Low-energy scattering parameters for resonance Coulomb systems

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A formula for the nuclear-Coulomb effective radius $r_l^{(sc)}$ has been obtained. This formula can be used for an arbitrary angular moment l and for establishing constraints on the quantity $r_l^{(cs)}$ which do not depend on the form of the strong potential. The scattering lengths and the effective radii for the lightest hadron systems have been calculated.

1. Study of low-energy resonances in very light hadronic systems (dt, $d\,^3$He, $\alpha\alpha$, ^{7}Li p, etc.) is of interest in nuclear physics, astrophysics, nuclear fusion (including μ catalysis), and in other fields. The cross sections for the fusion reaction $dt \to n\alpha$ and $d\,^3\,\mathrm{He} \to p\alpha$ have recently been measured very accurately near the s-wave resonances ^{5}He*$(3/2^+)$ and ^{5}Li*$(3/2^+)$ (see, e.g., the review article in Ref. 1). Reliable phase analysis of the elastic scattering so far, however, has not been carried out. The resonance wave ($l=0$, $J^P=3/2^+$) in these systems plays a dominant role in the elastic scattering and fusion reaction which determines primarily the reaction cross section σ_r. As was shown in Ref. 2, the amplitude of the elastic scattering of slow, charged particles (in contrast with the neutral particles) can be reconstructed from the data on the cross section $\sigma_r(E)$, which makes it possible to determine the complex scattering lengths a_{cs} and the effective radius r_{cs} $(l=0)$: $a_B/a_{cs} = \alpha_0 - i\beta_0$, $r_{cs}/a_B = 2(\alpha_1 - i\beta_1)$. The accuracy of determining each of these parameters is fairly high, on the order of several percent; however, the fitted variation of all four low-energy parameters, α_i and β_i, admits their variation over a fairly broad interval as χ^2 increases only slightly. This is illustrated in Table I (using the dt system as an example), in which the values of a_{cs} and r_{cs} vary by a factor of 2. Additional selection criteria must therefore be found.

Such a criterion might be a constraint imposed on r_{cs}, which is linked with the Coulomb renormalization of the scattering parameters which becomes, rather unexpectedly, quite important as $r_N > a_B$ (in contrast with the case $r_N/a_B \ll 1$, which was analyzed in Ref. 3, for example, in the theory of the $p\bar{p}$ atom). Here $a_B = \hbar^2/Z_1Z_2e^2m$ is the Bohr radius of the system, m is the reduced mass, and r_N is the effective range of nuclear forces (we are considering the case of Coulomb repulsion, i.e., $Z_1Z_2>0$).

2. The coefficients of expansion of the effective radius[1)] in powers of k^2 are the nucleus-Coulomb (cs) low-energy parameters:

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TABLE I.

α_0	$-a_{cs}$	r_{cs}	R_c	χ^2
0,20	$112+i31,0$	$6,4-i0,36$	11,3	2,00
0,21	$105+i31,2$	$6,2-i0,32$	10,1	1,28
0,22	$99+i31,3$	$6,0-i0,28$	9,2	0,85
0,23	$94+i31,4$	$5,8-i0,26$	8,4	0,66
0,238	$90+i31,4$	$5,7-i0,25$	7,8	0,62
0,26	$80+i31,2$	$5,2-i0,30$	6,2	0,67
0,28	$73+i30,9$	$4,6-i0,34$	5,0	0,82
0,30	$66+i30,5$	$3,9-i0,36$	3,7	0,90
0,32	$60+i30,3$	$3,2-i0,23$	2,7	1,08

Note. The sets of low-energy parameters of the dt system listed in this table correspond to the local minimum of χ^2 (per experimental data point) for a fixed value of α_0. The values of a_{cs}, r_{cs}, and R_c are given in fm.

$$K_l^{(cs)}(k^2) = -1/a_l^{(cs)} + \frac{1}{2} r_l^{(cs)} k^2 + O(k^4). \quad (1)$$

The effective radius $r_l^{(cs)}$ can be expressed in terms of the radial wave function with zero-point energy, $\varphi_1(r) \equiv rR_l(r,k=0)$:

$$r_l^{(cs)} = \frac{a_B^{1-2l}}{3(l!)^2} - 2[(2l-1)!!]^2 \int_0^\infty \{\varphi_l^2(r) + \frac{2}{a_l^{(cs)}} g_l(r) f_l(r) - \frac{1}{[a_l^{(cs)}]^2} f_l^2(r)\} dr. \quad (2)$$

