Phase diagram of a dissipative quantum system

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A phase diagram is constructed and the low-frequency asymptotic forms of the correlation functions are found for a dissipative quantum system in a periodic potential in the limit $T \to 0$. It is demonstrated that the system can be in a localized or delocalized state, depending on the magnitude of the coefficient of friction.

The quantum behavior of macroscopic objects in a medium with dissipation has recently attracted attention.[1–5] Of particular interest here is the influence of dissipation on the properties of a system in a potential with several minima. Such a system is described by an effective "Euclidean" (with "imaginary" time) functional[1–3,5,6]

$$Z = \int Dq e^{S_{eff}}, \quad S_{eff} = S_0 + S_{int}, \quad S_{int} = \int_{-1/2T}^{1/2T} d\tau V(q),$$

$$S_0 = \frac{1}{2} \int_{-1/2T}^{1/2T} d\tau \left[m\dot{q}^2 + \frac{\eta}{2\pi} \int_{-\infty}^{\infty} d\tau \left(\frac{q_\tau - q_{\tau'}}{\tau - \tau'} \right)^2 \right] = S_K + S_D, \tag{1}$$

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where m is the effective mass, η is the coefficient of friction, $V(q)$ is the potential, T is the temperature of the medium, and $q(-1/2T)=q(1/2T)$.

In the case of potentials with nondegenerate minima, a more accurate calculation of the effect of the dissipative term S_D on the decay of the metastable state is of principal interest.[1-3,6] In the case of potentials with degenerate (or nearly so) minima, an effective long-range interaction occurs between different tunneling trajectories, because of the possible multiple tunneling and nonlocality of S_D, thereby making the problem a multiparticle problem.[4,5]

We shall investigate the system (1) in the limit $T\to 0$ and in a periodic potential. This case has already been examined in a preliminary form in Ref. 7, where the possibility of the existence of two phases in the system was noted (see also Ref. 8): localized and delocalized phases. However, the phase diagram presented in Ref. 7 is not correct, because it is based on the phase diagram of a classical one-dimensional ln-gas, which contains neutral configurations only with alternating opposite charges.[9] This phase diagram differs from the phase diagram of the adequately studied ln-gas system with all neutral charge configurations.[8] We shall present the corrected phase diagram, and we shall also find the low-frequency asymptotic expressions for the correlation functions and mobilty in different phases.

Let $V(q)=-g\cos kq+Fq$, where F is, for the time being, an arbitrary force depending on τ. Since $S_D\equiv\frac{1}{2}\int\int q_\tau \alpha_{\tau\tau'} q_{\tau'} d\tau d\tau'>0$,we shall assume below, without changing $V(q)$, that

$$S_0=\frac{1}{2}\int\int q_\tau \Box_{\tau\tau'} q_{\tau'}\,d\tau\,d\tau'=\frac{1}{2}\int\frac{d\omega}{2\pi}|q_\omega|^2[m\omega^2+\eta|\omega|] \tag{2}$$

and we shall also make q dimensionless by incorporating k into it; in this case, $\eta\to\bar\eta=\eta/k^2$, $m\to\bar m=m/k^2$,$F\to\bar F=F/k$. It follows from (2) that the nonlocal, dissipative part of the "kinetic" energy S_D plays the main role in determining the asymptotic, with respect to τ, properties of the model (1). It corresponds to a propagator with the long-range kernel,

$$\Box^{-1}(\tau)\cong-(\pi\bar\eta)^{-1}\ln|\tau|,\ \tau\gg m/\eta.$$

For this reason, the low-frequency properties of the model (1) with a periodic potential coincide with the long-wavelength properties of the classical one-dimension ln-gas with charges $e=\pm 1$, which contains all neutral charge configurations[7,8] $\int d\tau\rho_N(\tau)\equiv\Sigma_1^N e_i\int\delta(\tau-\tau_i)d\tau$.

$$Z=\sum_0^\infty\frac{z^N}{N!}\sum_{\{\rho_N\}}{}'\int\prod_{i=1}^N d\tau_i\exp\left\{-\frac{1}{2}\int\int(\rho+iF)_\tau\Box^{-1}_{\tau\tau'}(\rho+iF)_{\tau'}\right\}, \tag{3}$$

where $z=g/2$, $\int d\tau F=0$, and $\Sigma'_{\{\rho_N\}}$ indicates summation over all neutral configurations. Here the inverse temperature of the ln-gas is $\beta=(\pi\bar\eta)^{-1}$.

