Certain aspects of a gauge theory

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(Submitted 4 November 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **39**, No. 5, 228–229 (10 March 1984)

An example is cited to counter the customary assumption that although the Green's functions depend on the gauge, the observable processes should not.

We consider a Yang-Mills field A_μ^a which is interacting with a real scalar field ϕ^a, which transforms by an associated representation. We insert "unity" in a path integral:

$$1 = \Delta \int (D\Omega) \prod_x \prod_{a=1}^{n^2-1} \delta\,(\partial^\mu (A_\mu^\Omega)^a - \alpha(\phi^\Omega)^a),$$

where the expression Δ is needed only for the fields that satisfy the condition $\partial^\mu A_\mu^a = \alpha\phi^a$ [we assume that $SU(n)$ is a gauge group]. Noting that

$$\prod_{a=1}^{n^2-1} \delta((\Omega^{-1}R\,\Omega)^a) \equiv \prod_{a=1}^{n^2-1} \delta(R^a)\ ,$$

we find that $\Delta(A)$ can be expanded in terms of the "ghost" field in precisely the same way as for the Lorentz gauge. We make the replacement $A \to A^{\Omega^{-1}}, \phi \to \phi^{\Omega^{-1}}$ in the path integral and as a result single out the volume of the gauge group. We now separate the longitudinal components from the transverse components of the field A_μ, inserting "unity,"

$$1 = \text{const} \int (D\lambda) \prod_x \prod_{a=1}^{n^2-1} \delta(\partial^\mu (A^a_\mu - \partial_\mu \lambda^a))\ ,$$

and we then make the replacement $A_\mu \to A_\mu + \partial_\mu \lambda$. An integration over λ^a then eliminates the δ-function $\delta(\partial^\mu A^a_\mu - \alpha\phi^a)$, and we find $\lambda^a = \Box^{-1}\alpha\phi^a$. Examining the resulting Lagrangian carefully, we see that it would be very interesting to set $\alpha = g^{1/2}\zeta$ (where g is the coupling constant, and ζ is a dimensional parameter) and examine the limit $g \to 0$. At a fixed g, the observables should apparently not depend on α, so that we can also choose α to be a function of g. In this limit, however, the Lagrangian becomes

$$L = \frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}) + \frac{1}{2}(\partial_\mu \phi^a)(\partial^\mu \phi^a) - \frac{m^2}{2}\phi^a\phi^a - \tilde{\lambda}(\phi^a\phi^a)^2$$

$$+ \frac{1}{4} f^{abc}(\partial_\mu \Box^{-1}\phi^b)(\partial_\nu \Box^{-1}\phi^c) f^{ab'c'}(\partial^\mu \Box^{-1}\phi^{b'})(\partial^\nu \Box^{-1}\phi^{c'}) + \frac{1}{2}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)$$

$$\times f^{abc}(\partial^\mu \Box^{-1}\phi^b)(\partial^\nu \Box^{-1}\phi^c)$$

i.e., we see interaction terms (in the limit $g = 0$!). It is not difficult to show that there actually are no processes which contain the field A in an exterior line (since $\Box A = 0$ for exterior lines). We are, however, left with terms of a fourth nonlocal interaction of the field ϕ (we are not interested in the term $\tilde{\lambda}\phi^4$). The equations of motion for the exterior lines, $\Box\phi = m^2\phi$, do not cause the interaction to disappear, if only because the classical field solution are different from those for the case of a free field.

Would we have an analogous effect in electrodynamics?

a) Let us consider spinor electrodynamics and the gauge

$$\partial^\mu A_\mu = \alpha f(\bar{\psi}, \psi)\cdot$$

Setting $\alpha = e^{-1}$ and taking the limit $e \to 0$, we then find a term

$$(\partial^\mu \Box^{-1} f(\bar{\psi}, \psi)) j_\mu,$$

and since $\partial^\mu j_\mu = 0$ we conclude that this term does not correspond to an interaction between particles.

b) We not consider scalar electrodynamics and the Duffin-Kemmer formalism. Again, no interaction arises.

c) We now consider scalar electrodynamics with the field A appearing linearly and quadratically in the interaction. In the limit $e \to 0$, an interaction remains [the gauge $\partial^\mu A_\mu = \alpha f(\phi^*, \phi)$, where f is gauge-invariant; $\alpha = e^{-1}; e \to 0$].

Turning to quantum chromodynamics, adopting the gauge

$$\partial^\mu A^a_\mu = \alpha \bar{\psi} t^a \psi ,$$

setting $\alpha = g^{-1/2}$, and letting $g \to 0$, we again find an interaction.

We thus see that an observable quantity (the presence or absence of an interaction) depends on the gauge.

Discussion

The transformation to these gauges depends in a complicated way on g (on e), and there is an essential singularity in g (in e) at the origin. We would expect this to be the case for Yang-Mills fields [with an instanton contribution $\sim \exp(-1/g^2)$], but if we consider 0 to be an instanton sector, then for fields $\phi^a(x)$, which fall off sufficiently rapidly at infinity, the transformation from $\partial^\mu A^a_\mu = 0$ to $\partial^\mu A^a_\mu = \alpha \phi^a$ would leave us in the same sector, so that the observed deviation from analyticity should not be assumed to be due to instantons. For scalar electrodynamics, the complicated dependence on e is again unrelated to instantons (with $m \neq 0$ and $\tilde{\lambda} \geqslant 0$, there are no instantons in this case). There is of course a change of variables which would again make our fields free, but any arbitrary Lagrangian expressed in terms of in(out) fields would take the form of a free field. An important point here is that we have gone over to interacting fields as a result of a gauge transformation, which should not alter the physics of the system. We will not take up questions of renormalization here, but we do not believe that they would alter the results.

I wish to thank V. Ya. Faĭnberg for a discussion of this paper.

Translated by Dave Parsons
Edited by S. J. Amoretty