Here $g_l(r)$ and $f_l(r)$ are the solutions of the Schrödinger equation with $E=0$ in the Coulomb repulsion field:

$$g_l = \frac{2^{2l}}{(2l)! a_B^l} \rho K_{2l+1}(\rho), \quad f_l = \frac{2^{l-2}(l!)^2}{(2l)!} a_B^{l+1} \rho I_{2l+1}(\rho), \quad (2')$$

$\rho = (8r/a_B)^{1/2}$, $K_{2l+1}(\rho)$, and $I_{2l+1}(\rho)$ are the Bessel functions of the imaginary argument, while the function $\varphi_l(r)$ is normalized by the asymptotic condition:

$$\varphi_l(r) = g_l(r) - \frac{1}{a_l^{(cs)}} f_l(r), \quad r \gg r_N \quad (2'')$$

[the derivation of Eq. (2) will be given in a more detailed paper].

At an exact resonance (i.e., at the time at which a bound l level forms) we have $a_l^{(cs)} = \infty$, $\varphi_l(r) \equiv \chi_l(r)$. Here $\chi_l(r) \propto r^{l+1}$ as $r \to 0$ and it decays exponentially at infinity:

$$\chi_l(r) = g_l(r) = \mathrm{const}\, r^{1/4} \exp\{-(8r/a_B)^{1/2}\}, \quad (3)$$

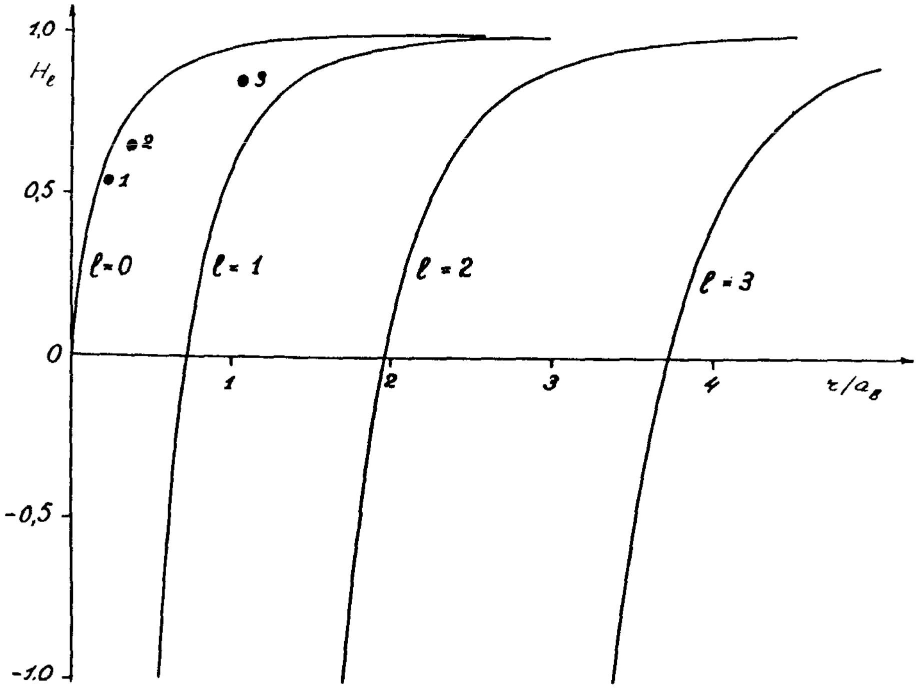

FIG. 1. The function H_l verus r/a_B. Data points 1, 2, and 3 correspond to dt, $d\,^3$He, and $\alpha\alpha$ systems (their abscissa is r_N/a_B and their ordinate is $3\tilde{r}_{cs}/a_B$).

which is attributable to the Coulomb barrier. Equation (2) in this case simplifies considerably; we denote the corresponding effective radius by $\tilde{r}_l^{(cs)}$.

3. Let us assume that R_c is the minimum distance, beginning at which the strong interaction is negligible compared with the Coulomb interaction. Using (2) and (3), we find

$$\tilde{r}_l^{(cs)} \leq \frac{a_B^{1-2l}}{3(l!)^2} H_l(R_c), \tag{4}$$

where $H_l(r) = 1 - 3/2 \int_\rho^\infty K_{2l+1}^2(x)x^3dx$ (Fig. 1) [the sign of equality in (4) is attained for the Breit model[6] of the strong potential $V(r)$].

