Nonequilibrium neutrinos and primordial nucleosynthesis

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We have found that the residual interactions between neutrinos and electrons, which have different temperatures after decoupling, result in appreciable spectral distortion of the order of 1% on the higher energy side of the distribution, when the temperature drops below 1 MeV. The resulting modification in the helium abundance, however, is small. It is only of the order of $\Delta Y \approx 1.3 \times 10^{-4}$.

The correct prediction of the cosmic abundances of light elements has been regarded as a great success of the standard hot universe model.[1–4] The only free parameter, the baryon-to-photon ratio N_B/N_γ, deduced from a comparison between the prediction and the observation of the primordial abundances of d, ^{3}He, ^{4}He, and ^{7}Li, now converges to a very narrow range. It is interesting to ask, however, to what accuracy such an agreement holds, when more precise estimates become available for the primoridal elemental abundances. For the primordial helium, for instance, the latest value from HII galaxies is $Y_{obs} = 0.229 \pm 0.004$ (Ref. 5), with a relative error at a 2% level. Taking a small error seriously, this is marginally consistent with the standard calculation with three neutrino species and with N_B/N_γ determined from $^3\text{He} + d$ and ^{7}Li: $Y = 0.236$–0.243 (Ref. 3).

After freezing the neutron-to-proton ratio, the calculation can be accurately carried out with the standard code, and the uncertainties in the nuclear reaction rates are small. In the calculation of the n/p ratio, however, all authors have assumed the equilibrium Fermi distribution for the electron neutrino spectrum. Let us examine this assumption: neutrinos decouple from the primeval plasma at a temperature $T \sim 3$ MeV for ν_e and at 5 MeV for ν_μ and ν_τ. In this epoch there is no doubt that the neutrino spectrum is described well by the Fermi distribution. After this epoch, however, the temperatures of neutrinos and of the $e^\pm$ and γ plasma change because of annihilation of e^+e^- pairs, which heats the electromagnetic component of the plasma. The relative temperature difference is about 0.9×10^{-3} at 3 MeV, about 1.6×10^{-2} at $T = 0.7$ MeV, and reaches eventually the well-known value of 29%.[4] Although equilibrium ceases at a few MeV, some thermal contact between electrons and neutrinos remains, especially in the high energy tail of the neutrino spectrum due to stronger interactions between them at a higher energy. This would distort the equilibrium Fermi distribution. In fact we find that this distortion amounts to as much as 1% for the higher-energy side of the spectrum. This motivates us to examine the change in the n/p ratio

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caused by this distortion. Few authors, found that the effect is due to the temperature difference between the photon and the neutrino components.[6-8] These authors, however, considered only average heating of the neutrino gas because of the residual interaction between electrons and neutrinos, and assumed that the effect is renormalized to a change in the effective neutrino temperature. What we really need to see is, however, the effect of the distorted spectrum, which cannot simply be absorbed into the temperature. In this paper we study the effect of nonequilibrium on the n/p ratio by directly solving the kinetic equations.

The kinetic equation that governs the ν_e phase space distribution in the expanding universe has the form

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) n_\nu(t,p) = S \ , \tag{1}$$

where $H = 1/2t$ is the expansion parameter $\dot{a}/a$, and $p = E$ is the neutrino momentum with the mass of neutrinos which is assumed to be negligible. The collision term S is given, for example, for $\nu\bar{\nu} \leftrightarrow e^+e^-$ by the integral

$$S = \frac{(2\pi)^4}{2p} \int d\tau(e^-)d\tau(e^+)d\tau(\bar{\nu})\delta^4(p_+ + p_- - p - \bar{p}) \mid A(\nu\bar{\nu} \leftrightarrow e^+e^-) \mid^2$$
$$[n_{e^+}n_{e^-}(1-n_\nu)(1-n_{\bar{\nu}}) - n_\nu n_{\bar{\nu}}(1-n_{e^+})(1-n_{e^-})] \ , \tag{2}$$

where p_+,p_-,p, and $\bar{p}$ are the momenta of e^+, e^-, ν, and $\bar{\nu}$ and $d\tau(e^-) = d^3p_-/(2\pi)^3 2E_-$, etc. is the phase space volume element for the respective particles. The amplitude in the integrand is written

$$\mid A(\nu\bar{\nu} \leftrightarrow e^+e^-) \mid^2 = 128G_F^2[g_L^2(pp_+)^2 + g_R^2(pp_-)^2 + g_L g_R m^2(p\bar{p})] \ , \tag{3}$$

where $G_F = 1.03\times10^{-5}/m_N^2$ is the Fermi coupling constant, $g_L = 1/2 + \sin^2\theta_W$, $g_R = \sin^2\theta_W (\sin^2 Q_W = 0.23)$, and m is the electron mass. There are also contributions to S from the elastic scattering $\nu e^\pm \leftrightarrow \nu e^\pm$ etc., which are taken into account below.

For the energy region that concerns us, the number densities of neutrinos and electrons are small enough, so that we can approximate the Fermi distribution by the Boltżmann distribution, especially when we are concerned with small correction terms. For electrons and positrons the Coulomb and Thomson scattering processes are fast enough, and their distribution is given by the equilibrium form

$$n_e = \exp(-E_e/T_\gamma) \simeq \exp(-E_e/T)\left(1 + \frac{E_e}{T}\frac{\Delta T}{T}\right) \ . \tag{4}$$

Here the temperature of the $e\gamma$ plasma, T_γ, differs from the neutrino temperature T by $\Delta T = T_\gamma - T$. We write the neutrino distribution in the form

$$n_\nu = \exp(-E_\nu/T)[1 + \delta(p,t)] \ , \tag{5}$$

where $\delta(p,t)$ is the spectral distortion due to neutrino heating by electrons and positrons.

