The reaction $e^+e^-\to\eta\pi^+\pi^-$

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At energies $\sqrt{s} = 1.0$–1.4 GeV we should expect a threshold enhancement of the cross section for the reaction $e^+e^-\to\eta\pi^+\pi^-$ because the reaction channel $e^+e^-\to\rho\to\eta\rho\to\eta\pi^+\pi^-$ comes into play. The width of the decay $\phi\to\eta\pi^+\pi^-$ has been calculated for the first time: $\mathrm{Br}(\phi\to\eta\pi^+\pi^-) = 0.35\times10^{-6}$. This width can be extracted from the interference pattern.

Experiments with a neutral detector are presently being carried out at energies 1–1.4 GeV on the VÉPP-2M accelerator at the Institute of Nuclear Physics, Novosibirsk, in order to study the radiative decays of the ϕ meson. The large statistical base and the comprehensive detection of γ rays make it possible to study processes with small cross sections. For example, results were reported in Ref. 1 on the decays $\phi\to\eta\gamma$ and $\phi\to\pi\gamma$, and the rare decay $\phi\to\eta e^+e^-$, with a cross section $\sigma(e^+e^-\to\eta e^+e^-)$, a peak value of $\sim$0.5 nb, and $\mathrm{Br}(\phi\to\eta e^+e^-) = 1.25\times10^{-4}$.

In this letter we discuss the question of experimentally studying the reaction $e^+e^-\to\eta\pi^+\pi^-$ with this apparatus.

This process can be used near the ϕ meson $(e^+e^-\to\phi\to\eta\pi^+\pi^-)$ to measure the rare decay $\phi\to\eta\pi^+\pi^-$, which breaks G parity. In the present letter we calculate the width of this decay for the first time: $\mathrm{Br}(\phi\to\eta\pi^+\pi^-) = 0.35\times10^{-6}$.

The reaction $e^+e^-\to\eta\pi^+\pi^-$ is of further interest for studying the isovector part of the electromagnetic current: $e^+e^-\to\rho+\rho'+...\to\eta\pi^+\pi^-$. We show in particular that in the reaction $e^+e^-\to\rho\to\eta\pi^+\pi^-$ we would expect a sharp enhancement, of the nature of a threshold, of the cross section at energies from 1 to 1.4 GeV because the channel $e^+e^-\to\rho\to\eta\rho\to\eta\pi^+\pi^-$ opens up. As the energy is changed from 1.01 to 1.3 GeV, for example, the cross section $\sigma(e^+e^-\to\eta\pi^+\pi^-)$ increases by a factor of 20 (Fig. 3).

Assuming that $\phi\to\eta\rho\to\eta\pi^+\pi^-$ is the main process, we find

$$\Gamma(\phi\to\eta\pi^+\pi^-) = \frac{1}{12\pi^2}\int\limits_{4m_\pi^2}^{(m_\phi-m_\eta)^2} \frac{\sqrt{q^2}\,\Gamma_\rho(q^2)\,p_\eta^3(m_\phi^2,q^2)}{(q^2-m_\rho^2)^2+(\sqrt{q^2}\,\Gamma_\rho(q^2))^2}\,|g_{\phi\rho\eta}|^2\,dq^2, \tag{1}$$

$$\sqrt{q^2}\,\Gamma_\rho(q^2) = m_\rho\Gamma_\rho(m_\rho^2)\left(\frac{p(q^2)}{p(m_\rho^2)}\right)^3\frac{m_\rho}{\sqrt{q^2}},$$

$$p_\eta^2(m_\phi^2,\ q^2) = \frac{1}{4m_\phi^2}\left((m_\phi^2-m_\eta^2-q^2)^2-4m_\eta^2q^2\right), \tag{2}$$

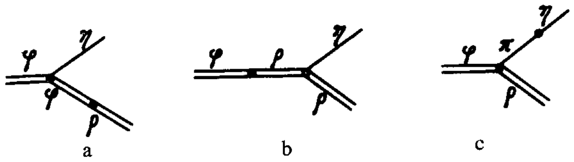

FIG. 1.

where q^2 is the invariant mass of the $\pi^+\pi^-$ system, and $p(q^2)$ is the pion momentum.

In general, the vertex $g_{\phi\rho\eta}$ is a sum of three components (Fig. 1). The most important diagram is that of Fig. 1a, which is enhanced by a factor m_ρ^2/q^2 due to the single-photon contribution to the $\phi\to\rho$ transition, since q^2 is small in this case $(4m_\pi^2 < q^2 < (m_\phi - m_\eta)^2)$. The effective enhancement is by a factor $m_\rho^2/q^2 \sim 4.4$. The other components cancel out to some extent, so that a large part of the width of the decay $\phi\to\eta\pi^+\pi^-$ is described by the diagram in Fig. 2a.

Working from data on the decay[2] $\phi\to\eta\gamma$ and the vector dominance model (VDM), we find from (1) $\mathrm{Br}(\phi\to\eta\pi^+\pi^-) = 0.35\times 10^{-6}$. The diagram in Fig. 2a gives us $\mathrm{Br}(\phi\to\eta\pi^+\pi^-) = 0.25\times 10^{-6}$, while "pure" electrodynamics would give us 0.11×10^{-6} (see the diagram in Fig. 2b). By way of comparison we note that the expected value is $\mathrm{Br}(\phi\to\eta\mu^+\mu^-) = 0.6\times 10^{-5}$.

