The $B \to D(D^*)e\nu$ decay amplitude factorization and the Isgur-Wise function calculation

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A new approach to the relativistic system of heavy and light quarks, in which a light quark is confined and a heavy quark behaves as an ordinary Fermi particle with a large mass, is formulated. The form factors for the $B \to D(D^*)e\nu$ decay are calculated on the basis of this approach. The Isgur–Wise function is obtained in the heavy quark limit $M_Q \to \infty$.

The physics of hardons containing one heavy quark has recently received more attention because of the development of new symmetry in the world of heavy quarks.[1] It was shown that in the limit of large heavy quark masses, first, the flavor of the heavy quarks becomes irrelevent; i.e., the heavy quarks can be rotated into each other. Second, the spin degrees of freedom of the heavy quark decouple from dynamics in this limit, because of color hyperfine interaction scales, inversely with the heavy quark mass. The relationships among the form factors describing the $B \to D(D^*)l\nu$ decays were obtained with the use of these symmetries. It was shown that all form factors can be expressed in terms of a single universal function, which depends on the product of the four-velocities of the initial and final heavy mesons, and which is normalized to unity at zero recoil.

Here we formulate a new approach to the relativistic system of heavy and light quarks, in which a light quark is confined and a heavy quark behaves as an ordinary Fermi particle with a large mass. We calculate the form factors for the $B \to D(D^*)e\nu$ decay on the basis of this approach. It is shown that the Isgur–Wise representation for the form factors is reproduced in the heavy-quark limit, and that the universal function is obtained.

We will describe heavy B and $D(D^*)$ mesons as the bound states of light $q(x)$ and heavy $Q(x)$ quarks using the compositeness condition $Z_3 = 0$ in the quantum field theory (see, e.g., Ref. 2 and the bibliography cited there). The starting point in this approach is the interaction Lagrangian describing a transition of a meson $H(x)$ into two quarks $q(x)\bar{Q}(x)$:

$$L_H(x) = g_H H(x)\bar{Q}(x)\Gamma_H q(x) + \text{c.c.} \tag{1}$$

Here Γ_H is a Dirac matrix: $i\gamma^5$ for pseudoscalar D, B and γ^μ for vector D^* mesons, respectively. A meson H is assumed to be a bound state of $q\bar{Q}$, which can be expressed

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in the *compositeness condition* that the renormalization constant Z_H for the meson wave function $H(x)$ is equal to zero:[2]

$$Z_H = 1 + h_H \tilde{\Pi}'_H(m_H^2) = 0, \tag{2}$$

where $h_H = 3g_H^2/(2\pi)^2$ is the effective coupling constant, and $\overline{\Pi}'_H$ is the derivative of the (renormalized) meson mass operator. Physically, this condition means that the probability of finding the meson H in a bare state is equal to zero. In other words, the meson H is a bound state in the system of two quarks. It is important to note that (i) the interaction Lagrangian (1), together with the compositeness condition (2), is equivalent to the heuristic QCD bozonization based on the ideas of the Nambu–Jona-Lasinio model[3] and (ii) the compositeness condition (2) allows one to determine the coupling constant h_H (or g_H) as a function of the physical meson mass. Such a procedure of hadronization was accepted in Ref. 4 as one of the crucial points of the quark confinement model (QCM). In the one-loop approximation the physical processes are described by the quark diagrams that contain a convolution of quark propagators.

It is widely accepted that the behavior of quarks at large distances is defined by their interactions with the vacuum gluon background. There exist vacuum configurations with constant strengths[5–7] which provide a quark confinement, i.e., allow a quark propagator to be a complete analytical function on the momentum plane. Physically, it could be understood as the absence of a quark with a definite value of mass in the observable hardon spectrum. As was shown in Ref. 4, the propagator of a light quark in the confined gluon background can be represented in the form

$$G(\not{p}) = \int\limits_L \frac{d\mu\rho(\mu)}{\mu - \not{p}} = \frac{1}{\Lambda_{conf}}[a(-\frac{p^2}{\Lambda^2_{conf}}) + \frac{\not{p}}{\Lambda}b(-p^2\Lambda^2_{conf})], \tag{3}$$

where the integration contour L, the density of the quark mass distribution $\rho(\mu)$, and hence the confinement functions $a(z)$, and $b(z)$, and the parameter $\Lambda_{\rm conf}$, which characterizes the scale of the confinement region, should be defined by solving an equation for the Green's function of a quark in the external gluon field. The representation (3) allows one to consider a confined quark propagator as a superposition of local quark propagators with a smeared constituent mass μ of a density $\rho(\mu)$. Here we will not restrict the analysis by using the concrete shapes for the confinement functions but investigate how sensitive are the final results for the form factors to their different forms. The only requirement imposed on the confined propagator (3) is that $p^2 \to -\infty$ in the Euclidean direction, in order to provide a convergence of all Feynman integrals. This means that the propagator of a confined quark is assumed to be localized near the origin $p \simeq 0$.

It is well known (see, for example, Ref. 8) that heavy quarks interact weakly with vacuum gluon fields. Therefore, it seems quite reasonable to use the local Dirac propagator with large mass for describing the behavior of a heavy quark at large distances:[9,10]

$$S(\not p)=\frac{1}{M_Q-\not p}. \tag{4}$$

Here M_Q is a constituent mass of a heavy quark. We will focus special attention on the heavy quark limit $M_Q\to\infty$.