The system (1) or (3) can be studied by the method of the renormalization group (RG) for $gm/\eta\ll 1$[8] where m/η plays the role of the scale a which cuts off the logarithmic behavior of $\Box^{-1}(\tau)$. The RG equations have the form $(F=0)$[8]

$$\frac{dv}{dl} = -v + \frac{2}{3\pi\bar{\eta}}u^2, \quad \frac{du}{dl} = xu - Bu^3, \quad l = da/a, \quad a(l=0) = m/\eta, \tag{4}$$

where $v = \bar{m}/a$, $u = ga$, $x = 1 - 1/2\pi\bar{\eta}$, $B = 16/3$. It follows from (4) that for $u \ll 1$ the variable v is imaginary and that $\bar{\eta}$ (or β) is not renormalized, in contrast to the case of a one-dimensional ln-gas with only alternating charges.[9] The properties of the system (1) are determined by the magnitude of the friction coefficient $\bar{\eta}$. For $\bar{\eta} < \bar{\eta}_c = 1/2\pi (x<0)$ the potential is imaginary and the behavior of the system in the limit $\tau \to \infty$ $(\omega \to 0)$ is determined by the "dissipatons" with the spectrum $\bar{\eta}|\omega|$ (they are analogous to the spin waves of the low-temperature phase of the XY model) and by the slowly decreasing correlation functions $(1/T \gg |\tau - \tau'| \gg m/\eta)$

$$\langle q_\tau q_{\tau'} \rangle \sim -(\pi\bar{\eta})^{-1} \ln(|\tau - \tau'| T). \tag{5}$$

The mobility of a particle $\mu(\omega) \equiv \bar{\eta}|\omega| \langle qq \rangle_\omega$[7] in this phase in the limit $\omega \to 0$ is $\mu(\omega) = 1 + O(|\omega|)$, which corresponds to its delocalization.

For $\bar{\eta} > \bar{\eta}_c$, the potential becomes real. In this region the low-frequency spectrum has the form $\bar{\eta}|\omega| + \bar{u}\xi^{-1}$, where $\xi \sim (m/\eta)e^{l^*}$ is the correlation length and

$$l^* = \begin{cases} \frac{1}{2x}[C + \ln(1 - \bar{u}^2/u_0^2)], & \text{for } u_0 > \bar{u},\ x \ll 1; \\ \frac{1}{x} \ln \bar{u}/u_0, & \text{for } u_0 \ll u; \end{cases} \tag{6}$$

where $u_0 = gm/\eta$, C is a constant of order 1, and $\bar{u}^2 = x/B$. Such a spectrum corresponds to the correlation function $(|\tau - \tau^*| \gg M^{-1} \equiv \bar{\eta}\xi/\bar{u})$

$$\langle q_\tau q_{\tau'} \rangle \sim -(\pi\bar{\eta})^{-1} [\, si(\tau M) \sin(\tau M) - ci(\tau M) \cos(\tau M)] \sim o((\tau M)^{-\alpha}), \quad \alpha < 1, \tag{7}$$

which, while oscillating, decreases in almost a power-law fashion. As $\omega \to 0$, $\mu(\omega) \cong |\omega|/M \to 0$ and, therefore, weak localization (i.e., one that is characterized by a power-law decrease of the correlation function) occurs in this phase.

As noted in Ref. 7, another region of parameters of the system (1) which can be studied is the region of applicability of the steepest-descent method $g\bar{m} \gg 1, \bar{\eta}^2$. In this region Z can be approximated by the grand partition function of a gas of tunneling trajectories—kinks[7]

$$q_{cl}(\tau) = \Sigma e_i f(\tau - \tau_i), \quad f(\tau) = 4 \arctan(e^{\omega_0 \tau}), \quad \omega_0 = (g/\bar{m})^{1/2}, \quad e_i = \pm 1, \quad \Sigma e_i = 0,$$

which is effectively equivalent to a gas of particles with charges $e_i = \pm 1$ which interact according to the logarithmic law[4,5,7]

$$Z = \sum_{N=0}^{\infty} \frac{z^N}{N!} \sum_{\{e_i\}}{}' \int \prod_{i=1}^{N} d\tau_i \exp\{-\frac{1}{2}\sum_{i \neq k} e_i e_k \tilde{\Delta}_{\tau_i \tau_k} - \int d\tau \bar{F} q_{cl}\}, \tag{8}$$

where $z = \gamma/2 = \omega_0 (s/2\pi)^{1/2} e^{-s}$, $s = 8\,(g\bar{m})^{1/2}$ is the action of a single kink,

$$\widetilde{\Delta}_\tau \equiv \Delta_\tau - \Delta_0 = \int \frac{d\omega}{2\pi} (e^{i\omega\tau} - 1)|f_\omega|^2 \alpha_\omega \cong -4\pi\bar{\eta} \ln(|\tau|\omega_0), (\omega_0|\tau|) \gg 1,$$

where $f_\omega = -2\pi i[\omega \mathrm{ch}(\pi\omega/2\omega_0)]^{-1}$. Equation (8) can be represented in the form

$$Z = \int D\tilde{q} e^{-S_{\text{eff}}}, \quad S_{\text{eff}} = \frac{1}{2} \int\int [\tilde{q}_\tau \Delta^{-1}_{\tau\tau'} \tilde{q}_{\tau'} - F_\tau \alpha^{-1}_{\tau\tau'} F_{\tau'}] + \int d\tau \widetilde{V}(\tilde{q}), \tag{9}$$

