Renormalization of 2D quantum gravity with matter in connection with black holes

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A single-loop renormalization of a 2D quantum gravity with matter is analyzed. The theory may be multiplicatively renormalizable at the single-loop level. Charged black holes which arise in the theory are discussed.

Witten's identification[1] of a 2D black hole in string theory has attracted considerable interest to black holes in 2D gravity. In particular, a classically solvable and multiplicatively renormalizable model of 2D gravity was studied in Ref. 2 as a "toy model" for the evaporation and production of black holes. A more general case of this model (a 2D gravity with matter) is described by the action

$$S=\int d^2x\sqrt{g}\exp(-2\phi)[R+\gamma g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{4}f(\phi)F_{\mu\nu}^2$$

$$+b(\phi)g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi+V(\phi,\psi)], \tag{1}$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, ϕ is a dilaton, $f(\phi)$ and $b(\phi)$ are arbitrary functions of ϕ, and ψ is a scalar. Particular cases of theory (1) describe the effective action of a boson string ($A_\mu=\psi=0$, $\gamma=4$, $V=\Lambda$) or of a heterotic string ($f=1$, $b=\text{const}$, $V=\Lambda$). A theory with action (1) can also be linked with a 4D Einstein-Maxwell theory by means of a compactification. Our purpose in the present letter is to study the renormalization of a theory with action (1).

For the discussion below we rewrite action (1) by means of the following transformations:

$$\tilde{\phi}=e^{-\phi},\qquad c_1\varphi=\gamma\tilde{\phi}^2,\qquad g_{\mu\nu}=e^{2\rho}\tilde{g}_{\mu\nu},$$

$$\rho=\frac{\gamma\tilde{\phi}^2}{4c_1}-\frac{1}{8\gamma}\ln\tilde{\phi}. \tag{2}$$

Omitting the tilde (˜) and introducing the functions f, b, and V, we can rewrite (1) as

$$S=\int d^2x\sqrt{g}[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi+c_1\varphi R-\frac{1}{4}f(\varphi)F_{\mu\nu}^2$$

$$+b(\varphi)g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi+V(\varphi,\psi)]. \tag{3}$$

Theory (3) is renormalizable in a generalized sense (in which changes in f, b, and V in the course of the renormalization are allowed).

We can evaluate the single-loop counterterms by the technique of Refs. 3. We also use the following choice of gauge conditions (in the electromagnetic and gravitational sector):

$$S_{GF} = -\frac{c_1}{2} \int d^2x \sqrt{g} (\nabla_\nu h^\nu_\mu - \frac{1}{2} \nabla_\mu h - \frac{1}{\phi} \nabla_\mu \varphi) \phi$$

$$\times (\nabla_\rho h^{\rho\mu} - \frac{1}{2} \nabla^\mu h - \frac{1}{\phi} \nabla^\mu \varphi) - \frac{1}{2} \int d^2x \sqrt{g}\, f(\phi) (\nabla_\mu A^\mu)^2, \tag{4}$$

where ϕ is a background dilaton, and $h_{\mu\nu}$, φ, and A_μ are quantum fields. As a result of this (exceedingly tedious) calculation,[1)] the single-loop renormalized action can be written

$$S_R = \int d^2x \sqrt{g} \left\{ \frac{1}{2} \left[1 - \frac{6}{\epsilon \varphi^2} - \frac{1}{4\epsilon} \left(\frac{b^{(1)}(\varphi)}{b(\varphi)} \right)^2 \right] \right.$$

$$\times g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + c_1 \varphi R + V(\varphi, \psi) \left(1 - \frac{1}{\epsilon c_1 \varphi} \right) - \frac{V^{(1)}(\varphi, \psi)}{\epsilon c_1}$$

$$\left. - \frac{\ddot{V}(\varphi, \psi)}{2\epsilon c_1} - \frac{f(\varphi)}{4} F^2_{\mu\nu} \left(1 - \frac{1}{\epsilon c_1^2} - \frac{f^{(1)}(\varphi)}{\epsilon c_1 f(\varphi)} \right) + b(\varphi) g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \right\}, \tag{5}$$

where $V^{(1)} \equiv \partial V / \partial \varphi$, $\dot{V} \equiv \partial V / \partial \psi$, and $\epsilon = 2\pi(n-2)$. The result in the dilaton-gravity sector is the same as that derived in Ref. 3.

