Quantum oscillations in an anyon gas in a magnetic field

I. E. Aronov
Institute of Radiophysics and Electronics, Academy of Sciences of The Ukraine, 310085, Kharkov, The Ukraine

É. N. Bogachek and I. V. Krive
Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukraine, 310164, Kharkov, The Ukraine

S. A. Naftulin
Institute of Single Crystals, Academy of Sciences of the Ukraine, 310141, Kharkov, The Ukraine

(Submitted 12 August 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **56**, No. 5, 285–288 (10 September 1992)

The thermodynamics of an anyon gas above the superconducting transition temperature in a magnetic field is analyzed. The magnetic permeability is calculated as a function of the temperature. The magnetic oscillations in anyon systems have distinctive features: a nonmonotonic temperature dependence of the oscillation amplitude and a sensitivity of the oscillation characteristics to the field direction.

Hypothetical particles with a fractional statistics (anyons), which were first described by Wilczek in the early 1980s,[1] have found widespread applications in $(2+1)$-dimensional models in quantum field theory and in solid state physics (Ref. 2, for example). Although arguments regarding an anomalous statistics are now generally accepted in the theory of the fractional quantum Hall effect,[3–5] and although an anyon mechanism for high-T_c superconductivity is being discussed in the description of the properties of layered superconductors,[6–9] anyons remain exotic particles and are invoked only to analyze anomalous states of matter. It is thus interesting to examine certain properties of an anyon gas in the normal phase (i.e., the nonsuperconducting phase) which might be identified comparatively easily in the laboratory. Since an "intrinsic diamagnetism" is a distinctive feature of such systems, it is natural to examine the response of an anyonic medium to an external magnetic field.

Anyons with an approximately normal statistics (with a statistical parameter $\Theta = \pi/N$, where N is a large integer) are known to be in a superconducting phase at temperatures $T < T_c = \pi\rho/2N^2m$ (ρ is the density of anyons, and m is their mass; we set $\hbar = c = 1$ here and below).[10] One might expect that the properties of anyonic matter in the normal phase ($T > T_c$) would be much like those of a $2D$ metal in a quantizing magnetic field. For anyons, however, this "magnetic" field $b = \rho\Theta/e$ (e is the charge of the anyon) is a fictitious field, which simulates exchange forces (and which is therefore proportional to the density) in the Hartree–Fock approximation. Above the temperature T_c of the phase transition (which is of the Kosterlitz–Thouless type), the properties of the normal phase are described by an equilibrium distribution of particles and holes in Landau levels with a "cyclotron" frequency $\Delta = 2\pi\rho/Nm$. An anyon diamagnetism should therefore be sensitive to temperature changes. We show below that the magnetic permeability $\mu(T)$ is a rapidly increasing function of the temperature at $T_c < T < \Delta$ and that the thermodynamic and kinetic characteristics of the system undergo anomalous oscillations as a function of the amplitude of the external magnetic field H and of the density ρ in this temperature interval.

Using a $1/N$ expansion, and treating the external magnetic field of induction $B(x)$ by perturbation theory, we can construct an effective Lagrangian of the field B in the anyonic medium:

$$\mathcal{L}_{eff}(B) = -\frac{e^2\rho}{4\pi m\alpha_2(T)}B(x)\int d^2x' K_0\{[2\pi\rho\alpha(T)/\alpha_2(T)]^{1/2}|x-x'|\}B(x'). \quad (1)$$

Here $K_0(z)$ is the modified Bessel function, $\alpha = (\alpha_3 + \alpha_4^2/\alpha_1)$, and the coefficients α_j ($j = 1,...,4$) determine the expansion of the thermodynamic potential in fluctuations of the statistical field:[10,11]

$$\delta\Omega = \frac{e^2}{4\pi m}\alpha_1(T)b^2 + \frac{me^2}{8\pi^2\rho}\alpha_2(T)\vec{\epsilon}^2 + \frac{me^2}{4\pi}\alpha_3(T)\alpha_0^2 + \frac{ie^2}{2\pi}\alpha_4(T)\alpha_0 b + ... \quad (2)$$

