Bound states near a short-range impurity in crossed magnetic and electric fields

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The problem of quantum scattering of an electron by a short-range impurity in a two-dimensional electron gas in the presence of a perpendicular magnetic field and a longitudinal electric field is solved. It is shown that in addition to the bound state, which exists in the case of zero electric field, there are in an electric field N novel, nondegenerate, quasi-bound states with energies close to the Nth Landau level, irrespective of whether the impurity is attractive or repulsive.

In analytical and numerical studies, which address magnetokinetic phenomena in bulk semiconductors,[1] the quantum Hall effect,[2,3] the conductance of a microconstriction,[4] and variable-range hopping magnetoresistance,[5] it was important to investigate the electron scattering by a single impurity in a magnetic field. In these studies, the scatterer is of short-range type. In Ref. 2, the scattering by an impurity of the drifting electron is considered for the case of crossed electric and magnetic fields, but the bound states near the lowest Landau level (LL) was mainly considered. In the present article we also consider more carefully the possible bound states around the higher Landau levels. In Ref. 3, the intersubband tunneling of drifting electrons via an impurity was studied, and the scattering potential was assumed to be a δ-function. The use of the 2D δ-function as a scattering potential was found to be incorrect in Refs. 4–6. In Refs. 7–9, we used a method which allows to circumvent this difficulty. Additionally, our method turned out to be very efficient for the investigation of the structure of bound states. Hence, further concise calculations are in the context of this method.

Let us assume that the magnetic field H is uniform and directed perpendicularly to the plane of the two-dimensional electron gas (2DEG), and that the uniform electric field E lies in the plane of the 2DEG and is directed along the y axis. It is known that the electron under such circumstances drifts along the x axis with the velocity $v = cE/H$. In the Landau gauge, the wavefunctions of that motion, which are normalized to unity $\int d^2r\,\Psi_{nk}(r)\Psi^*_{n'k'}(r) = \delta(k-k')\delta_{nn'}$, have the form

$$\Psi_{nk}(r) = \frac{1}{(2\pi)^{3/4}\sqrt{n!}} \times e^{ik\xi} \times D_n(s - 2k + 2\alpha). \tag{1}$$

Here D_n are the functions of the parabolic cylinder with integer indices, and $n = 0,1,2,...$ (Ref. 10). We introduced the dimensionless coordinate $r = (\xi, s)$ with

$\xi = \sqrt{2}/a_H \cdot x$, $s = \sqrt{2}/a_H \cdot y$; the momentum $k = a_H/\sqrt{2} \cdot p_x/\hbar$, the drift velocity $2\alpha = \sqrt{2}/a_H \omega_H \cdot v$, where the magnetic length is $a_H = \sqrt{\hbar/m\omega_H}$, and the cyclotron frequency $\omega_H = |e|H/mc$. The energy of the drifting electron, in units of $\hbar\omega_H$, $\varepsilon_{nk} = n + \frac{1}{2} + 2\alpha k - \alpha^2$, consists of the kinetic energy of the cyclotron motion $n + \frac{1}{2}$, the kinetic energy of the drifting motion α^2, and the potential energy $2\alpha(k - \alpha)$.

At a fixed energy ε, there is an infinite set of drifting states which are separated in space if $\alpha \ll 1/\sqrt{n}$. Let us also assume that such a drift state with a wave function $\Psi_{Nk_N}(r)$ and $k_N = (\varepsilon - N - \frac{1}{2} + \alpha^2)/2\alpha$ is scattered by a short-range impurity (which is an s-scatterer) situated in r_0. The scattered field for such an impurity is[8]

$$\psi'(r) = -2\pi \Psi_{Nk_N}(r_0) \times \frac{G_\varepsilon(r, r_0)}{D_\varepsilon(r_0)}. \tag{2}$$

Here $G_\varepsilon(r,r_0)$ is the Green's function of the outgoing waves of the Schrödinger equation without the scattering potential. The Green's function $G_\varepsilon(r,r_0)$ can be expanded in eigenfunctions (1) (see Ref. 11). Additionally, in a further expansion of the small parameter α the Nth LL is the main contribution:

$$G_\varepsilon(r, r_0)|_{r_0=0} = \frac{1}{(2\pi)^{3/2} N! \cdot \alpha} \times \int_{-\infty}^{+\infty} dk \quad e^{ik\xi} \times \frac{D_N(s - 2k) D_N(-2k)}{2k - \varepsilon_N/\alpha - i \cdot 0}. \tag{3}$$

Here for convenience we set $r_0 = 0$ and omit the gauge factor $e^{i\xi\alpha}$ (we can eliminate it by slightly varying the gauge), and $\varepsilon_N = \varepsilon - N - \frac{1}{2} - \alpha^2$. The expression (3) is valid everywhere except the region near the impurity, where the Green's function diverges:

$$G_\varepsilon(r, r_0)|_{r \to r_0} = \frac{1}{2\pi} \left[\ln\left(\frac{1}{|r - r_0|} \right) + K_\varepsilon(r_0) \right]. \tag{4}$$