Substituting Eqs. (4) and (5) into Eq. (2), we obtain a kinetic equation for δ. It is a linear integrodifferential equation which is not easy to handle, but we shall see in what follows that δ is a small quantity in the temperature range responsible for determining the neutron-to-proton ratio and the δ term, which appears in the second term on the right-hand side, can be ignored in a first approximation.

We should also take into account the heating of neutrinos by elastic νe^- and νe^+ scattering. These processes conserve the neutrino numbers, but modify the spectrum. The kinetic equation can then be written as follows:

$$\left(\frac{\partial}{\partial t} - HE\frac{\partial}{\partial E}\right)\delta(E,t) \approx \frac{16G_F^2(g_L^2+g_R^2)}{3\pi^3}\frac{\Delta T}{T}T^3E[E+4T+\frac{7}{4}(E-4T)] , \quad (6)$$

where the terms proportional to δ are ignored on the right-hand side. Since $\dot{T} = -HT$, we can easily integrate Eq. (6) and obtain

$$\delta(E/T,T) \approx 0.031\frac{E}{T}\left(\frac{11}{4}\frac{E}{T}-3\right)\int_\eta^{\eta_i} d\eta\eta^2\frac{\Delta\eta}{\eta} , \quad (7)$$

with use of $t=(45/32\pi^3 g)^{1/2}m_{pl}T^{-2}$ (g is the number of relativistic degrees of freedom, and m_{pl} is Planck's mass). Here η is the temperature (in units of MeV and n_i is its initial value corresponding to the decoupling of ν_e from the plasma. $\Delta\eta/\eta \equiv \Delta T/T$ is given in Ref. 4. We note that $(\Delta T/T)T^2 \simeq 0.60\times 10^{-2}$ $(\mathrm{MeV})^2$ for wide range of T, from 3 MeV to 0.5 MeV (within 3%; the error is only 10% even at 0.3 MeV). With $T_i \approx 3$–4 MeV (see below), this yields approximately

$$\delta \approx 6\times 10^{-4}(E/T)(11E/4T-3) \quad (8)$$

at $T \simeq 0.6$ MeV. The corrections due to the inverse processes and the dependence of the freezing temperature on the neutrino energy have been taken into account in our detailed calculations and proved to be small.

The helium abundance is basically determined by the neutron-to-proton ratio n/p, which is fixed by the competition of $n+\nu \leftrightarrow p+e^-$ and $n+e^+ \leftrightarrow p+\bar{\nu}$ with the expansion rate of the universe. The kinetic equation that governs the evolution of neutron number density with n_ν, given by Eq. (5), can be written as follows:

$$\frac{dr_n}{dT} = -0.05T^2\int_0^\infty dx x^2\left(x+\frac{\Delta M}{T}\right)^2 e^{-x}\{e^{-\Delta M/T}[1+\frac{1}{2}\delta(E+\Delta M)]$$
$$-r_n[1+\frac{\delta(E)}{2}+e^{-\Delta M/T}\left(1+\frac{\delta(E+\Delta M)}{2}\right)]\} , \quad (9)$$

where $r_n = r_n(t)$ is the fraction of the neutron number against the total number of baryons (so that $r_n + r_p = 1$), $\Delta M = 1.29$ MeV is the neutron-proton mass difference, $x = E/T$ and $\delta(x,T)$ are given by Eq. (7) or (9), and $\Delta T/T$ is given by Eq. (8). We integrated Eq. (21) numerically with the fourth-order Runge–Kutta algorithm. Below 0.3 MeV, the approximation $(\Delta T/T)T^2 = \mathrm{const}$ is not valid, but the effect is small and it can be easily taken into account in the final answer. We thus find the deviation of r_n from the standard value, $r_n\,(\varepsilon=0)$, to be 0.9×10^{-4} at low enough temperatures. This is indeed a very small number, compared with the number which we naively expect from the deviation of the neutrino spectrum from the Fermi distribution.

Accordingly, the influence of the nonequililbrium distribution of neutrinos to the neutron-to-proton ratio is very small and it changes the helium abundance only by the amount $\Delta T= -1.3\times10^{-4}$. This value may nominally be compared with those obtained by Dicus *et al.*,[6] $+3\times10^{-4}$, and by Herrera and Hacyan,[7] -2×10^{-4} (note that the signs do not agree with each other), and also by Rana and Mitra,[8] -3×10^{-3}, which is too large. All authors estimated the effect as a shift of the effective neutrino temperature, and hence of the freezing temperature of the beta equilibrium. Our emphasis here is on the point that one cannot absorb the effect into the shift of the freezing temperature. This may be demonstrated by the fact that the correction to the n/p ratio is temperature dependent. For instance, according to our calculations, ΔY would be $+1.1\times10^{-4}$ if we adopt r_n near the freezing temperature, usually accepted for beta equilibrium, $T\approx0.7$ MeV. Anyway, the effect seesm too small ($\Delta Y/Y= -0.05\%$) to be observationally relevant. The standard calculation, assuming the equilibrium distribution, is sufficiently accurate as a matter of fact for the purpose of estimating the helium abundance. If the discrepancy between the prediction and the observation is actually present for the primordial helium abundance, we must look for the reason elsewhere.

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