Envelope solitons with different carrier frequencies in a dispersive nonlinear medium

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A scattering transformation method is used to solve the problem of two-frequency solitons in an optical fiber with a quadratic dispersion and a Kerr nonlinearity. Some "soliton spectral flashes" at sum and difference frequencies are observed. The energy of the solitons is reversibly transformed into these flashes.

A topic which has attracted considerable interest in the nonlinear optics of optical fibers is the propagation of solitons with different frequencies or different polarization states through a fiber.[1–9] However, even when there are only two components, the system of coupled equations for the envelopes $E_n(z,t)$, which can be written in the form

$$i(E_n' + v_n^{-1}\dot{E}_n) + \ddot{E}_n/2 + E_n(|E_n|^2 + 2|E_{3-n}|^2) = 0, \qquad n = 1,2 \ , \tag{1}$$

cannot be integrated exactly. This situation was explained in Ref. 10 on the basis that the Painlevé test is not satisfied for (1). Numerical simulation has shown that the solitons of system (1) lose energy through radiation in the course of collisions and may be destroyed.[4]

We believe that a description by means of Eqs. (1) is incomplete, since it ignores the appearance of field components at sum and difference frequencies as a result of the nonlinear polarization of the medium.

We write the field $E(z,t)$ as a superposition of fields at the initial frequencies ω_1 and ω_2 and at the sum and difference frequencies $\omega_{n,m} = \omega_n + 2m(\omega_2 - \omega_1)$:

$$E(z,t) = \mathrm{Re}(E_1(z,t)\exp(i(\omega_1 t - k_1 z)) + E_2(z,t)\exp(i(\omega_2 t - k_2 z))),$$

$$E_n = \sum_m E_{n,m}(z,t)\exp(im\Omega), \quad \Omega = 2((\omega_2 - \omega_1)t - (k_2 - k_1)z),$$

$$n = 1,2: \quad m = 0, \pm 1, \pm 2, \dots \ .$$

We assume that $E_{n,m}$ vary slowly over the scale $(\omega_2 - \omega_1)$, and we adopt a quadratic linear-dispersion law:

$$k(\omega) = v_0^{-1}\omega + \omega^2/2. \tag{2}$$

Taking into account the nonlinear response of the medium which is cubic in the field, we then find the following equations, in dimensionless variables, from Maxwell's equations:

$$i(E_n' + v_n^{-1}\dot{E}_n) + \ddot{E}_n/2 + E_n(|E_n|^2 + 2|E_{3-n}|^2) + E_n^* E_{3-n}^2 \exp(i(3-2n)\Omega) = 0,$$

$$n = 1, 2; \quad v_n^{-1} = v_0^{-1} + \omega_n. \tag{3}$$

Equations (3) for E_1 and E_2 contain, in compact form, equations for all the components $E_{1,m}$ and $E_{2,m}$.

Let us consider the overdetermined system of linear equations $\dot{\psi} = U\psi$, $\psi' = V\psi$ with the compatibility condition[11]

$$U' - \dot{V} + [U, V] = 0. \tag{4}$$

We choose

$$U = \lambda D + F(z,t),$$

$$D = \mathrm{diag}(1, -1, -1, 1), \quad F(z,t) = \begin{pmatrix} 0 & \Phi_1 & \Phi_2 & 0 \\ \Phi_8 & 0 & 0 & \Phi_3 \\ \Phi_7 & 0 & 0 & \Phi_4 \\ 0 & \Phi_6 & \Phi_5 & 0 \end{pmatrix},$$

$$V = (i\lambda - v_0^{-1})U + iD(\dot{F} - F^2)/2.$$

Then Eq. (4) becomes

$$i(F' + v_0^{-1}\dot{F}) + D(\ddot{F}/2 - F^3) = 0. \tag{5}$$

Equations (5) consist of eight equations for Φ_k and can be reduced: $\Phi_1 = \Phi_5 = -\Phi_4^* = -\Phi_8^* = E_1 \exp(i(\omega_1 t - k_1 z))$, $\Phi_2 = \Phi_6 = -\Phi_3^* = -\Phi_7^* = E_2 \exp(i(\omega_2 t - k_2 z))$. Using (2), we then find (3). The existence of representation (4) for (3) is sufficient for solving (3) by the scattering-transformation method.