The quantity $K_\varepsilon(r_0)$ plays in important role in the analysis of the structure of bound states because it appears in the denominator of the scattering amplitude in (2).

$$D_\varepsilon(r_0) = \Lambda + K_\varepsilon(r_0), \qquad \Lambda = \ln\left(\frac{a_H}{\sqrt{2} \cdot a} \right), \tag{5}$$

where a is the 2D scattering length of the impurity scattering potential. The exact formula for $K_\varepsilon(r_0)$ takes the form[1)]

$$K_\varepsilon(r_0)|_{r_0=0} = \frac{1}{2} \sum_{n=0}^{\infty} \left[\frac{1}{\alpha \cdot i^{n-1}} \times D_n(\varepsilon_n/\alpha) D_{-n-1}(-i\varepsilon_n/\alpha) + \frac{1}{\varepsilon_n} \right] + K_{\varepsilon,\alpha=0} \equiv K_\varepsilon, \tag{6}$$

where

$$K_{\varepsilon,\alpha=0} = -\frac{1}{2} \psi(1/2 - \varepsilon) - C + \ln 2. \tag{7}$$

Here $\psi(1/2 - \varepsilon)$ is the digamma function,[10] and $C = 0.577...$ is Euler's constant. Equation (7) can be obtained[10,12] from the standard Green's function of an electron in

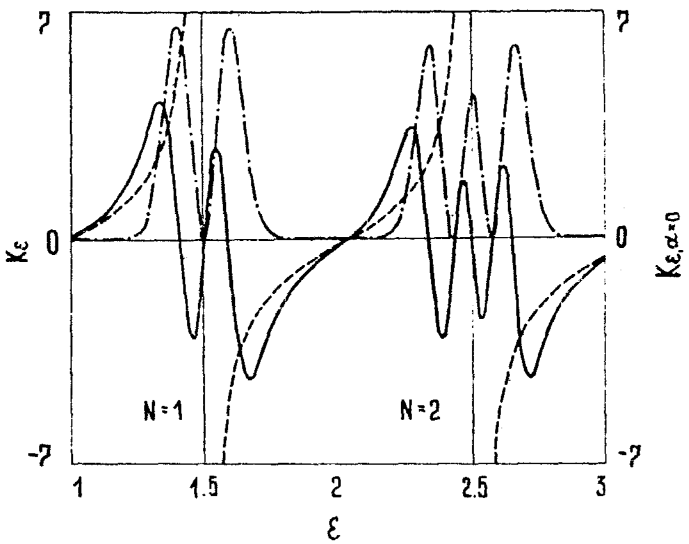

FIG. 1. Plots of Re K_ε (—), Im K_ε (- · -), and K_ε, $\alpha = 0$ (- - -) for $N = 1$ and $N = 2$. α is equal to 0.07. The separation of oscillations belonging to the neighboring Landau levels is clearly seen.

a magnetic field.[13] The plots of the functions K_ε and $K_{\varepsilon,\alpha=0}$ are given in Fig. 1. The presence of an electric field leads to a nonvanishing imaginary part of the Green's function, and to oscillations of both the real part and the imaginary part of K_ε. These oscillations, which are localized near each LL, do not overlap with those of the neighboring levels if $\alpha \ll 1/\sqrt{N}$. The amplitude of the oscillations of K_ε increases with decreasing electric field as $1/\alpha$. In this case it is convenient to rewrite (6) in the form

$$K_\epsilon = \frac{1}{2i^{N-1}\cdot\alpha} \times D_N(\varepsilon_N/\alpha) D_{-N-1}(-i\varepsilon_N/\alpha) + P, \tag{8}$$

where P is a quantity on the order of unity which weakly depends on N and on the energy in the vicinity of the Nth LL. If the condition $\alpha \ll 1/N$ is satisfied, the imaginary part of P is exponentially small. We can thus assume P to be a real constant.

Bound states are defined as the scattering amplitude poles $\bar{\varepsilon} - i\Gamma$, which are located near the real energy axis. These poles are the solutions of the equation

$$D_\epsilon = \tilde{\Lambda} + \frac{1}{2i^{N-1}\cdot\alpha} \times D_N(\varepsilon_N/\alpha) D_{-N-1}(-i\varepsilon_N/\alpha) = 0, \qquad \tilde{\Lambda} = P + \Lambda. \tag{9}$$

Since Γ is assumed to be small, we can write this equation in the form

$$\Gamma = \left[\frac{\mathrm{Im} K_\epsilon}{d/d\epsilon \mathrm{Re} K_\epsilon}\right]_{\epsilon=\bar{\epsilon}}. \tag{10}$$

From Fig. 1 and the general form of Eq. (9) immediately follows the first important result: bound states exist only if the condition

$$|\tilde{\Lambda}|\alpha \ll 1 \tag{11}$$

is satisfied. In the absence of an electric field ($\alpha = 0$), the bound states always exist for each impurity.

