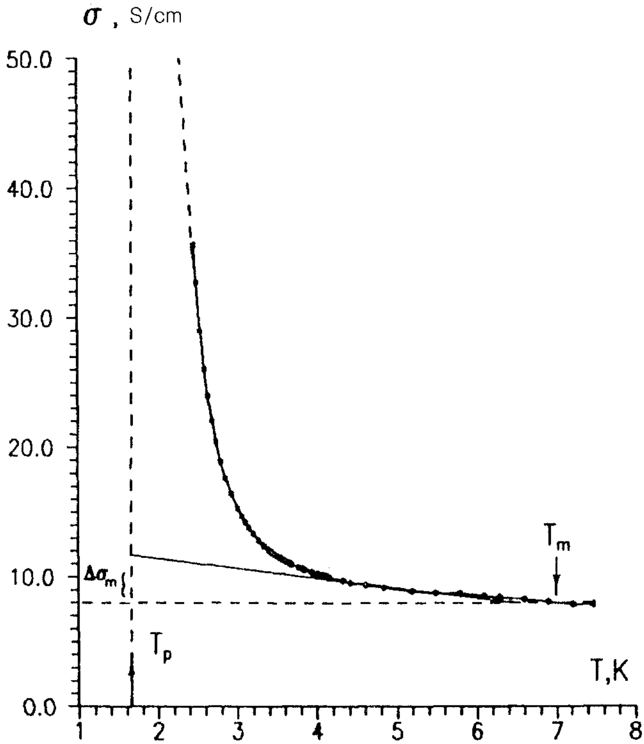

FIG. 3. Linear asymptotic behavior of the conductivity in the limit of a low concentration of superconducting inclusions; transition of the $\sigma(T)$ curve to the critical behavior described by (2) ($I = 0.5$ mA).

and the quantity n, which are usually quite difficult to determine experimentally. On the other hand, the use of a numerical differentiation presupposes highly accurate measurements of $\rho(T)$ (even at $\rho \approx 0$). In our case, we achieved this high accuracy by using a special computer-controlled data-acquisition system designed for the purpose.

3. Figure 2 shows the experimental $\rho(T)$ dependence at $T < T_m$ in the coordinates $\ln[(1/\rho)(\partial\rho/\partial T)] = f(\ln\rho)$. We see a long linear section for $\rho \leqslant 6\times10^{-2}$ $\Omega\cdot$cm, which corresponds to the asymptotic behavior in (2). The value of q turns out to be $q = 1 \pm 0.01$, in agreement with the Éfros–Shklovskiĭ hypothesis[1] and in contradiction of the Coniglio–Stanley model.[3] For the experimental value $q = 1$ we find $d_f \approx 3.27 > d = 3$ from (3), and the fractal dimension is considerably greater than the dimensionality of the space. We regard this situation as extremely unlikely.

It follows from Fig. 2 that relation (2) is violated with a further lowering of the temperature, $T \to T_p$. At $\rho < 4.5\times10^{-3}$ $\Omega\cdot$cm, the quantity $\ln[(1/\rho)(\partial\rho/\partial T)]$ reaches saturation, indicating that a functional dependence of different type is characteristic of this region. We believe that this circumstance may be a result of an effect of the weak links between superconducting inclusions. As we have shown previously,[6] the existence of such links is confirmed by measurements of the current dependence $\rho(I)$ for $T \sim T_p \sim 2$ K (see the inset in Fig. 2). Near T_p we see, in addition to the region with a sharp increase in $\rho(I)$ at $I > 10^{-2}$ A (this increase corresponds to Joule heating of the medium and to a current-induced destruction of the superconductivity[6]), a region in which ρ increases smoothly ($I < 10^{-2}$ A). This region is not related to the factors mentioned above; it is instead a consequence of a disruption of the weak links by the current. As can be seen in the inset in Fig. 2, the effect of the weak links strengthens with decreasing temperature. It is thus natural to expect the power-law behavior in (2) to give way to a stronger dependence, e.g., an exponential dependence $\rho \sim \rho_0 \exp(-\alpha n)$, at $\rho < 4.5 \times 10^{-3}$ $\Omega \cdot$cm ($T < 2.8$ K). It is easy to see that in this case we would have $\ln[(1/\rho)(\partial\rho/\partial T)]$, and for a slowly varying $\phi(T)$ we would have $\ln[(1/\rho)(\partial\rho/\partial T)] \approx$const (this is what we observe experimentally). Weak links were ignored in the derivation of expression (2) (Refs. 1, 3, and 4).

We turn now to the region $\rho < 6 \times 10^{-2}$ $\Omega \cdot$cm ($T \sim 4$ K), where the concentration of inclusions is low and where, according to Ref. 7, the conductivity of the medium is given by

$$\sigma = \sigma_0(1 + 3\beta n), \tag{6}$$

where β is a geometric factor which depends on the shape of the inclusions.[7] It follows from (6) that for $\phi(T) \approx$const we would have a linear asymptotic behavior as $T \to T_m \sigma(T)$ (Fig. 3). This behavior would generally be different from that described by (2) as $n \to 0$. Extrapolating the linear region of $\sigma(T)$ (Fig. 3) to the value $T \to T_p$, we find $\Delta\sigma_m = 3\beta\sigma_0 n_c$. It follows from Fig. 3 that we have $\Delta\sigma_m/\sigma_0 \sim 0.6$ for *a*-GaSb. This value is extremely close to the theoretical value $\Delta\sigma_m/\sigma_0 \sim 0.51$ for spherical superconducting inclusions ($\beta = 1$, $n_c = 0.17$).

In summary, we have shown that the conductivity of a real system with superconducting inclusions can be described by the existing theories[1,7] for inhomogeneous media, except in a narrow region near the percolation threshold. In this region, effects stemming from a weak superconductivity are apparently important. The critical region is described best by the Éfros–Shklovskiĭ model.[1]

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