Rotational 2D and 3D solitons in magnetic materials

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Stable, localized 2D and 3D solitons can exist in magnetic materials with a nonuniaxial symmetry. They are stabilized by conservation of the angular momentum of the magnetization field.

Static 2D and 3D solitons of a nonlinear field, in particular, of the magnetization field **M** in a ferromagnet with an energy

$$E = \int \{\alpha/2(\nabla \mathbf{M})^2 + w_a(\mathbf{M})\} d\mathbf{r} \tag{1}$$

(α is the inhomogeneous-exchange constant, and w_a is the anisotropy energy), can be stabilized only if there exists some integral of motion in addition to the energy. Otherwise, localized solitons are unstable[1] ("localized" in this case means that the direction of the magnetization **M** far from the soliton is constant). Examples of 2D and 3D solitons were constructed in Refs. 2–6 in uniaxial ferromagnets, in which the anisotropy energy $w_a = w_a(\theta)$ is independent of the azimuthal angle φ [θ and φ are angular variables; $M_x + iM_y = M_0 \sin\theta \exp(i\varphi)$; and z is the easy axis of the ferromagnet]. All these solitons were stabilized by conservation of the z projection of the resultant magnetization, $I_z = \int M_z d\mathbf{r}$. They were characterized by a precession of the magnetization at a constant frequency ω [$\varphi = \varphi_0(\mathbf{r}) + \omega t$] and an amplitude $\theta = \theta(\mathbf{r})$ which depends on the coordinate. The same behavior prevails for several nonlinear models of a complex field,[7] antiferromagnets,[8,9] etc. If the anisotropy energy instead does depend on the angle φ (as in, for example, biaxial ferromagnets and uniaxial ferromagnets with an anisotropy in the xy basal plane), the projection I_z is no longer an integral of motion, and the precession solitons discussed above do not exist.

Solitons can be stabilized, even under the conditions $\partial w_a/\partial\varphi \neq 0$ and $I_z \neq \text{const}$, as a result of the conservation of another quantity: the orbital angular momentum of the magnetization field, **L** (Refs. 3–6, 1):

$$\mathbf{L} = (M_0/g) \int (1 - \cos\theta)[\nabla\varphi, \mathbf{r}] d\mathbf{r}. \tag{2}$$

The **L** conservation condition is associated with an isotropy in coordinate space, rather than in spin space. Specifically, if the tensor α_{ij} in the general formula for the inhomogeneous-exchange energy, $\alpha_{ij}(\partial M/\partial x_i)(\partial M/\partial x_j)$, is proportional to δ_{ik} [see (1)], then all three projections of **L** are conserved. This assertion applies, for example, to cubic ferromagnets, and it also applies to many biaxial magnetic materials, e.g., orthoferrites.[10] In uniaxial magnetic materials with an arbitrary anisotropy in the basal plane we would have $\alpha_{ij} = \text{diag}(\alpha,\alpha,\alpha_z)$, and L_z would be conserved.

If $\mathbf{L} = \text{const}$ or $L_z = \text{const}$, one can show that the Landau–Lifshitz equations for $\mathbf{M}$ have dynamic solutions of the form

$$\theta = \theta(\mathbf{r}'), \quad \varphi = \varphi(\mathbf{r}'), \quad \partial \mathbf{r}'/\partial t = [\vec{\omega}, \mathbf{r}'], \tag{3}$$

where $\vec{\omega}$ is a constant vector. In the case $\alpha_{ik} = \text{diag}(\alpha,\alpha,\alpha_z)$ we have $\alpha_{ik} = \text{diag}(\alpha,\alpha,\alpha_z)\,\vec{\omega}\| e_z$; in the case $\alpha_{ik} \sim \delta_{ik}$, the orientation of $\vec{\omega}$ is arbitrary. Adopting $\vec{\omega} = \omega e_z$ for definiteness, we transform (3) to the form $\theta = \theta(x',y',z)$, $\varphi = \varphi(x',y',z)$, where $x' = x \cos \omega t - y \sin \omega t$, $y' = x \sin \omega t + y \cos \omega t$. For the functions θ and φ we find from the general Landau–Lifshitz equation an equation which is "stationary" in a rotating coordinate system:

$$\begin{aligned} &\alpha \nabla^2 \theta - \sin\theta \cos\theta (\nabla \varphi)^2 - \partial w_a/\partial \theta = -(\omega/gM_0)\sin\theta(\partial\varphi/\partial\chi), \\ &\alpha \nabla(\sin^2\theta \nabla\varphi) - \partial w_a/\partial\varphi = (\omega/gM_0)\sin\theta(\partial\theta/\partial\chi), \end{aligned} \tag{4}$$

where $\nabla = \partial/\partial r'$, $x' = r \cos\chi$, $y' = r \sin\chi$. These equations are the conditions for an extremum of the functional

$$F = E - \omega L_z = M_0^2 \int d\mathbf{r}\{\alpha/2[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2] + w_a(\theta,\varphi) - \omega/gM_0(1-\cos\theta)\partial\varphi/\partial\chi\}. \tag{5}$$

The condition $\delta F = 0$ can be thought of as a condition for a minimum of the energy of the ferromagnet, E, for a given value of L_z; the quantity ω would have the meaning of a Langrange multiplier; and we would have $dE/dL_z = \omega$.

