# Confinement of acoustic modes due to electron–phonon coupling in the electron sheet

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The confinement of acoustic modes within 2DEG due to only the electron–phonon coupling has been studied. The confined modes split off from the bulk phonons even at uniform lattice parameters.

One of the current topics of the semiconductor physics is the confinement of phonons in seminconductor heterostructures.[1–3] This phenomenon is interesting because of the fundamental aspects, and because of its effect on the electron transport that is very important for applications. It is known that the phonon confinement in the heterostructures is due to the different lattice characteristics of the semiconductor compounds that form the heterostructure (various lattice constants, lattice forces, symmetry, etc.). On the other hand, existence of free carriers in these layers is not considered as the main reason for the confinement.

In this paper we predict and study the phonon confinement that originates from the electron–phonon interaction. We show that confinement of acoustic modes appears due to only the electron–phonon interaction if there is an electron gas sheet (3D or 2D electron layer). This effect exists even in the case of uniform lattice characteristics. Such a physical situation and the electron layers can be realized by modulation doping, for example, under $\delta$-doping.[4]

The following two known phenomena can be the basis for understanding of the proposed mechanism for the phonon confinement. First, the electrons bring about the renormalization of phonon spectrum and always lead to a reduction of the elastic modulus and a softening of the lattice. Second, the embedded layer characterized by a decreased elastic modulus always splits the bulk acoustic spectrum into bulk-like modes and localized modes. The latter are confined to, or are near, the embedded layer, and propagate along the layer. Therefore we can expect that the electron–phonon interaction under the localization of electrons within the electron sheet would lead to the phonon confinement effect.

We will describe the long-range acoustic vibrations of the lattice by an equation for the sound waves:[5]

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}, \tag{1}$$

where $u_i$ are components of the displacement vector $\vec{u}$ of the medium $\rho$ is its density, and $\sigma_{ik}$ is the stress tensor. For simplicity, we consider the isotropic elastic medium

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and assume that electrons are characterized by isotropic energy law. The contribution of the lattice and the electrons to the stress tensor $\sigma_{ik}$ will then be[5,6]

$$\sigma_{ik} = \sigma_{ik}^{(L)} + \sigma_{ik}^{(E)} = (\lambda + \frac{2}{3}\mu)u_{ll}\delta_{ik} + 2\mu(u_{ik} - \frac{1}{3}\delta_{ik}u_{ll}) + bn\delta_{ik}, \tag{2}$$

where $\lambda$ and $\mu$ are Lamé coefficients, and $u_{ik}$ is the strain tensor. The electron–phonon interaction will then be described by only one constant of deformation potential $b$ (Ref. 6), and $n$ is the concentration of the electrons.

We assume that the electrons are confined to a sheet of thickness $d$ by a corresponding potential (for example) by the electrostatic potential of the positive charge of the donor sheet). The phonon wave vector $q$ is restricted in such a way that the characteristic decay length of the modes outside the sheet, $k_{ch}^{-1}(q)$, is much larger than the layer thickness $d$: $k_{ch}(q)d \ll 1$.

In such a case it is possible to consider the electrons as confined in a plane (for example, in the $z = 0$ plane). Hence, the concentration of the electrons can be written as $n(\vec{r},t) = n_s(x,y)\delta(z)$, where $n_s$ is the "surface" concentration of the electrons. We assume that the electrons follow adiabatically the vibrations of the lattice and are redistributed in the potential of the acoustic wave. The inequality $\bar{\epsilon} \gg \hbar\omega$, which is a necessary condition of this adiabatic approximation, always holds for the semiconductors ($\bar{\epsilon}$ is the characteristic electron energy, and $\omega$ is the phonon frequency). The potential induced by acoustic wave is $h(\vec{r}) = bu_{ll} - e\varphi$. Here $\varphi$ is the electrostatic potential produced as a result of the nonuniform redistribution of the electrons in space. This potential is governed by the Poisson equation