where $\widetilde{V}(\tilde{q}) = -\gamma \cos\tilde{q} - \bar{q}\bar{F}$, $\bar{q}_\tau = \int \frac{d\omega}{2\pi} e^{i\omega\tau} \tilde{q}_\omega f_\omega^{-1} \alpha_\omega^{-1}$. It follows from (3) and (8) or (1) and (9) that system (1) in the low-frequency limit is self-dual with respect to the transformations[7]

$$2\pi\bar{\eta} \rightarrow 1/2\pi\bar{\eta}, \quad gm/\eta \rightarrow \gamma/\omega_0, \tag{10}$$

which have the stationary point $\bar{\eta}^* = 1/2\pi$, $s^* \cong 1.5$,[7] which, since $g^*m^*/\eta^* \cong 0.2$, lies outside the region of applicability of expansion (8) but on the boundary of applicability of expansion (3).

Application of the RG to (8) and (9), which is valid for $\gamma/\omega_0 \ll 1$, gives for the correlation length, spectrum, and correlation function of the effective coordinate $\tilde{q}$ expressions coinciding with those obtained above, with allowance for the substitution (10). Here the value $\bar{\eta}_c = \bar{\eta}^*$ remains unchanged and,therefore, the critical line $\bar{\eta} = 1/2\pi$ transforms into itself under the duality transformation. To find the properties of system (1) it is necessary to establish a relation between the correlation functions $\langle qq \rangle_\omega$ and $\langle \tilde{q}\tilde{q} \rangle_\omega$. This relation has the form

$$\langle qq \rangle_\omega = \alpha^{-1}_\omega [1 - \alpha^{-1}_\omega |f_\omega|^{-2} \langle \tilde{q}\tilde{q} \rangle_\omega]; \quad \mu(\omega) \underset{\omega \to 0}{\cong} 1 - \tilde{\mu}(\omega). \tag{11}$$

For $\bar{\eta} > \bar{\eta}_c$, we obtain

$$\langle qq \rangle_\omega \cong O(1/\eta) \quad \mu(\omega) \cong O(|\omega|). \tag{12}$$

Thus, in terms of the starting system (1), this phase is localized. Effectively, each tunneling in it is accompanied by reverse tunneling, and the probability of any uncompensated tunneling approaches zero $\gamma(T) \sim \gamma_0(T/\omega_0)^{-2\pi\bar{\eta}}$. As a result, the particle becomes stuck in the vicinity of some minimum. For $\bar{\eta} < \bar{\eta}_c$ we have

$$\langle qq \rangle_\omega = (\bar{\eta}|\omega|)^{-1} [1 - (4\pi^2 M)^{-1} |\omega| + \ldots], \quad \mu(0) = 1 \tag{13}$$

and therefore the particle is delocalized. This is a result of the fact that in this phase the probability of uncompensated tunneling increases with increasing scale. We note that the static mobilities found coincide with the mobilities proposed in Ref. 7.

As a result, a phase diagram is obtained (Fig. 1). It is worth noting that although the RG analysis presented above says nothing about the presence of an additional structure in the phase diagram, this possibility cannot be completely ruled out in the vicinity of the stationary point $(\bar{\eta}^*, s^*)$. If, however, additional structure does exist, then it must be symmetrical with respect to duality transformation (10).

In conclusion let us see how the parameters of the Josephson functions are distributed in the diagram. In this case, $g = I_c/2e$, $\bar{\eta} = \hbar/4e^2R$, $\bar{m} = \hbar C/4e^2$, where I_c, R, and C are the critical current, shunting resistance and the capacitance of the junc-

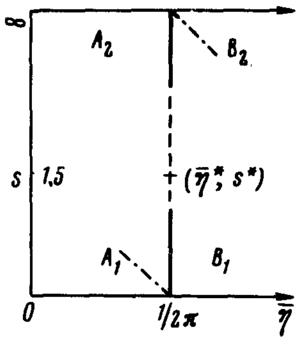

FIG. 1. Phase diagram. The phases A_1 and A_2 are delocalized; the phases B_1 and B_2 are localized. The dashed line is an interpolation of the phase boundaries between the phases in the vicinity of the stationary point ($\bar{\eta}^*$, s^*); the dot-dashed line show the phase boundaries between the phases according to Schmid.[7]

tion. The intensity of damping in the junctions is usually characterized by the ratio $\omega_0/\omega_c = \bar{\eta}(g\bar{m})^{-1/2}$. It follows that the regions of appalicability of expansions (3) and (8) with $\bar{\eta} \sim 1$ correspond to regimes of strong and weak damping. For tunnel junctions we would have $\omega_0/\omega_c \cong 10^{-1}$–$10^{-3}$, while for point contacts we would have $\omega_0/\omega_c \gg 1$.

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