Spontaneous electrical polarization of vortices in superfluid ^{3}He

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Quantum vortices in ^{3}He have a spontaneous electrical polarization which is concentrated in the core of the vortex and oriented along its axis. The polarization is caused by spontaneous parity violation in the core of the vortex and arises due to the flexoelectrical effect.

Vortices in ^{3}He-B have several unique properties related to the structure of their core, whose size is of the order of several coherence lengths ξ, i.e., $\sim 10^{-5}$ cm. In addition to a structural phase transition occurring in the core of the vortex at $T = 0.6T_c$,[1] a magnetic moment concentrated in the core has also been observed experimentally.[2] A symmetry analysis of the structure of the core[3] showed that a vortex with a single quantum of circulation and with an axially symmetrical distribution of the order parameter can assume five different states which differ by the discrete symmetry which are breaking and denoted as o,u,v,w,uvw. It is assumed that the vortices observed at $T < 0.6T_c$ are in the v state. These v vortices have a superfluid core, within which the A and B phases with ferromagnetic ordering of nuclear spins are found.[3] It turns out that due to parity violation in the core, the v vortex has a spontaneous electrical polarization oriented along the axis. This facilitates identifying the vortices experimentally, since the o,u and w vortices do not have an electric dipole moment, although there is a spontaneous superfluid flow in the core of a w vortex along the axis, while a uvw vortex has both properties.

Among the five vortices, the o vortex has a maximum discrete summetry group $Z_2 \times Z_2$. If the elements of the gauge symmetry, which is important for the order parameter, but not for the observed quantities, are ignored, then this group, in addition to the identity transformation, contains three elements $P_1 = P$, $\mathrm{P}_2 = \mathrm{PTU}_2$, $\mathrm{P}_3 = \mathrm{TU}_2$, where P is the parity, T is the time-inversion operator, and U_2 is a rotation by an angle π around an axis orthogonal to the vortex. The u, v, and w vortices retain

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only a single symmetry element each, P_1, P_2, and P_3, respectively, and discrete symmetry breaks down completely in the *uvw* vortex. Let us examine the projection of the electric dipole moment **d** on the vortex axis $\hat{\Omega}$, which coincides at equilibrium with the direction of rotation of the vessel. The quantity $\mathbf{d}\hat{\Omega}$ changes sign under the action of both P_1 and P_3, so that it is nonzero only in the vortices *v* and *uvw*, where both of these symmetries break down. The sign of the quantity $\mathbf{d}\hat{\Omega}$ in these vortices is arbitrary, reflecting the degeneracy that accompanies symmetry breaking. Analogously, spontaneous flow of the superfluid component along the axis of the vortex $\mathbf{j}_s\hat{\Omega}$ occurs with the breaking of P_1 and P_2 symmetry, i.e., in the *w* and *uvw* vortices. The density of the flow in the cores of these vortices is evidently of the order of $\rho_s(\hbar/m_3\xi)$, where ρ_s is the density of the superfluid component. We shall estimate the dipole moment in the *v* and *uvw* vortices.

As in ordinary liquid crystals, a dipole moment appears in the superfluid phases of ^{3}He under deformation of the liquid-crystalline axes (see the so-called flexoelectrical effect[4]). Since the cores of the *v* and *uvw* vortices consist primarily of the *A* phase, we shall limit ourselves, for purposes of estimation, to an examination of the flexoelectrical effect in the *A* phase, which has three flexoelectrical coefficients, relating the polarization *P* to the gradients of the anisotropy axis **l** and the superfluid velocity

$$\mathbf{P} = \beta_1 \mathbf{l}(\vec{\nabla}\mathbf{l}) + \beta_2 \frac{m_3}{\hbar}[\mathbf{l}, \mathbf{v}_s - \mathbf{v}_n] + \beta_3(\mathbf{l}\vec{\nabla})\mathbf{l}. \tag{1}$$

The integral of (1) over the cross section of the vortex given an estimate for the dipole moment per unit length of the vortex: $\mathbf{d}\hat{\Omega} = \int d\,\mathbf{SP} \sim \beta_2\xi \sim (\beta_1 - \beta_3)\xi$.

The flexoelectrical parameters β can be estimated from a microscopic analysis. Helium atoms are not polar, so that the dipole moment arises only due to the neutral polarization of the atoms. Pair interaction of atoms leads to an induced dipole moment for a pair of atoms, which has an identical magnitude, but opposite orientation. In the *A* phase, thee dipole moments, which participate in the rotation of Cooper pairs around **l**, create an electronic ferromagnetic moment oriented along **l**.[5] The average dipole moment in a diatomic molecule consisting of identical nonpolar ^{3}He atoms is zero, so that this effect cannot give an integral moment of the vortex, leading only to a term in the polarization with $\beta_1 + \beta_3$, which is a total derivative. A three body interaction is required for a nonzero integral dipole moment. A molecule consisting of three atoms situated at the points $\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3$, has a dipole moment $\mathbf{U}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$, different from zero if the arrangement of the atoms does not contain a symmetry. For this reason, the total dipole moment of the liquid is expressed in terms of a three-particle correlation function:

$$\int d^3r\,\mathbf{P}(\mathbf{r}) = \int d^3r_1 d^3r_2 d^3r_3 \mathbf{U}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \langle \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\rho(\mathbf{r}_3) \rangle . \tag{2}$$