Here $\vec{\epsilon}$ and b are the electric and magnetic parts of the stress tensor;

$$\alpha_2 = 1 - \frac{\Delta}{2NT}\sum_{n=0}^{\infty}(2n+1)f_n(1-f_n), \quad (3)$$

$$\alpha_3 = \frac{\Delta}{N^2T}\sum_{n=0}^{\infty}f_n(1-f_n), \quad (4)$$

$$\alpha_2 = 1 - N^2\alpha_3, \qquad \alpha_4 = N\alpha_3; \quad (5)$$

and f_n is the one-particle distribution function

$$f_n = \left\{1 + \exp\left[\frac{\Delta}{T}\left(n+\frac{1}{2}\right) - \frac{\zeta}{T}\right]\right\}^{-1}, \quad (6)$$

where ζ is the chemical potential. In the absence of an external field, this distribution function is determined from the normalization condition $\Sigma_{n=0}^{\infty}f_n = N$. At $T=0$, this condition reduces to the following well-known rule (Ref. 9, for example): For anyons with a statistical phase $\Theta = \pi/N$, exactly N Landau levels are occupied in the mean-field approximation.

In deriving (1)–(5) we ignored the Coulomb interaction between particles and holes in the Landau levels. This simplification is justified only at $T > T_c$. In a superconducting phase, we would need to set $T=0$ in expressions (3)–(6). In this case, Lagrangian (1) becomes

$$\mathcal{L}_{eff}^{(0)} = -\frac{e^2\rho}{2mc^2}A^2, \qquad B = \operatorname{curl} A, \quad (7)$$

demonstrating a Meissner effect. In the normal phase, a uniform field $B = \text{const}$ penetrates into the medium, so the magnetic permeability can be calculated in the standard way:

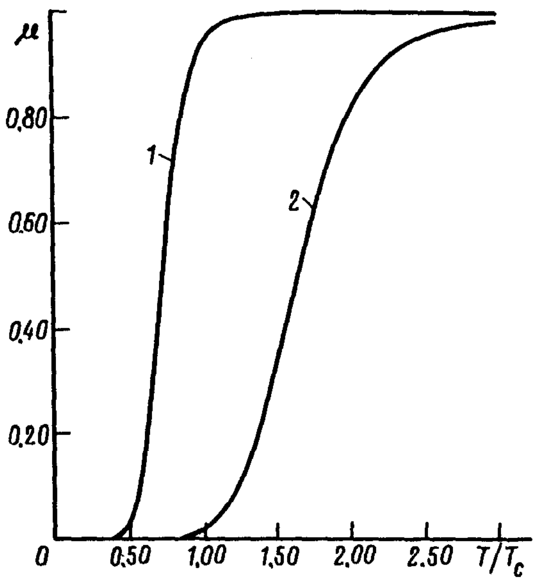

FIG. 1. Magnetic permeability μ as a function of the temperature T for the value $\kappa \sim 10^5$ (κ is a dimensionless parameter). *1*—$N=5$; *2*—$N=10$.

$$\mu = \kappa\alpha(T)\{[(1+2/\kappa\alpha(T)]^{1/2} - 1\} \qquad \kappa = mc^2/4e^2 \tag{8}$$

(d is the thickness of the $2D$ layer). Figure 1 shows a plot of $\mu(T)$ for $\kappa \sim 10^5$ and for various values of N. Over a fairly broad temperature range the permeability $\mu(T)$ increases monotonically, from $\mu = 0$ at $T < T_c$ to a value $\mu \simeq 1$ (at $T \sim \Delta$), which is characteristic of normal metals. Our results are only slightly sensitive to variations in κ. The dependence of μ on the density ρ can be reconstructed easily by noting that the temperature and the density appear in the ratio ρ/T in expressions (3)–(6). In particular, the Coulomb interaction becomes progressively more important as ρ increases. At a given temperature T there is therefore a critical density $\rho_c = 2N^2mT/\pi$, at which there is a transition to the superconducting phase.