$$\nabla^2\varphi = \frac{4\pi e}{\epsilon_0}\delta n_s(x,y)\delta(z), \tag{3}$$

where $\epsilon_0$ is the dielectric constant of the crystal. We assume that the dependence of all variables on the plane coordinates $(x,y)$ has the form $u_i, \varphi, h, \delta n, \propto e^{i\vec{q}\cdot\vec{r}}\|$, where $\vec{q}$ lies in the plane of the electron sheet. Change in the electron concentration, $\delta n$, can be calculated by using the perturbation theory for the density matrix

$$\delta n_s(x,y) = h(x,y,z=0)P(\vec{q},T). \tag{4}$$

Here, $P(\vec{q},T)$ is the static polarization of the electron subsystem, including the quantization of the electron motion in the $z$ direction.[7] If the number of occupied 2D subbands is large, the electron motion within the sheet is nearly three-dimensional. In the opposite case, the polarization $P(\vec{q},T)$ corresponds to the electron gas with a reduced dimensionality. The set of the relations mentioned above is sufficient to account for the acoustic modes that are localized near the electron layer.

It is easy to see that only longitudinal acoustic waves interact with electrons in our model. Therefore, it is convenient to consider the equation for the relative volume change $u_{ll}$, instead of the several components of the displacement of the lattice. We seek the solutions which satisfy the following boundary conditions far away from the

sheet: $u_{ll}$, $\phi \to 0$, $z \to \pm \infty$. The solutions can be written for both $u_{ll}$ and $\phi = -e\varphi$ as follows:

$$u_{ll} = Ae^{-\kappa|z|}, \qquad \kappa = \sqrt{q^2 - \frac{\omega^2}{c_l^2}} \qquad \phi = Be^{-q|z|}. \tag{5}$$

The relationship between the magnitudes of the acoustic wave and the electrostatic potential is

$$B = -A\, b\, \frac{4\pi e^2 P(\vec{q}, T)/\epsilon_0}{2q + 4\pi e^2 P(\vec{q}, T)/\epsilon_0}. \tag{6}$$

The expression for $\kappa$ is

$$\kappa = \sqrt{q^2 - \frac{\omega^2}{c_l^2}} = \frac{b^2 P(\vec{q}, T)}{\lambda + 2\mu} \frac{q^3}{2q + 4\pi e^2 P(\vec{q}, T)/\epsilon_0}. \tag{7}$$

The right-hand side of Eq. (7) is always positive. This means that the solutions decay exponentially far away from the layer. The same expression gives dispersion relation for the confined acoustical modes:

$$\omega^2 = q^2 c_l^2 \left(1 - \left(\frac{b^2 P(\vec{q}, T)}{\lambda + 2\mu} \frac{q^2}{2q + 4\pi e^2 P(\vec{q}, T)/\epsilon_0}\right)^2\right). \tag{8}$$

It is evident from (8) that the frequencies of the confined phonons are always lower than the frequencies of the bulk phonons. The splitting value of the frequencies depends on the fourth power of the coupling constant $b$. The distinction between the bulk phonons and the confined phonons becomes more pronounced with increasing $q$: The degree of the confinement becomes larger, and can be seen from relation (7), and the dispersion relation falls off from the linear behavior.

The term $4\pi e^2 P(\vec{q},T)/\epsilon_0$ in denominator of expressions (6) and (7) evidently describes the screening effect of the electron charge which is redistributed in the electron sheet. We can introduce the characteristic wave vector $q_{sc}$ by the equality $q_{sc} = 2\pi e P(q_{sc})/\epsilon_0$. In the case $q < q_{sc}$, the total potential $h(\vec{r})$ induced by the acoustic wave is small, because the change of the bottom of the conduction band $bu_{ll}$ and the electrostatic energy $-e\varphi$ cancel each other. In this limiting case the dispersion relation becomes $\omega^2 = q^2 c_l^2\ (1 - (b^2 \epsilon_0 q^2/4\pi e^2(\lambda + 2\mu))^2)$ and does not depend on the parameters of the electron band, on the quantization to the sheet, on the temperature, etc. This simple expression is valid under certain conditions indicated above. In this limit, the decay length of the acoustic mode outside the sheet $\kappa^{-1}$ is proportional to $q^{-3}$.

