Optical bistability in the case of broadening by intrinsic pressure

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The so-called optical bistability, i.e., the ambiguity of the steady state of the medium in an electromagnetic field, has recently attracted considerable attention. Attention is focused here on the active systems (amplifying medium) and on the passive systems (absorbing medium) (H. M. Gibbs, Optical Bistability: Controlling Light with Light, Academic Press, Orlando, 1985). In the familiar passive systems the bistability is established by various feedbacks produced by basically complex devices. In contrast, a case is described in the present article, in which the optical bistability "at the atomic level" is attributed to the properties of strictly the absorbing atomic gas.

Let us consider a two-level atom (of energy $E_m > E_n$) which interacts with a monochromatic field. We will use a model of relaxation constants and nondegenerate states and we will restrict the analysis to a resonance approximation. The system of equations for nondiagonal element and diagonal elements (ρ_m, ρ_n) of the density matrix will then have the form

$$\rho_m + \rho_n = N, \tag{1}$$

$$\dot{\rho}_m = -\gamma_m \rho_m + 2\mathrm{Re}(iG\rho), \tag{2}$$

$$\dot{\rho} = -[\frac{1}{2}\gamma_m + \gamma_1 N - i\gamma_2(\rho_n - \rho_m) + i\Omega]\rho + iG^*(\rho_n - \rho_m), \tag{3}$$

$$G = d_{mn}E/2\hbar, \quad \Omega = \omega - \omega_{mn}. \tag{4}$$

Here E and ω are the strength and frequency of the field, N is the total number of particles (in cm^3), ω_{mn} and d_{mn} are the Bohr frequency and the matrix element of the dipole moment of the m–n transition, and γ_m is the rate of the spontaneous $m \to n$ decay. The coefficients γ_1 and γ_2 specify the line broadening and line shift as a result of interaction. Here the line broadening is proportional to the total number of particles $\rho_m + \rho_n = N$, and the line shift is proportional to the difference in the populations,[2] $\rho_n - \rho_m$. The system of equations (1)–(3) disregards the motion of particles, the Doppler broadening, and the inelastic processes due to the collisions.

The steady state $(\dot{\rho}_m = 0, \dot{\rho} = 0)$ is described by the equations

$$\rho = -\frac{iG^* N}{\gamma_1 N + \gamma_m/2} y, \quad y = \frac{1 - 2x}{1 + i(\epsilon + \eta x)} = \frac{1 - i(\epsilon + \eta x)}{1 + \kappa + (\epsilon + \eta x)^2}, \tag{5}$$

$$\rho_m/N = x = \frac{\kappa/2}{1+\kappa+(\epsilon+\eta x)^2}, \tag{6}$$

$$\kappa = \frac{4|G|^2}{\gamma_m(\gamma_1 N+\gamma_m/2)}, \quad \epsilon = \frac{\Omega-\gamma_2 N}{\gamma_1 N+\gamma_m/2}, \quad \eta = \frac{2\gamma_2 N}{\gamma_1 N+\gamma_m/2}. \tag{7}$$

Equations (5) and (6) essentially are the same as the results obtained by Karplus and Schwinger.[3] The exceptions are the line shift and line broadening due to the intrinsic pressure of the absorbing gas. The "nonlinear shift" [the term ηx in Eqs. (5) and (6)] accounts for the qualitative features.

Relation (6) should be treated as an equation (a cubic equation) for $x=\rho_m/N$, whose roots specify the steady-state values of ρ_m. We will consider the real, positive roots.

At small and large light intensities the real root is the same:

$$x = \kappa/[2(1+\epsilon^2)], \quad \kappa \ll 1+\epsilon^2, \tag{8}$$

$$x = 1/2, \quad \kappa \gg 1+\epsilon^2. \tag{9}$$

Relation (8) describes the "linear absorption" and relation (9) describes the complete equalization of the combined levels. Three positive roots can exist in the intermediate range of values $\kappa \sim 1$.

The plots of $x(\kappa)$ for several values of η and ϵ are shown in Fig. 1. At $\eta=1$, $\epsilon=\pm 1$ the curves are qualitatively the same as the case $\eta=0$, $\epsilon=0$. With an increase in the value of η ($\eta=5$, 8, 10), the behavior of the plots changes and, as can be seen in the case $\eta=10$, $\epsilon=-4$, Eq. (6) can have three positive roots, i.e., we have an optical bistability.