It is easy to see that the single-loop renormalization of the fields can be chosen in the form

$$\varphi = \varphi_R, \quad c_1 = c_{1R}, \quad g_{\mu\nu} = e^{2\sigma(\varphi, c_1, \epsilon)} \tilde{g}_{\mu\nu},$$

$$\partial_\nu \sigma(\varphi, c_1, \epsilon) = \frac{1}{4c_1\epsilon} \left[6 \partial_\nu \varphi^{-1} - \frac{1}{4} \left(\frac{b^{(1)}}{b} \right)^2 \partial_\nu \varphi \right]. \tag{6}$$

The single-loop effective action in (5) then becomes finite if we make the following choice (for example):

$$V(\varphi, \psi) = V_1(\varphi) \exp(\sqrt{a(\varphi)} \psi) + V_2(\varphi) \exp(-\sqrt{a(\varphi)} \psi),$$

$$b(\varphi) = b \equiv \text{const}, \qquad a(\varphi) = \text{const} \tag{7}$$

and

$$f(\varphi) = \frac{f_0}{\varphi^3} \exp\left[- \left(\frac{1}{c_1} + \tilde{b} \right) \varphi \right], \qquad f_0 = \left(1 - \frac{\tilde{b}}{\epsilon c_1} \right) f_0^R,$$

$$V(\varphi) = \mu \varphi^2 \exp\left[- \left(\frac{a}{2} + \tilde{a} \right) \varphi \right], \qquad \mu = \left(1 - \frac{\tilde{a}}{\epsilon c_1} \right) \mu_R, \tag{8}$$

where $\tilde{a}$ and b are arbitrary constants. We have thus written out explicitly some dilaton potentials for which the theory is not only renormalizable in a generalized sense but also multiplicatively renormalizable in the single-loop approximation.

In terms of our original action, (1), the theory is multiplicatively renormalizable for

$$V(\phi) = \mu' e^{\alpha'\phi} e^{A' e^{-2\phi}}, \qquad b(\phi) = b e^{2\phi}, \qquad f(\phi) = f' e^{-(\alpha'-6)\phi} e^{B' e^{-2\phi}}. \tag{9}$$

When $e^{-\phi}$ is expanded in a series, the potential V in (9) corresponds to the particular case of the dilaton potential which would be expected from the loop corrections in closed strings.[4]

For static spherical configurations $F_{\mu\nu} = \tilde{f}(r) dr \Lambda dt$, $\phi(r)$, $\psi(r)$, with the asymptotically planar metric

$$ds^2 = -g(r)dt^2 + g^{-1}(r)dr^2, \quad g(r) \underset{r\to\infty}{\longrightarrow} 1 , \tag{10}$$

and $\psi(r) = \text{const}$, and for functions (9), we can derive the following solution of the classical field equations for (1) (see also Refs. 4 and 5):

$$\tilde{f}(r) = \tilde{f}_0 f^{-1}(\phi(r)) e^{2\phi(r)},$$

$$\phi(r) = \begin{cases} \phi_0 + \frac{\alpha}{2} \ln r, & \gamma \neq 4 \\ \phi_0 - \frac{Q}{2} r, & \gamma = 4, \end{cases} \tag{11}$$

where $\alpha = 4/\gamma - 4$, ϕ_0, and Q are arbitrary constants. For $g(r)$ we find

$$g(r) = \begin{cases} r^{\alpha+1}[-2m - \frac{1}{\alpha} \int^r ds\, s^{-\alpha} W(\phi(s)), & \gamma \neq 4 \\ e^{-Qr}[-2m + \frac{1}{Q} \int^r ds e^{Qs} W(\phi(s)), & \gamma = 4, \end{cases} \tag{12}$$

where $W(\phi) = V(\phi) - \tilde{f}_0^2 e^{4\phi}/2f(\phi)$, and m is a new constant, which plays the role of the mass of a black hole. Solutions (10) and (11) correspond to charged black holes with a multiple horizon.[2)]

In summary, we have shown that a 2D dilaton gravity with matter is multiplicatively renormalizable (for a certain choice of dilaton potentials) and that it allows solutions corresponding to charged black holes.

[1)]The details of these calculations will be published separately.
[2)]The physical properties of these solutions will be analyzed separately.

[1]E. Witten, Phys. Rev. **44**, 314 (1991).
[2]C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. D **45**, 1005 (1992).
[3]S. D. Odintsov and I. L. Shapiro, Phys. Lett. B **263**, 183 (1991); Europhys. Lett. **15**, 575 (1991); Yad. Fiz. **55** (1992) [Sov. J. Nucl. Phys. **55** (1992)] (in press); Pis'ma Zh. Eksp. Teor. Fiz. **54**, 205 (1991) [JETP Lett. **54**, 200 (1991)].
[4]O. Lechtenfeld and C. Nappi, Princeton Preprint 92/22, 1992.
[5]E. Elizalde and S. D. Odintsov, Barcelona Univ. Preprint 92/17, 1992.

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