In opposite case $q > q_{sc}$, the screening is not essential and the dispersion relation takes the form

$$\omega^2 = q^2 c_t^2 \left(1 - \left(\frac{b^2 P(\vec{q},T)}{2(\lambda+2\mu)} q\right)^2\right). \tag{9}$$

In this case, the magnitude of the mode outside the electron sheet decays with $\kappa^{-1} \sim q^{-2}$. Analysis of the polarization $P(\vec{q},T)$ shows that the parameter $q_{sc}$ and the confinement effect increase with decreasing temperature. Because of this circumstance, we will consider the low-temperature case in detail. The following analysis will clarify the actual situation, $q_{sc} < q$, better. The value $q_{sc}$ is always small compared with $k_F^{(n)}$ (i.e., the Fermi vector of the $n$th subband) for semiconductors with a large dielectric constant $\epsilon_0$ (for example, the IV–VI compounds). This means that in the region $q \sim k_F$, where the confinement effect is more pronounced, the screening does not suppress the effect. In general, for the semiconductors with a modest $\epsilon_0$ the inequality $q_{sc} < q$ also holds. In fact, the maximum value of $q_{sc}$ is of the order of the inverse Bohr radius $a_B$ for semiconductors. Let us assume that the electron sheet is created by means of doping. It is necessary to dope the semiconductor to such concentrations, however, that $n_s a_B^2 > 1$, in order to obtain the free carriers and the conductivity.[4] This criterion, however, is equivalent to the inequality mentioned above. When this criterion holds, we can assume that the case $q \simeq k_F$, $q_{sc} < q$ is valid. The decay length of the acoustic wave outside the electron sheet, $\kappa^{-1} = 2(\lambda + 2\mu)/b^2 P(\vec{q},T) q^2$, is much larger than the wavelength $2\pi/q$ even for $q \sim k_F$ and $q \sim 1/d$ for actual semiconductor parameters (see the estimates below). Since the inequalitites $\kappa_{ch}(q) d \ll 1, q_{sc} < q$, and $q_{sc} < k_F$ are compatible, the expression for the dispersion relation (7) holds in the region $q \sim k_F$, the electrostatic potential is not essential, and the splitting of the confined acoustic mode has a maximum at $q = 2k_F$. At $q > 2k_F$ the magnitude of $P(\vec{q})q$, which determines $\omega$, is proportional to $q^{-1}$, so the splitting decreases with increasing $q$. The behavior of $\omega(q)$ for the confined acoustic mode is shown in Fig. 1a.

It is known that not only the heterostructures with one electron sheet but also with many-layered system can be fabricated.[4] The distance between these electron sheets, $2L$, can be varied artificially. If $L$ is of the order of the characteristic scale $\kappa_{ch}^{-1}$, the effect of interaction of these sheets appears. In the example of a two-electron-sheet structure, we show that the interaction of the sheets leads to a splitting off of the additional acoustic waves which are characterized by other features. Specifically, the case shows the two sorts of confined modes (symmetric and antisymmetric). Although the mode differs from the single-electron-sheet solutions by the magnitude of the splitting, by the degree of confinement, etc., it shows the same physical trends. However, the antisymmetric modes are considerably different at small values of $\kappa$ and $q$. This solution is separated from the bulk solution after a finite value of $q = q_c$. Since $\kappa_{\text{antisym}}$ is small for $q \sim q_c$, we can find the equation for $q_c$:

$$\frac{1}{L} = R(q_c) = \frac{b^2 P(\vec{q_c},T)}{\lambda+2\mu} \frac{q_c^3(1+\coth q_c L)}{2q_c(1+\coth q_c L) + 4\pi e^2 P(\vec{q_c},T)/\epsilon_0}. \tag{10}$$

This equation always has a single root. Near $q_c$, the dispersion relation for the antisymmetric modes is