Carrying out the inverse scattering transformation for a reflectionless potential containing N and M solitons with carrier frequencies ω_1 and ω_2, we obtain the $(N+M)$-soliton solution

$$E_1(z,t) = (a + ib)\exp(i(k_1 z - \omega_1 t))/2, \quad E_2(z,t) = (a - ib)\exp(i(k_2 z - \omega_2 t))/2, \tag{6}$$

$$a = \sum_{j,l}^{N+M} A_{jl}/\det[a_{j,l}], \quad b = \sum_{j,l}^{N+M} B_{j,l}/\det[b_{j,l}],$$

where $A_{j,l}$ is the signed minor of the element $a_{j,l}$, and det is the determinant of the matrices with the elements

$$C_{j,l} = (g_j^{-1} + g_l^*)/(h_j + h_l^*), \quad h_j = A_j + i(\Delta_j + \omega_j), \quad j,l = 1, 2, \ldots (N+M)$$

$$g_j = \exp(h_j(t - v_0^{-1}z - t_j) + ih_j^2 z/2 + i(\varphi_j + \delta)).$$

Here $\omega_j = \omega_1$ for $j \leqslant N$ and $\omega_j = \omega_2$ for $j > N$. We have $\delta = 0$ for the matrix $a_{j,l}$; $\delta = -\pi/2$ for $j \leqslant N$, and $\delta = \pi/2$ for $j > N$ for the matrix $b_{j,l}$. The $4(N+M)$ arbitrary real constants specify the amplitude A_j, the retardation parameter Δ_j, the initial retardation t_j, and the initial phase φ_j of the solitons.

The components $E_{n,m}$ are extracted from solution (6) by direct and inverse Fourier transforms, in a process in which the frequency interval near $\omega_{n,m}$ is singled out

$$E_{n,m}(z,t) = (2\pi)^{-1} \int_{\omega_L}^{\omega_U} d\omega \exp(i\omega t) \int_{-\infty}^{\infty} \exp(-i\omega\tau) E_n(z,\tau) d\tau,$$

where $\omega_L = \omega_n + 2(2m - 1/2)(\omega_2 - \omega_1)$, $\omega_U = \omega_n + (2m + 1/2)(\omega_2 - \omega_1)$.

Analysis of the $(1+1)$-soliton solution shows that the components $E_{1,0}$ and $E_{2,0}$, which correspond to slow amplitudes at the frequencies ω_1 and ω_2, asymptotically (as $z \to \infty$) become Schrödinger solitons:[12]

$$\lim_{z \to \infty} E_{n,o} = A_n \operatorname{sech} A_n (t - v_n^{-1} z) \exp(i A_n^2 z/2).$$

In a collision of solitons, these components become distorted. The components $E_{1,m}$ and $E_{2,m}$, corresponding to sum and difference frequencies, appear when the solitons approach each other, and they decrease exponentially when the solitons move apart. Accordingly, some "soliton spectral flashes" arise: Energy is briefly pumped from the fundamental frequencies of the spectrum to the sum and difference frequencies, and then the process reverses direction. With increasing distance from the original frequencies (with increasing value of the index m), the duration and amplitude of the components decrease: $E_{n,m} \propto (A_1 A_2/(\omega_2 - \omega_1)^2)^{-2|m|}$. A transformation of energy into the sum and difference frequencies is probably possible for a wide class of nonlinear media, but a complete restoration of energy to the original frequencies may not always occur. In this case, condition (3) would have to be satisfied, and the original pulses would have to have a soliton shape.

With these results in mind, let us look at the case in which light is propagating without loss near the frequencies ω_1 and ω_2, while the losses at other frequencies are so high that the field at the sum and difference frequencies can be ignored. In this case we can omit terms of the type $E_n^* E_{3-n}^2$ from (3), and (3) becomes (1). In this case, energy is again transformed into the sum and difference frequencies (the nonlinear properties of the medium have not changed), but the field at the sum and difference frequencies is negligible because of the rapid absorption by the medium, so the inverse process does not occur. Equations (1), which are widely used, thus actually correspond to the case of strong absorption at the sum and difference frequencies. This case does not always hold for quartz optical fibers. Specifically, for some typical experimental conditions[13]—a soliton length ~ 10 ps, an amplitude ~ 1 W, and wavelengths $\lambda_1 = 1.48$ μm and $\lambda_2 = 1.50$ μm—the number of nearest sum and difference frequencies falling in the transmission region of the fiber is ~ 10. An experimental observation of soliton spectral flashes would require inserting a delay between the pulses designed to cause the pulses to collide near the exit end of the fiber. By varying the delay, one could then observe various stages in the soliton spectral flashes.

It follows from these results that the transmission capability of optical-fiber data-transmission systems can be increased by making use of solitons with different carrier frequencies. If losses and crosstalk are to be suppressed, the nearest sum and difference frequencies must lie in the transmission region.

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