Let us consider the effectiveness of this constraint. Table I gives the optimal variants of the fit of the low-energy parameters α_0, β_0, α_1, and β_1, all of which run through the criterion χ^2 (except, perhaps, the first set). The absolute minimum of χ^2 corresponds to the set with $\alpha_0 = 0.238$ which leads, however, to a physically unacceptable[2)] value $R_c \approx 8$ fm (especially since the first three sets of parameters are eliminated from Table I). It is reasonable to assume that $R_c - r_N \sim \delta \approx 1$ fm, where δ is the diffusivity of the edge of the nucleus. As can be seen in Table I, this condition, along with (4), sharply decreases the indeterminacy in the extraction of a_{cs} and r_{cs} from the experimental data on the reaction cross section.

TABLE II. Parameters of the resonance Coulomb systems.

System	Nucleus	l	a_B, fm	$\frac{r_N}{a_B}$	E_R, keV	$-a_B^{2l+1}/a_l^{(cs)}$	$a_B^{2l-1}\cdot r_l^{(cs)}$
dt	$^5He^*(3/2^+)$	0	24,0	0,15	$46,0-i37,7$	$0,280+i0,119$	$0,191-i0,015$
d^3He	$^5Li^*(3/2^+)$	0	12,0	0,33	$194-i127$	$0,200+i0,025$	$0,256-i0,014$
^7Lip	$^8Be^*(3^-)$	1	11,0	0,30	$386-i5,4$	14,5	−6,83
^7Bep	$^8B^*(2^-)$	1	8,24	0,40	$637-i19$	5,22	−2,30
$\alpha\alpha$	$^8Be(0^+)$	0	3,63	0,91	$92,1-i1,8\times10^{-3}$	$3,05\times10^{-3}$	0,282

Note. As r_N we used the sum of the charged particle radii of the system.[1] The dimensionless expansion coefficients (1) are listed in the last two columns (in Coulomb units, $a_B=1$).

The results of the calculations are given in Table II. The radii r_{cs} for the s-wave resonances satisfy the minimum condition for χ^2 and the inequality (4). It is evident from Fig. 1 that the latter constraint is important, since the data points[3)] calculated by us are close to the upper limit of (4), which corresponds to the solid curve $l=0$. On the other hand, inequality (4) is satisfied with a wide margin for a few resonances with $l\neq0$ which we examined.

In Table II we give the positions of the right-hand [in the k plane, $k=(2E)^{1/2}$] poles of the S matrix, $E_R=E_r-i\Gamma/2$ and also the parameters of the expansion (1), which completely determine the analytic structure of the scattering amplitude near the elastic threshold ($k=0$), including two Coulomb pole series which bunch together toward the threshold.[2] The quantities $a_l^{(cs)}$ and $r_l^{(cs)}$ are real in the last three examples in Table II. As a result, the poles of the scattering amplitude are arranged symmetrically relative to the imaginary axis of the kth plane. In the remaining cases this symmetry is broken.

It can be shown that the radius $\tilde{r}_l^{(cs)}$ at the time of the formation of the l level determines the localization of the system:

$$w_<(R)=1-\frac{1-H_l(R)}{1-3(l!)^2a_B^{2l-1}\tilde{r}_l^{(cs)}},\quad R>R_c, \tag{5}$$

where $w_<(R)$ is the probability for finding the system at a distance $r<R$. For the dt system, for example, we have $w_<$ (5 fm)$\approx$0.05 and $w_<$ (10 fm)$\approx$0.50, while for $\alpha\alpha-w_<$ (4 fm)$\approx$0.75. With an increase in the ratio r_N/a_B, the Coulomb system "adjusts" to the effective range of the nuclear forces.

We note in conclusion that the constraint (4) holds for any manifestation of the strong potential $V_s(r)$ and for any value of the ratio r_N/a_B. It would be relevant to determine its application to the states with a nonvanishing angular momentum l.

We wish to thank D. Popov for assistance with the numerical calculations.

[1)]Its standard definition for an arbitrary moment is found, for example, in Refs. 4 and 5.

[2)]In the R-matrix approach, the radius of a charged channel for a dt system is usually assumed to be $R_c=5$ fm.

[3)]In the calculation of these points for dt and $d\,^3$He we introduced a correction for the incomplete connectedness of these systems, i.e., for the case in which $1/a_{cs}$ is nonvanishing (the corresponding points conform to the asymptotic curve [Eq. (4)] if this correction is ignored).

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