We have restricted the discussion to the linear-response approximation up to this point, but it is a simple matter to generalize our results to the case of a uniform magnetic field of finite amplitude. Specifically, the external field $B = \mu H$, like the fictitious field b, should enter the equations only through the cyclotron frequency. Replacing Δ by $\tilde{\Delta}$, where

$$\tilde{\Delta} = |\Delta \pm e\mu(\tilde{\Delta})H/m|, \tag{9}$$

and changing the normalization of the distribution function, we therefore find self-consistent equations for the nonlinear permeability. In principle, these equations can be solved numerically. The $\pm$ in (9) correspond to the two possible directions of the external magnetic field with respect to the microscopically fixed direction of the fictitious field b (the chirality of the ground state).

In the nonlinear case, oscillations appear in the magnetization as a function of the amplitude of the external magnetic field and of the anyon density. The physical reason for these oscillations is the same as in a normal metal: The Fermi energy is intersected by Landau levels. In our case, however, these levels are produced by the self-consistent field $b \pm \mu H$. The condition for the applicability of the mean-field approximation, $N \gg 1$, leads to the limitation $\tilde{\Delta} < N\Delta$ (for external fields which are feasible in practice we would have $\tilde{\Delta} \sim \Delta$). At $T \to T_c$, the permeability decreases, and we find $\tilde{\Delta} \to \Delta$. The chemical potential coincides with the last filled Landau level, $\zeta \simeq N\Delta$, and the oscillations disappear. The coupling condition $eb = \Theta\rho$ forbids a crossing of the Fermi boundary by the Landau levels. As the temperature is raised, the oscillation amplitude first increases; after saturation of the susceptibility ($\mu \simeq 1$) at $T \sim \tilde{\Delta}$, the oscillation amplitude falls off again because of a smearing of the edge of the Fermi distribution:

$$\mu \simeq 1 - \frac{1}{2\kappa(12 + 1/N^2)} - \frac{4\pi^2 N^2 \Delta^2 T}{\kappa \tilde{\Delta}^3} \exp(-2\pi^2 T/\tilde{\Delta}) \cos(2\pi N \Delta/\tilde{\Delta}), \quad T \sim \tilde{\Delta}. \tag{10}$$

As $N \to \infty$, the monotonic part of the susceptibility of an anyon gas [the second term in (10)] becomes the standard expression for the diamagnetic susceptibility of a $2D$ electron gas ($\chi = -e^2/24\pi mc^2 d$).

The characteristic "period" of the oscillations along the scale of the amplitude of the external field, H,

$$\delta H \simeq [c/2\pi\hbar\rho e\mu(T)][2\pi\hbar\rho/N \pm e\mu(T)H/c], \tag{11}$$

depends on the temperature, the strength of the field, and the direction of the field. The quasiperiodic nature of the oscillations, the nonmonotonic temperature dependence of their amplitude, and the dependence on the direction of the applied field are all distinctive features of the de Haas–van Alphen effect in an anyonic medium. We might add that there are oscillations as a function of the density, with a characteristic period

$$\delta\rho \simeq [c/2\pi\hbar e\mu(T)H][2\pi\hbar\rho/H \pm e\mu(T)H/c]^2, \tag{12}$$

in an anyon gas.

We note in conclusion that corresponding features are present in the quantum oscillations in the transport characteristics of normal anyon systems.

We wish to thank T. N. Antsygina for much assistance in the numerical calculations.

[1]F. Wilczek, Phys. Rev. Lett. **48**, 1144 (1982); **49**, 957 (1982).
[2]F. Wilczek, *Fractional Statistics and Anyon Superconductivity*, World Scientific, Singapore, 1990.
[3]B. I. Halperin, Phys. Rev. Lett. **53**, 1583 (1984).
[4]D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984).
[5]D. H. Lee, Int. J. Mod. Phys. B **5**, 403 (1991).
[6]R. B. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988).
[7]A. L. Fetter, C. B. Hanna, and R. B. Laughlin, Phys. Rev. B **39**, 9679 (1989).
[8]Y. H. Chen, F. Wilczek, E. Witten, and B. I. Halperin, Int. J. Mod. Phys. B **3**, 1001 (1989).
[9]J. D. Lykken, J. Sonnenschein, and N. Weiss, Int. J. Mod. Phys. A **6**, 1335 (1991).
[10]Y. Kitazawa and H. Murayama, Phys. Rev. B **41**, 11101 (1990).
[11]Y. Kitazawa, Phys. Rev. Lett. **65**, 1275 (1990).

Translated by D. Parsons