Quantum corrections to the thermoelectromotive force of dirty conductors

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Quantum corrections to the thermoelectric coefficient of a disordered two-dimensional conductor due to the so-called fan diagrams are analyzed. The dependence of these corrections on the magnetic field H is analyzed for various relaxation mechanisms.

Let us examine the quantum corrections to the thermoelectric coefficient η (in the expression $\mathbf{j} = -\eta\nabla T$ for the current density) of a two-dimensional ($2D$) "dirty" conductor in the case $p_F l/\hbar \gg 1$, where p_F is the Fermi momentum, and l is the electron mean free path. We consider the "fan" diagrams[1,2] (Fig. 1) in the case in which the electron-electron interaction can be ignored. Corresponding calculations were first carried out by Ting *et al.*,[3] who concluded that the relative corrections to η and to the conductivity σ are equal. As a result, there would be no correction to the differential thermoelectromotive force η/σ. Under these circumstances, an experimental study of η would reveal nothing new.

Our result is quite different: We conclude that the corrections to η, i.e., $\Delta_C\eta$, are far smaller than would follow from Ref. 3. This correction can nevertheless be extracted from the specific dependence on the magnetic field. A study of this correction can yield further information on the relaxation processes which determine the quantum corrections (information on the times τ_i and τ_ϕ, as discussed below). We believe it is worthwhile, with the corresponding experimental data available, to compare the values found for τ_i and τ_ϕ from studies of the thermoelectromotive force and the electrical conductivity.

We calculate η by the Π approach, as in Ref. 3: We calculate the coefficient $\vec{\Pi}$ in the proportionality between the heat flux density $\mathbf{Q}$ and the electric field $\mathbf{E}$ (the Peltier coefficient) and then use the Onsager relaxation $\eta = \Pi/T$. As a result of these calculations, we find[1)]

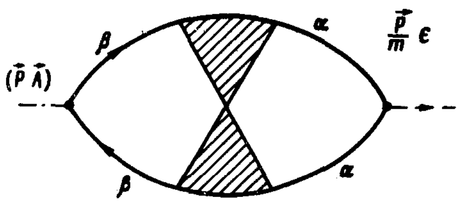

FIG. 1.

$$\Delta_C \eta = \frac{4e}{T\hbar} \int_{-\infty}^{\infty} d\epsilon \frac{\partial n_0}{\partial \epsilon} \epsilon D(\epsilon) C(\epsilon), \qquad (1)$$

were $n_0(\epsilon) = [\exp(\epsilon/T) - 1]^{-1}$, $D(\epsilon)$ is the electron diffusion coefficient, averaged over a surface of constant energy ϵ (reckoned from the Fermi level μ), and C is the contribution of the fan diagrams.[2)] The explicit expression for this contribution depends on the scattering mechanism which eliminates the divergence of $C(\epsilon)$. Let us consider two such mechanisms.

We first consider the spin-spin scattering of electrons by magnetic impurities combined with spin-orbit scattering.[4] In this case we would have

$$C(\epsilon) = C_1(\epsilon) + \frac{1}{2} C_2(\epsilon) - \frac{1}{2} C_3(\epsilon), \qquad (2)$$

where

$$C_i(\epsilon) = \int \frac{(dq)}{D(\epsilon) q^2 + 1/\tau_i}, \qquad (3)$$

and the times τ_i appear in the pole expressions in Ref. 4. We then find

$$\Delta_C \eta = -\frac{\pi}{3} \frac{eT}{\hbar} S, \quad S = \frac{\partial}{\partial \epsilon} \left[\ln \frac{L_1}{l} \sqrt{\frac{L_2}{L_3}} \right]_{\epsilon = 0}, \qquad (4)$$

where $L_i^2(\epsilon) = D(\epsilon) \tau_i(\epsilon)$. We see that the resulting expression is not proportional to a large logarithm.