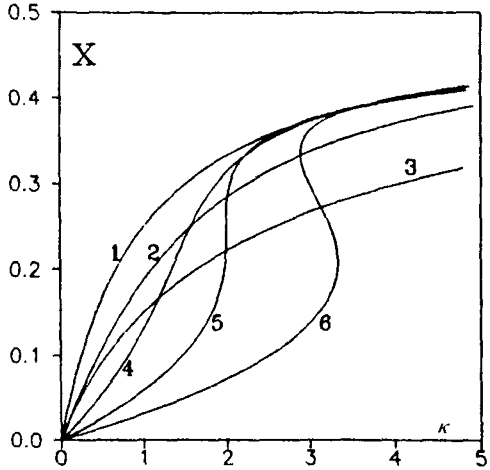

FIG. 1. Filling of the upper level (x) versus the light intensity (κ) for the following values of η and ϵ: 1) $\eta=0$, $\epsilon=0$; 2) $\eta=1$, $\epsilon=-1$; 3) $\eta=1$, $\epsilon=1$; 4) $\eta=5$, $\epsilon=-2$; 5) $\eta=8$, $\epsilon=-3$; 6) $\eta=10$, $\epsilon=-4$.

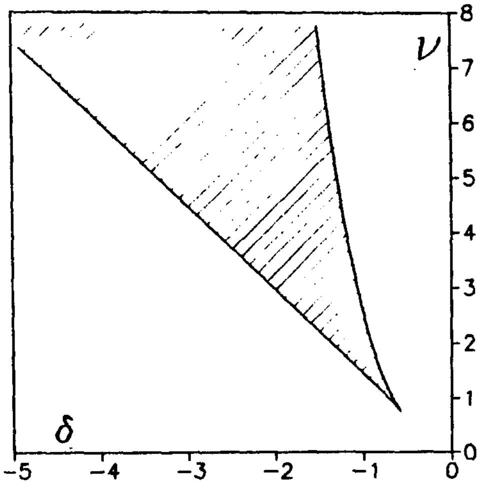

FIG. 2. The hatching shows the bistability region.

The physical cause of the bistability is obvious. Under nonresonance conditions ($|\epsilon| \gg 1$), a considerable power is required to even slightly fill the upper level. If in addition we have $\eta \gg 1$, then even a small value of x could drive the system from the nonresontant state to a resonant state, which would greatly increase the filling factor. The point at which the "anomalous plots" come in contact with the plot $\eta = 0$, $\epsilon = 0$ corresponds to the resonance condition, $\epsilon + \eta x = 0$.

It is easy to show that one of the roots corresponds to the unstable state, while the other two roots correspond to the stable state. The optical bistability therefore accounts for the hysteresis phenomena.[1,4] The optical bistability can also manifest itself in many other ways; specifically, it can manifest itself at all values which are in some way connected with the filling ρ_m, ρ_n of the m, n levels or with the dipole moment induced in the m–n transition. In particular, this case applies to the dependence of the absorption and the refractive index on the frequency ϵ, to the absorption and scattering spectra of the probing field which is at resonance with the m–n transition or with the adjacent m–l and n–j transitions, during the diffusion of the excited atoms, etc.

Using standard procedures, we easily see that Eq. (6) has three real roots if the following conditions are satisfied:

$$-\delta(1+\delta^2) - (\delta^2 - 1/3)^{3/2} < \nu < -\delta(1+\delta^2) + (\delta^2 - 1/3)^{3/2}, \tag{10}$$

where we have introduced the parameters

$$\nu = 1/4\kappa\eta/(1+\kappa)^{3/2}, \quad \delta = \frac{1}{3}\epsilon/\sqrt{1+\kappa}. \tag{11}$$

In Fig. 2 the hatching shows the region of the δ, ν plane, where the conditions (10) are satisfied. The starting point of the region

$$\nu = 4\sqrt{3}/9, \quad \delta = -1/\sqrt{3}\ ,$$

with $\kappa = 2$ corresponds to the values $\eta = 8$ and $\epsilon = -3$ (Fig. 1). The lower limit on the bistability region roughly corresponds to the line segment $-3\delta/2$, and the upper limit is the cubic parabola $-\delta(2\delta^2 + 1/2)$.

The usual interactions, which account for the broadening and shift of the spectral lines, give the values $\eta \approx 2$, while $\eta \gtrsim 8$, in my judgment, are not known. Large values of η are nonetheless consistent with the general laws of physics, and entities with the indicated values of η probably can exist. At any rate, the system which we described here has a legitimate claim as a model system.

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[1]H. M. Gibbs, *Optical Bistability: Controlling Light with Light*, Academic Press, Orlando, 1985.
[2]A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **51**, 1751 (1966) [Sov. Phys. JETP **24**, 1183 (1966)].
[3]R. Karplus and J. Schwinger, Phys. Rev. **73**, 1020 (1948).
[4]A. P. Kazantsev, S. G. Rautian, and G. I. Surdutovich, Zh. Eksp. Teor. Fiz. **54**, 1409 (1968) [Sov. Phys. JETP **27**, 756 (1968)].

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