Dynamic quantum Hall effect

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(Submitted 9 December 1983; resubmitted 11 January 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 4, 153–155 (25 February 1984)

The Hall response $\sigma_H(\omega)$ of a two-dimensional electron impurity system in a quantizing magnetic field at a frequency $\omega \neq 0$ is analyzed. The response $\sigma_H(\omega)$ is predicted to have a dip in its functional dependence on the concentration. The dip results from the filling of localized states and may actually consist of two dips. When the Landau level is completely filled, $\sigma_H(\omega)$ is approximately the same as the response of the pure system.

Reaching a better understanding of the systems in which a quantization of the Hall resistance has been discovered[1] requires studying not only the static but also the dynamic responses of such systems. In this letter we analyze the Hall response of a two-dimensional ($2D$) electron impurity system in a quantizing magnetic field at the frequency $\omega \neq 0$. To derive exact analytic expressions, we work from a simple model with δ-shaped impurities.[2–4]

An important point for calculating the linear response of a system is to consider transitions from localized states to delocalized states, which have not previously been studied. It is also important to use the exact sum rules; the approach of ignoring transitions between Landau levels—the customary approach in the case of strong magnetic fields—is justified in a study of the spectra of the system, but it may lead to a qualitatively incorrect result for the response.

In the absence of an electric field, our system has states of two types: 1) delocalized states, which are not split off from the Landau level, and 2) states which are split off and which are localized at impurities.[2–4] Since the matrix elements of the Hall currents between unperturbed degenerate states vanish, we can use the theory of the linear response in an electric field here. According to this theory, the Hall conductivity at the frequency ω is

$$\sigma_H(\omega) = \frac{e}{S} \sum_{n\alpha} f_{n\alpha} \sum_{k\beta}' \left\{ \frac{< n\alpha \,|j_y|\, k\beta >< k\beta \,|x|\, n\alpha >}{E_{k\beta} - E_{n\alpha} + \omega - i\epsilon} + \text{H.a.}(-\omega) \right\} , \quad (1)$$

where S is the area of the system, the index α specifies the delocalized and localized states, n is the index of the Landau level, and $f_{n\alpha}$ is the Fermi distribution function.

Substituting the exact energy levels and the exact wave functions of the unperturbed system into (1), and making use of the completeness of these wave functions,[4] we find an exact expression for the real part:

$$\sigma'_H(\omega) = \sigma^D_H(\omega) + \sigma^L_H(\omega) ,$$

where D and L are the contributions from the delocalized and localized states. It is important to note that σ_H^D, for example, reflects not only virtual transitions between delocalized states but also transitions from delocalized states to localized states (an analogous assertion can be made for σ_H^L). Their sum provides the ideal quantization at $\omega = 0$.

For $\sigma_H^L(\omega)$, for example, we have the following expression (here and below, we write the results for a single δ-shaped impurity):

$$\sigma_H^L = -\frac{e^2}{4\pi\hbar N_0}\sum_n \frac{f_{nL}}{\psi'(-E_{nL})}\left\{\frac{\psi(-E_{nL}+\hbar\omega)-\psi(-E_{nL}-\hbar\omega)-2\hbar\omega\,\psi'(-E_{nL})}{\hbar\omega(1-\omega^2/\omega_c^2)}\right.$$

$$+\frac{(4E_{nL}+2)\,[\psi(-E_{nL}+\hbar\omega)+\psi(-E_{nL}-\hbar\omega)-2\psi(-E_{nL})]}{[\hbar\omega_c(1-\omega^2/\omega_c^2)]^2}$$

$$\left.-\frac{2\omega\,[\psi(-E_{nL}+\hbar\omega)-\psi(-E_{nL}-\hbar\omega)]}{\hbar\omega_c^2(1-\omega^2/\omega_c^2)^2}\right\}, \qquad (2)$$

where ψ and ψ' are the digamma and trigamma functions, N_0 is the degeneracy of the Landau level, and ω_c is the cyclotron frequency. In the static limit, in the case of completely filled n^* Landau levels (it is sufficient to have only the delocalized states filled), we find the ideal quantization of the Hall conductivity from these expressions at $T=0$: $\sigma_H^D(0) = e^2 n^*/2\pi\hbar$ and $\sigma_H^L(0) = 0$. These values are also found for an arbitrary number of impurities positioned arbitrarily far from the boundary (provided that the number of impurities is lower than the degeneracy of the Landau level). Here are the results for $\omega \neq 0$ and for $\hbar\omega/\epsilon_n \ll 1$:

$$\sigma_H'(\omega) = \frac{e^2}{2\pi\hbar(1-\omega^2/\omega_c^2)}\sum_n f_n + \frac{e^2\omega^2}{2\pi\hbar N_0\omega_c^2}\sum_n (f_n - f_{nL})$$

$$\times\left\{2+\frac{1}{\psi'(-E_{nL})}\left[\frac{(\hbar\omega_c)^2}{\epsilon_n^4}+\frac{\hbar\omega_c(4n+2)}{\epsilon_n^3}\right]\right\}, \qquad (3)$$

where $\epsilon_n = E_{nL} - E_n$, $\epsilon_n < \hbar\omega_c/2$. If the Landau level is completely filled, the second term in (3) vanishes, so that at the accuracy level of these calculations (in terms of ω), the dynamic Hall response is the same as the response of a pure system: $\sigma_H^0(\omega) = e^2/2\pi\hbar(1-\omega^2/\omega_c^2)\Sigma_n f_n$. If $\lambda > 0$(λ is the constant of the interaction with the impurity) we have $\sigma_H^L(\omega) < 0$ so that $\sigma_H'(\omega)$ decreases when the localized state is filled. If $\lambda < 0$, however, then at $|\lambda| > \lambda_c$, with $|\epsilon_n| > \hbar\omega_c/(4n+2)$, the contribution of the n-th localized state to $\sigma_H'(\omega)$ becomes positive.

Let us examine in a qualitative way the case of many impurities with positive and negative vlaues of λ under the condition $N_i \ll N_0$ (N_i is the number of impurities). In

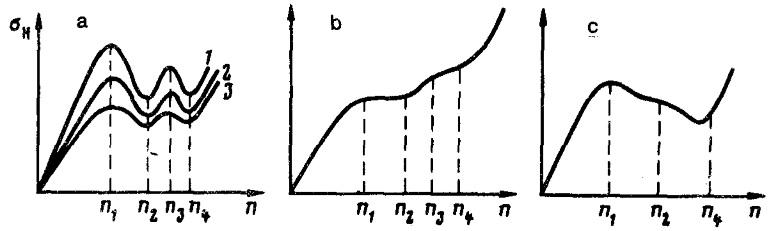

FIG. 1. a—Sketch of the Hall conductivity σ_H vs the density n. The points $n_i (i = 1–3)$ correspond to the complete filling of, respectively, 1) the delocalized states, 2) the entire Landau level, 3) the localized states which are split off downward from the following Landau level and which satisfy the condition $|\epsilon| > \epsilon_c$. 4) The beginning of the filling of the delocalized states (curves 1, 2, and 3 correspond to frequencies $\omega_1 > \omega_2 > \omega_3$.) b—A structure without a dip arises only if there are impurities with $\lambda < 0$ and $|\epsilon| > \epsilon_c$. c—In all other cases there is a single-dip structure.

this case, localized states split off both upward and downward from each Landau level. As the localized states which are split off upward from the $(n-1)$th Landau level are filled, the conductivity decreases, and it reaches the values in the pure system when the level is completley filled. With a further filling of the states which are split off downward from the n-th Landau level, the conductivity increases while the condition holds $|\epsilon_n| > \hbar\omega_c/(4n+2)$ and then decreases again when this condition no longer holds. We thus find a two-dip structure between the two successive Landau levels in the region of localized states on a plot of the conductivity against the electron density or the chemical potential (the control voltage), if there are states split off downward which satisfy the condition $|\epsilon_n| > \hbar\omega_c/(4n+2)$. Alternatively, there is a simple dip if there are no such states (Fig. 1). As the interaction with the impurities becomes weaker, the double-hump structure in the Hall conductivity should fade away. This situation can be arranged experimentally, for example, by applying a voltage to the base of a metal-insulator-semiconductor structure to drive the inversion layer away from the boundary and to thereby weaken the interaction with impurity states (which are concentrated for the most part at the surface).

We wish to thank L. V. Keldysh, D. A. Kirzhnits, and É. I. Rashba for useful discussions and comments.

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Translated by Dave Parsons
Edited by S. J. Amoretty