First, let us calculate the mass operator of pseudoscalar heavy mesons which is defined by the self-energy diagram. We have

$$\Pi_{HP}(p^2)=-\int\frac{d^4k}{4\pi^2 i}\mathrm{tr}\{\gamma^5 G(\not k)\gamma^5\frac{1}{M_Q-(\not k+\not p)}\}\Pi_{HP}(p^2)=-\Lambda^2_{conf}I_{HP}(p^2), \tag{5}$$

where

$$I_{HP}(p^2)=\frac{1}{2}\int\limits_0^\infty du u b(u)+\int\limits_0^\infty du C(u,p^2,M_Q^2)\{M_Q a(u)+\frac{1}{2}(p^2-M_Q^2+u)b(u)\},$$

$$C(u,x,z)=\frac{\sqrt{(u+z-x)^2+4ux}-(u+z-x)]}{2x}.$$

Here and below we assume for simplicity that all momenta and masses are given in units of Λ_{conf}.

Substituting (6) into the compositeness condition (2), we obtain the following expression for the coupling constant g_{HP} of a pseudoscalar heavy meson:

$$g_{HP}=\frac{2\pi}{\sqrt{3}}\frac{1}{\sqrt{I'_{HP}(m_H^2)}}. \tag{6}$$

The coupling constant of a vector heavy meson is calculated by analogy with the above procedure.

In the heavy-quark limit $m_H=M_Q\to\infty$, we obtain

$$g_{HP}=g_{HV}\to\frac{2\pi}{\sqrt{3}}\sqrt{\frac{2M_Q}{G_0}}, \tag{7}$$

where

$$G_0=2\int\limits_0^\infty dE_4E_4\{\mathrm{Re}G(iE_4)+\mathrm{Im}G(iE_4)\}=\int\limits_0^\infty du a(u)+\int\limits_0^\infty du\sqrt{u}b(u).$$

The amplitudes of the $B\to D(D^*)e\nu$ decay are written in the form

$$M^{\mu}(p,p') = g_B g_D \frac{3}{(2\pi)^2} \int \frac{d^4k}{4\pi^2 i} \mathrm{tr}[\frac{1}{M_c - \not{k} - \not{p}'} O^{\mu} \frac{1}{M_b - \not{k} - \not{p}'} \gamma^5 G(\not{k}) \gamma^5],$$

$$M^{\mu\nu}(p,p') = g_B g_D \cdot \frac{3}{(2\pi)^2} \int \frac{d^4k}{4\pi^2 i} \mathrm{tr}[\frac{1}{M_c - \not{k} - \not{p}'} O^{\mu} \frac{1}{M_b - \not{k} - \not{p}} \gamma^5 G(\not{k}) \gamma^{\nu}], \tag{8}$$

where p and p' are momenta of the initial and final states. Here we consider the heavy quark limit $m_H = M_Q \to \infty$. It is easy to obtain the asymptotic expression for the amplitudes in the leading order in $1/M_Q$:

$$M^{\mu}(p,p') = \sqrt{M_b M_c} \xi(w)(v + v')^{\mu},$$

$$M^{\mu\nu}(p,p') = \sqrt{M_b M_c} \xi(w)\{-g^{\mu\nu}(1 + vv') + (v')^{\mu} v^{\nu} - i\varepsilon^{\mu\nu\alpha\beta}(v')^{\alpha} v^{\beta}\}, \tag{9}$$

where w is the dot product of the four velocities of the initial particle and the final particle:

$$v = \frac{p}{M_b} \qquad v' = \frac{p'}{M_c} \qquad w = vv' = \frac{M_b^2 + M_c^2 - (p - p')^2}{2M_b M_c}.$$

The function $\Phi(w)$ is

$$\Phi(w) = \frac{1}{\sqrt{w^2 - 1}} \ln[w + \sqrt{w^2 - 1}]. \tag{10}$$

It exactly coincides with the Isgur-Wise representation. The Isgur-Wise function is

$$\xi(w) = \frac{\Phi(w) + \frac{2}{(1+w)} R}{1 + R}, \tag{11}$$

where

$$R = \frac{\int\limits_0^{\infty} dE_4 E_4 \mathrm{Im} G(iE_4)}{\int\limits_0^{\infty} dE_4 E_4 \mathrm{Re} G(iE_4)} = \frac{\int\limits_0^{\infty} du \sqrt{u} b(u)}{\int\limits_0^{\infty} du a(u)}. \tag{12}$$

It can be seen that the only model parameter R corresponds to the expression for $\xi(\omega)$. The value of this parameter is defined by the integral of the light quark propagator $G(z)$ over the Euclidean direction.

The behavior of $\xi(w)$ for different values of the model parameter R in (12) ($0 \leqslant R \leqslant \infty$) is shown in Fig. 1. One can see that the behavior of $\xi(\omega)$ depends only

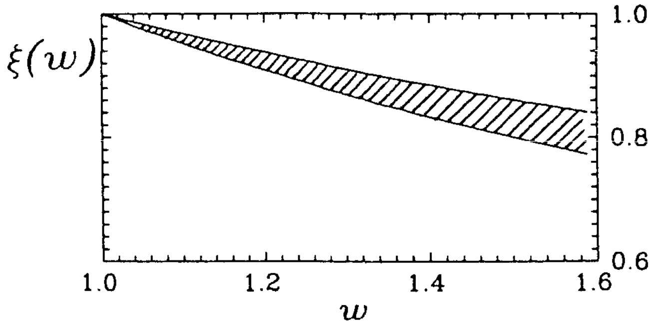

FIG. 1. The Isgur–Wise function. $R=0$—The lower bound; $R=\infty$—the upper bound.

slightly on this parameter. This means that the form factors of the $B\to D(D^*)e\nu$ decay depend very slightly on the behavior of the light quark at large distances.

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