In an inhomogeneous liquid, the polarization **P** can be calculated as a response function with respect to the gradient of the order parameter, for example, with respect to the superfluid velocity. Taking into account the fact that *U* arises as a result of the polarizing action of the liquid on the atom from the side of the dipole moments of Cooper pairs, which can be expressed in terms of the density of the electronic ferromagnetic moment $M \sim 10^{-9}(1 - T/T_c)$G/cm,[3,5] we obtain the following estimate for β

($\alpha \sim 3\times 10^{-3}$ is the polarizability of liquid ^{3}He, c is the velocity of light, and a is the interatomic distance).

$$\beta_1 - \beta_3 \sim \beta_2 \sim Mc\,\alpha a^2 m_3/\hbar\left(1 - T/T_c\right) \sim 10^{-10}\text{–}10^{-9}\left(1 - T/T_c\right)^2 \text{ Volts.} \quad (3)$$

A detailed calculation of the effect cannot yet be performed because the structure of the liquid at atomic distances is not known. The quantity $\beta_1 + \beta_3$ from the pair interaction does not contain the polarizability α and is therefore two to three orders of magnitude larger.

Let us discuss the experimentally observable consequences of the polarization of vortices. At the location where a vortex emerges onto the surface of the vessel, there is a surface density of electric charge $\sigma = P$, i.e., the point of termination of the vortex on the surface of the B phase has an electrical charge $e^* \sim \xi^2\sigma \sim 10^{-7}e$, where e is the charge of an electron. The signs of the charges in the flux line lattice are disordered because the interaction of the vortex cores is very small: for the velocities of rotation $\Omega \sim 1$ rad/s, the vortex density $n_V = 4m_3\Omega/h \sim 10^4$ cm^{-2} and the distance between the vortices is three orders of magnitude greater than the size of the core. The same charges can occur on the flux line within the fluid at points that separate the sections of th flux line with different polarizations. Vortices can be polarized identically by applying for some time a strong electric field along the axis of rotation. Identical charges will then appear on the surface with average density $e^* n_v$, which will create an average polarization field $4\pi e^* n_V \sim 10^{-8}$ V/cm. This field is ten orders of magnitude greater than the spontaneous electric field produced in the B phase as a result of parity violation in weak interactions due to neutral currents.[6]

If the polarization of the vortices is ordered, then the parity violation acquires a macroscopic nature, as in a liquid crystal with excess chiral molecules of one sign of chirality.[4] This leads to terms of the type $\eta \hat{z}_i R_{\alpha i} \nabla_k R_{\alpha k}$, where $\eta \sim \rho_s (\hbar/m_3)^2 \xi n_V$, which are linear with respect to the gradient of the order parameter $R_{\alpha i}$, in the macroscopic energy of the B phase. As a result, the texture of the B phase in the vessel will change after an ordering electric field is switched on for some time, which can be observed in *NMR* experiments.

Spontaneous polarization can also appear in a rotating A phase. We shall examine the analytical Anderson–Toulouse–Chechetkin vortex[7] with continuous distribution of $\mathbf{v}_s$ and $\mathbf{l}$ ($\mathbf{z}, \hat{\mathbf{r}}$, and $\hat{\varphi}$ are the axes of a cylindrical coordinate system with $\hat{\mathbf{z}} = \hat{\Omega}$, and η varies from $\pi/2$ on the axis of the vortex to $-\pi/2$ at the periphery):

$$\mathbf{l} = \hat{\mathbf{z}} \sin\eta(r) \pm \hat{\mathbf{r}} \cos\eta(r), \qquad \mathbf{v}_s = \frac{\hbar}{2m_3 r}(1 - \sin\eta(r))\,\vec{\hat{\varphi}}\,. \quad (4)$$

This vortex has the discrete symmetry of the v vortex in the B phase, and it therefore has a dipole moment, which, according to (1) has the form

$$\mathbf{d} = \pm\,\hat{\Omega}\,2\pi \int_0^\infty dr \left\{ \frac{1}{2}\beta_2 \cos\eta\,(1 - \sin\eta) + (\beta_3 - \beta_1)\, r \frac{\partial\eta}{\partial r} \cos^2\eta \right\}. \quad (5)$$

Since $\mathbf{d}$ is proportional to the radius of the core of the vortex and is equal to the intervortex distance for an analytical vortex in the absence of a magnetic field, **d** is three orders of magnitude greater than in the *B* phase. Since the interaction of cores in this case is large, the polarization of vortices must be ordered "ferromagnetically" or "antiferromagnetically." These vortices also have an electric charge in the core with linear density $2\pi\beta_2$, since the polarization **P** drops off as $\beta_2\hat{\mathbf{r}}/\mathbf{r}$ far from the core.

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