In a magnetic field directed normal to the surface of the sample and satisfying the condition $a_H = (c\hbar/eH)^{1/2} \ll l$, the quantity S in (4) is replaced by $S = -(\Phi_1 + \Phi_2/2 - \Phi_3/2)$, where

$$\Phi_i = \gamma_i'(0) \zeta(2, \gamma_i + 1/2); \qquad \gamma_i = a_H^2 / 4L_i^2(0), \qquad (5)$$

as can be shown by calculations analogous to those of Ref. 5. Here $\zeta(q,x)$ is the Riemann ζ function. In strong magnetic fields, $a_H/L_i \ll l$, the correction $\Delta_C \eta$ falls off as $1/H$.

As the second mechanism we consider the inelastic scattering by the tunneling states characteristic of amorphous metals. For this mechanism we find an expression similar to (3) but with $1/\tau_i$ replaced by $1/\tau_\phi$,where

$$\frac{1}{\tau_\varphi(\epsilon, T)} = \frac{2\pi}{\hbar} \beta(\epsilon) \epsilon \operatorname{cth} \frac{\epsilon}{2T}. \qquad (6)$$

The dimensionless quantity $\beta(\epsilon)$ is proportional to the constant of the interaction of electrons with two-level systems and to the concentration of these systems. In typical amorphous metals it is 10^{-2}–10^{-3} in order to magnitude. The characteristic value in its functional dependence on ϵ is $\mu \gg T$.

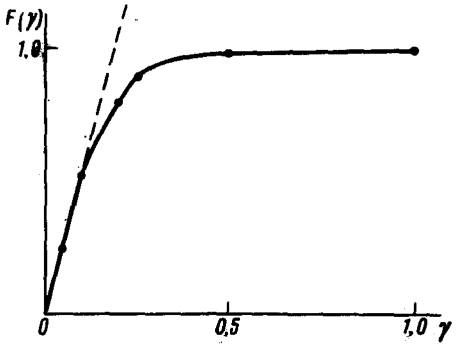

FIG. 2.

Calculations for this case yield Eq. (4) with

$$S = \frac{D'(0)}{D(0)} - \frac{\beta'(0)}{\beta(0)} , \tag{7}$$

where the prime denotes a derivative with respect to ϵ. In a magnetic field the function $S(H)$ becomes

$$S = \left[\frac{D'(0)}{D(0)} - \frac{\beta'(0)}{\beta(0)}\right] F(\gamma), \qquad \gamma = \frac{2\pi a_H^2 \beta(0)}{\hbar D(0)} ; \tag{8}$$

$$F(\gamma) = \frac{3}{\pi^2} \int_0^\infty \frac{(\eta/2\gamma)d\eta}{\mathrm{sh}(\eta/2\gamma)} \int_0^\infty \frac{x^3 dx}{\mathrm{sh}\, x} \exp\left(-\eta\, x\, \mathrm{cth}\frac{x}{2}\right) . \tag{9}$$

Figure 2 is a plot of the function $F(\gamma)$.

The reason why expression (9) is more complicated than (5) is that for inelastic processes the time $\tau_\phi(\epsilon,T)$ depends strongly on the energy ϵ in a region with a width of order T, while the same region plays a role in the integral that determines $\Delta_C\eta$. We wish to emphasize that this energy region is important for any inelastic process that determines τ_ϕ, including (for example) the scattering of electrons by phonons. Experiments on the $\Delta_C\eta(H,T)$ dependence can thus yield information on the energy dependence of the time τ_ϕ and thus on the nature of the mechanism for the inelastic relaxation.

In conclusion we wish to emphasize that we have studied only the "diffusion" part of the thermoelectromotive force here. In principle, there is also a contribution to the coefficient η from the phonon drag of electrons. In interpreting experimental data we would like to make sure that this contribution is small. A suitable criterion might be the temperature dependence of the main part of the thermoelectromotive coefficient η.

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[1)]The calculation procedure will be described in detail in a separate paper.
[2]As far as we can see, the dependence of this quantity on ϵ makes our results different from those of Ref. 3.

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[1]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].
[2]E. Abrahams and T. V. Ramakrishnan, J. Non-Cryst. Solids **35/36**, 15 (1980).
[3]C. S. Ting, A. Houghton, and J. R. Senna, Phys. Rev. **B25**, 1439 (1982).
[4]S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).
[5]B. L. Al'tshuler, D. E. Khmel'nitskiĭ, A. I. Larkin, and P. A. Lee, Phys. Rev. **B22**, 5142 (1980).

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