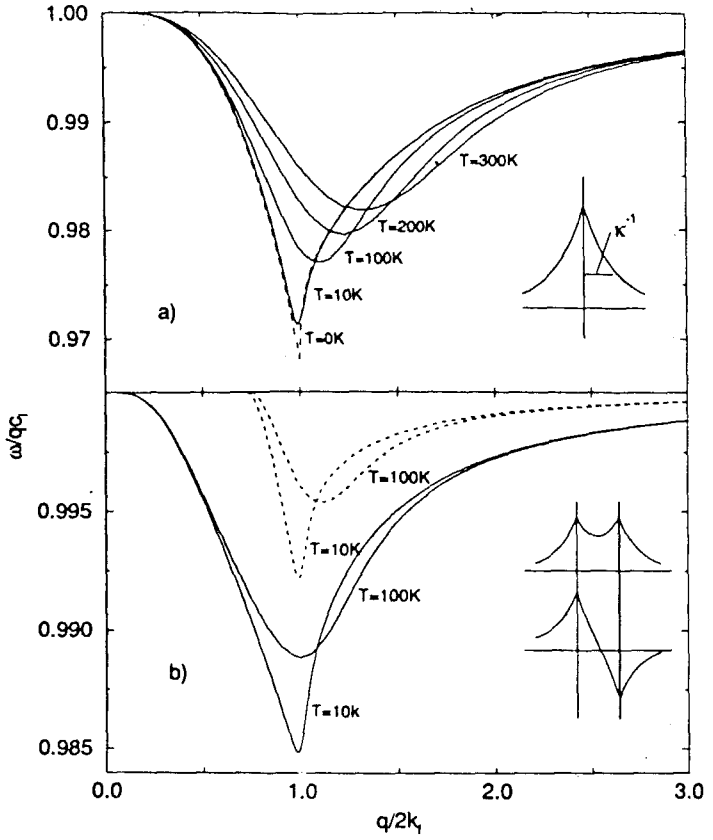


FIG. 1. Phase velocity $\omega qc_t$ of the confined modes as a function of the wave vector $q$ at different temperatures for (a) single electron sheet and (b) two electron sheets. The insets show the form of the solutions.

$$\omega^2 = c_t^2 q^2 \left(1 - \frac{(R'(q_c))^2}{c_t^2 q_c^2}(q - q_c)^2\right) \quad , \quad q > q_c. \tag{11}$$

Analysis shows that the splitting between bulk phonons and antisymmetric confined modes increases when $q$ increases to $2k_F$; the splitting then decreases. The antisymmetric branch is always between the bulk phonons and the symmetric modes. The phonon spectrum for the two-electron sheet is shown in Fig. 1b.

In conclusion, we have shown that the electron sheet, in particular, a 2D electron gas, localizes the acoustic modes due to only the electron–phonon coupling even in the case of uniform lattice characteristics. The confined modes propagate along the sheet and waves. The splitting of the modes from the bulk phonons increases when the wave vector $q$ increases and reaches a maximum at $q = 2k_F$; the split modes then converge to the bulk phonons with increasing $q$. Additional features of the confined modes arise for the case of several electron sheets.

From the above results it follows that the confinement effect is significant in the media with strong electron–phonon coupling, large effective mass of electrons, high concentration, and low temperature.

We use the following typical parameters of the semiconductors to estimate the value of the confinement effect: $\lambda + 2\mu = 10^{12}$ gr/cmsec$^2$, $b = 15$ eV, $m = 0.5m_0$ ($p$-material), $\epsilon_0 = 15$. Then, for a typical electron concentration for a $\delta$-doping layer $n = 6.7 \times 10^{12}$ cm$^{-2}$ ($k_F = 6.488 \times 10^6$ cm$^{-1}$ [4]), we find the results shown in Figs. 1a and 1b. For semiconductors with a large dielectric constant (the IV–VI compounds) the splitting increases by as much as a factor of 3.

The confinement of the acoustic modes by the electron sheet can thus be considerable. It can be investigated by acoustical measurements of the semiconductors with $\delta$-doping. The scattering on the confined modes can affect the electron transport in these materials.


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