The simplest soliton, which is stabilized by L_z conservation, is a 2D soliton with a topological charge ν (Refs. 3–5):

$$\nu = (1/4\pi) \int \sin\theta d\theta d\varphi = \pm 1, \pm 2, \ldots .$$

Corresponding to this case are the boundary conditions $\theta(0,\chi) = \pi$, $\theta(\infty,\chi) = 0$, $\varphi(r,\chi + 2\pi) = 2\pi\nu + \varphi(r,\chi)$. Although it is not possible to find an explicit solution of (4) with $w_a = w_a(\theta,\varphi)$ in this case, we can work from physical considerations to construct trial functions. Calculating $F(a_i)$ and determining the corresponding parameter a_i from the condition $\partial F/\partial a_i = 0$, we can then determine the functions $E(L_z)$ and $\omega(L_z)$ which characterize the soliton.

We choose the trial functions to have the structure $\varphi = \nu\chi + f(\chi)$, $f(0) = f(2\pi)$, $\theta = \theta(\xi)$, $\xi = \xi(r,\chi)$. For an orthorhombic ferromagnet with $w_a = (\beta/2)\sin^2\theta(1 + \epsilon\cos^2\varphi)$, we have $\xi = [(x'/c)^2 + (y'/b)^2]^{1/2}$ and $f(\chi) = B \sin 2\nu\chi$. If $\epsilon \ll 1$, then we have $B \propto \epsilon$ and $(c - b) \propto \epsilon$. The functional dependence $\theta(\xi)$ is chosen to be analogous to $\theta(r)$ for a radially symmetric soliton.[1,4,5,9] In the limiting cases of large- and small-radius solitons, with $\xi_0 \gg (\alpha/\beta)^{1/2}$ and $\xi_0 \ll (\alpha/\beta)^{1/2}$, where

$\theta(\xi_0) = \pi/2$, the properties of the soliton can be described by analytic functions. For example, we can write

$$\omega(L_z) = \omega_0 \begin{cases} (1/\nu^2) f_\nu(L_z/\hbar N_0), & L_z \ll \hbar N_0, \\ (\hbar N_0/\nu L_z)^{1/2}, & L_z \gg \hbar N_0, \end{cases} \tag{6}$$

where $N_0 = 2\pi s(\alpha/\beta a^2) \gg 1$, s is the spin of the atom, a is the lattice constant, and $\omega_0 = gM_0\beta(1+\epsilon)^{1/2}$ is the magnon activation frequency. As $x \to 0$ we have $f_\nu(x) \to 1$. This function is analytic for $\nu > 1$ and has a singularity in its derivative for $\nu = 1$. The reader interested in more details on the choice of the trial functions and on the construction of the functions $\omega(L_z)$ is referred to Refs. 5 and 9. These functions are qualitatively the same as for precession solitons, but they contain other powers of the topological charge ν.

The magnetization distribution in the soliton and its time evolution are sketched in Fig. 1. As $\epsilon \to 0$, the lines of $\theta = \text{const}$ transform into circles, and the rotational soliton becomes indistinguishable from a precession soliton.

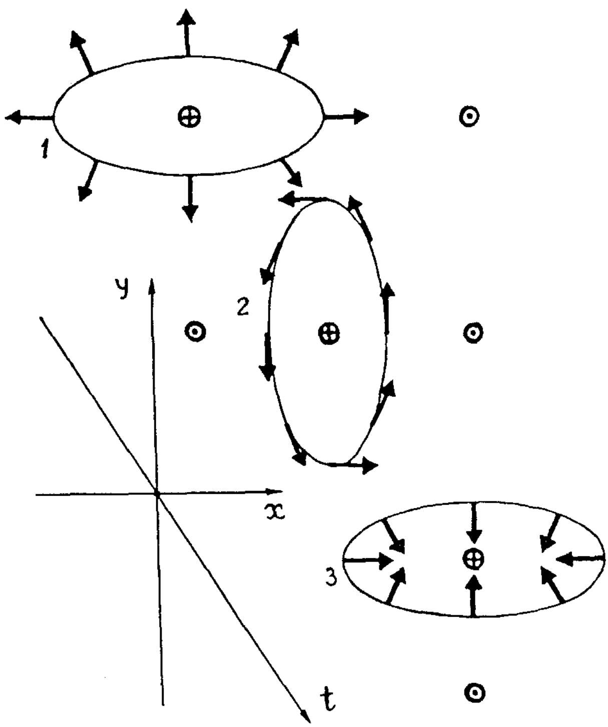

FIG. 1. Distribution of the magnetization (arrows) at certain points of a rotational soliton at several times: 1—$t = 0$; 2—$t = T/4$; 3—$t = T/2$, where $T = 2\pi/\omega$ is the magnetization rotation period. On the ellipse we have $\theta = \pi/2$; at the center of the soliton we have $\theta = \pi$ ($\times$); and far from the soliton we have $\theta = 0$ ($\odot$). The x and y axes are the hardest axis and the intermediate axis in the xy basal plane; the z axis is the easy axis.

In a corresponding way we could construct a rotational soliton which is localized in three dimensions. Its topological properties are determined by the Hopf invariant $h = \pm 1, \pm 2$ (Refs. 6 and 11). For such a soliton we have $L_z \sim h \neq 0$, but its structure and the functions $\omega(L_z)$ and $E(L_z)$ are more difficult to analyze; these topics lie outside the scope of the present letter. Rotational solitons may exist in nonlinear models other than ferromagnets. A particularly interesting possibililty is that they exist in real scalar or vector Lorentz-invariant models [such as the sine-Gordon and φ^4 models, for which we have $L_z \sim \int (\partial\varphi/\partial t)[\vec{r},\nabla\varphi]d\vec{r}$, or antiferromagnets[1,9]].

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