If the condition (11) is satisfied, there is at least one bound state, which is fully equivalent to that at $\alpha = 0$. Using the asymptotic form[12] of the functions D_N and D_{-N-1} in the limit $|\varepsilon_N/\alpha| \to \infty$, we find from Ref. 10 the bound state energy relative to the Nth LL

$$\varepsilon_N = \frac{2}{\tilde{\Lambda}} \tag{12}$$

and their width

$$\Gamma_N \sim \frac{\alpha}{(\alpha\tilde{\Lambda})^{2N+2}} \times \exp\left[-\frac{2}{(\alpha\tilde{\Lambda})^2}\right]. \tag{13}$$

As expected, the width of this bound state rapidly vanishes for $\alpha \to 0$ and the states become nondecaying. This bound state can occur below a LL (if $\tilde{\Lambda} < 0$) and also above it (if $\tilde{\Lambda} > 0$).

The presence of the electric field gives a set of new nondegenerate states near each LL with $N > 0$. Their number for the Nth LL is equal to N, in correspondence with the number of zeros in the imaginary part of K_ε:

$$\mathrm{Im} K_\varepsilon = \sqrt{\frac{\pi}{8}} \times \frac{[D_N(\varepsilon_N/\alpha)]^2}{N! \cdot \alpha}. \tag{14}$$

The twofold degenerate zeros of Im K_ε lie very close to the simple zeros of the real part of K_ε:

$$\mathrm{Re} K_\varepsilon = \frac{1}{2 \cdot \alpha} \times D_N(\varepsilon_N/\alpha) \times \left[i^{1-N} D_{-N-1}(-i\varepsilon_N/\alpha) - \frac{\sqrt{\pi/2}}{N!} \times D_N(\varepsilon_N/\alpha) \right] + P. \tag{15}$$

This circumstance gives rise to the existence of a set of narrow bound states. Using (14), (15), and the Wronskian relation $W[D_N(z), D_{-N-1}(-iz)] = i^{N+1}$, we obtain the energies of the bound states up to the second order in α

$$\varepsilon_N^m = \alpha\sigma_N^m - 2\alpha^2 \cdot \tilde{\Lambda}, \qquad m = 1, 2, .., N. \tag{16}$$

Their widths are

$$\Gamma_N^m \sim \alpha^3 \tilde{\Lambda}^2. \tag{17}$$

Here σ_N^m are the zeros of the Nth Hermite polynomial.

From (17) and (9) follows that the existence of nondecaying bound states

($\Gamma_N^{in} = 0$) is possible for a moderate strength of the impurity, $|\Lambda| = |P| \sim 1$. The wave function of these bound states is proportional to the scattered field (2), and hence to the Green's function (3). As expected, the wave function of these bound states is localized. Indeed, if $\widetilde{\Lambda} = 0$, then $\varepsilon_N^m/\alpha = \sigma_N^m$ and the zero in the denominator of the integrand in (3) cancels the zero of $D_N(-2k)$. In this case, the right-hand side of (3), a Fourier integral of a smooth function, becomes exponentially small in the limit $|\xi| \to \infty$.

It is obvious that these bound states exist not only in a uniform electric field but they are also characteristic of any smooth potential in which the drifting states are present (for example, in a parabolic confinement potential[9]).

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[1)]The principal methods of obtaining (6) and a similar result are given in Ref. 9.

[1]S. P. Andreyev and S. V. Tkachenko, Zh. Eksp. Teor. Fiz. **83**, 1816 (1982), and the references cited there.
[2]R. E. Prange, Phys. Rev. B **23**, 4802 (1981).
[3]V. L. Pokrovsky, L. P. Prjadko, and A. L. Talapov, Zh. Eksp. Teor. Fiz. **95**, 668 (1989) [Sov. Phys. JETP **68**, 376 (1989)].
[4]H. Tamura and T. Ando, Phys. Rev. B **44**, 1792 (1991).
[5]M. Ya. Azbel', Phys. Rev. B **43**, 2435 (1991).
[6]J. F. Perez and F. A. B. Coutinho, Amer. J. Phys. **59**, 52 (1991).
[7]C. Kunze and R. Lenk, paper to appear in Sol. State Commun.
[8]Y. B. Levinson, M. I. Lubin, and E. V. Sukhorukov, Phys. Rev. B **45**, 11936 (1992); Pis'ma Zh. Eksp. Teor. Fiz. **54**, 405 (1991) [JETP Lett. **54**, 401 (1991)]; Fiz. Nizk. Temp. **18**, to be published (1992) [Sov. J. Low Temp. Fiz. **18**, to be published (1992)].
[9]Y. B. Levinson, M. I. Lubin, E. V. Sukhorukov, and C. Kunze, unpublished.
[10]Handbook of Mathematical Functions, Eds. M. Abramowitz and A. Stegun, NBS, 1964.
[11]P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*, McGraw-Hill, 1953.
[12]*Higher Transcendental Functions*, Eds. H. Bateman and A. Erdelyi, McGraw-Hill, 1953, Vol. 2.
[13]V. V. Dodonov, I. A. Malkin, and V. I. Man'ko, Phys. Lett. A **51**, 133 (1975), see also Refs. 5 and 6.

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