In the region of the ϕ meson, the amplitude of the process of interest here can be represented as the sum of two terms: the actual resonant amplitude for the production of the ϕ meson and a background amplitude. The background for the amplitude $e^+e^-\to\phi\to\eta\pi^+\pi^-$ is the amplitude $e^+e^-\to\rho\to\eta\pi^+\pi^-$, in which the transition $\rho\to\eta\rho\to\eta\pi^+\pi^-$ is allowed by strong interactions. The cross section is

$$\frac{d\sigma(e^+e^-\to\eta\pi^+\pi^-)}{d\Omega_\pi\, d\Omega_\rho\, dq^2} = \frac{d\sigma}{dq^2}\,(1+\cos^2\theta_\rho)\, W(\theta_\rho,\theta_\pi,\phi_\pi), \tag{3}$$

$$W(\theta_\rho,\theta_\pi,\phi_\pi) = \frac{9}{128\pi^2}\left(\sin^2\theta_\pi + \frac{\sin^2\theta_\pi\cos 2\phi_\pi\sin^2\theta_\rho}{1+\cos^2\theta_\rho}\right), \tag{4}$$

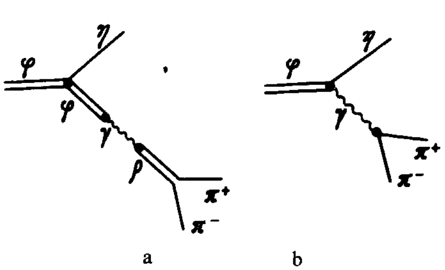

FIG. 2.

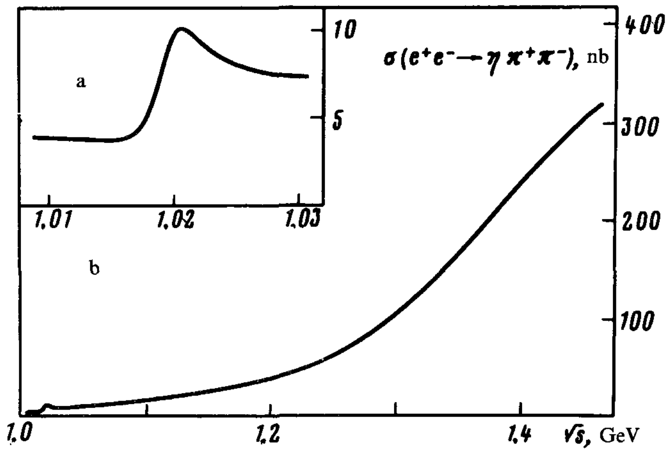

FIG. 3.

$$\frac{d\sigma}{dq^2} = \frac{4\alpha^2}{3}\frac{1}{s\sqrt{s}}\frac{\sqrt{q^2}\,\Gamma_\rho(q^2)p_\eta^3(s,q^2)}{(q^2-m_\rho^2)^2+(\sqrt{q^2}\,\Gamma_\rho(q^2))^2}\,|F(s)|^2\,, \qquad (5)$$

$$F(s) = \frac{m_\rho^2}{g_{\rho\gamma}}\,\frac{g_{\rho\rho\eta}}{s-m_\rho^2+i\sqrt{s}\,\Gamma_\rho(s)} + \frac{m_\phi^2}{g_{\phi\gamma}}\,\frac{g_{\phi\rho\eta}}{s-m_\phi^2+i\sqrt{s}\,\Gamma_\phi(s)}\,, \qquad (6)$$

where θ_ρ is the angle through which the ρ meson is scattered in the c.m. frame of the beams, and θ_π and ϕ_π are the polar and azimuthal angles of the π^+ in the rest frame of the $\pi^+\pi^-$ pair when the z axis is directed along the momentum of the ρ meson in the c.m. frame of the beams, while the azimuthal angle is reckoned from the reaction plane.

Figure 3 shows the cross section

$$\sigma(e^+e^-\to\eta\pi^+\pi^-) = \int_{4m_\pi^2}^{(\sqrt{s}-m_\eta)^2}\frac{d\sigma}{dq^2}\,dq^2\,. \qquad (7)$$

We can find the vertex $g_{\rho\rho\eta}$ by means of the VDM from the value[2] $\Gamma_{\rho\to\eta\gamma} = 56$ keV, which is the same as the prediction of the quark model if we use the well-known value of $\Gamma_{\omega\to\pi\gamma}$. In Fig. 3a we see an interference pattern near the ϕ meson, a consequence of the decay $\phi\to\eta\pi^+\pi^-$.

Figure 3b demonstrates the impressive threshold enhancement of the cross section for the process $e^+e^-\to\eta\pi^+\pi^-$ which stems from the opening of the channel $e^+e^-\to\rho\to\eta\rho\to\eta\pi^+\pi^-$.

The expected background from the two-photon annihilation $e^+e^- \rightarrow f \rightarrow \eta\pi^+\pi^-$ is of the order of 1%.

We wish to emphasize that there may be a manifestation of $\rho'(1250)$, whose existence has been under discussion for many years, in this energy range.

To observe the decay $\phi \rightarrow \eta\pi^+\pi^-$ on the VÉPP-2M accelerator with a neutral detector is a difficult problem although solvable in principle. As for the threshold effect in $\eta\rho$ production, we note that it can be studied in the very near future.

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[1]A. D. Bukin *et al.*, Preprint INP 83-80, Institute of Nuclear Physics, Novosibirsk, 1983.
[2]Particle Data Group, Phys. Lett. **111B**, 1 (1982).

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