Axial-anomaly pole and spontaneous breaking of chiral symmetry

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There is no factorization of vertices for an axial-anomaly pole. Consequently, a Goldstone boson coupled with a spontaneous breaking of chiral symmetry cannot reproduce an axial-anomaly pole. The possibility of introducing a massless Goldstone pseudoscalar boson, along with a massless pseudovector boson, is discussed.

The classic problem of the axial anomaly[1,2] continues to hold the focus of theoreticians.

We recently reported[3] analytic expressions, free of kinematic singularities, for the invariant amplitudes of the single-loop triangle diagrams (Fig. 1) for the transition (axial vector current) $\rightarrow q\bar{q}\rightarrow\gamma(k_1)\gamma(k_2)$ with $k_1^2=0$, $k_2^2\neq 0$. It was shown that an axial-anomaly pole[4] arises only in the limit of massless fermions, and only for real photons ($k_2^2=0$), despite widespread opinion to the contrary (Ref. 5, for example).

In the present letter we analyze the problem of the spontaneous breaking of chiral symmetry, in which (as it appears to us) some new subtleties arise in connection with the results of Ref. 3.

The amplitude for the transition (axial vector current) $\rightarrow q\bar{q}\rightarrow$ (conserved vector current) $\times$ (conserved vector current) is[6] (Fig. 1)

$$T_{\alpha\beta\mu}=\sum_i A_i t^i_{\alpha\beta\mu}=A_1k_1^\sigma\epsilon_{\sigma\alpha\beta\mu}+A_2k_2^\sigma\epsilon_{\sigma\alpha\beta\mu}+A_3k_{1\beta}k_1^\delta k_2^\sigma\epsilon_{\delta\sigma\alpha\mu}$$
$$+A_4k_{2\beta}k_1^\delta k_2^\sigma\epsilon_{\delta\sigma\alpha\mu}+A_5k_{1\alpha}k_1^\beta k_2^\sigma\epsilon_{\delta\sigma\beta\mu}+A_6k_{2\alpha}k_1^\delta k_2^\sigma\epsilon_{\delta\sigma\beta\mu}\,. \tag{1}$$

Gauge invariance (the condition for conservation of vector current),

$$k_1^\sigma T_{\alpha\beta\mu}=k_2^\beta T_{\alpha\beta\mu}=0, \tag{2}$$

is ensured by the relations

$$A_1=k_2^2A_4+(k_1k_2)A_3\,,\quad A_2=k_1^2A_5+(k_1k_2)A_6\,. \tag{3}$$

In addition, we have

$$A_3=(k_1,k_2)=-A_6(k_2,k_1),\quad A_4(k_1,k_2)=-A_5(k_2,k_1). \tag{4}$$

The invariant amplitudes A_3, A_4, A_5, and A_6 do not have kinematic singularities, and they have been determined well.[6] With $k_1^2=0$ (or $k_2^2=0$) they can be calculated in analytic form.[3]

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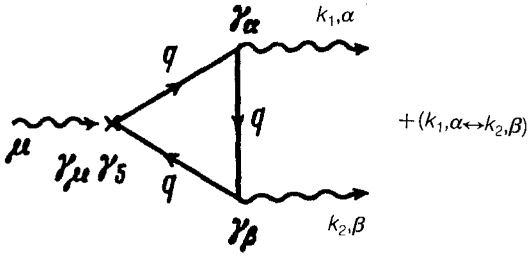

FIG. 1.

The amplitudes A_5 and A_4 do not contribute directly to physical quantities [not through relations (3)], since $k_{1\alpha}$ and $k_{2\beta}$ in (1) convolute only with the polarization vectors $[k_{1\alpha}e^{\alpha}(k_1)]=0$ and $[k_{2\beta}e^{\beta}(k_2)]=0$, or only with the conserved vector currents $[k_{1\alpha}j^{\alpha}(k_1)]=0$ and $[k_{2\beta}j^{\beta}(k_2)]=0$. In our case ($k_1^2=0$, $k_2^2\neq 0$), we can completely ignore A_5, as can be seen from (3).

In the limit $m_q\to 0$ (the chiral limit) the expressions for the amplitudes A_i are[3]

$$A_1=\frac{1}{4\pi^2}\left\{\frac{Q^2}{Q^2-W^2}\ \ln\frac{Q^2}{W^2}+1\right\},$$

$$A_2=\frac{1}{4\pi^2}\left\{\frac{Q^2}{Q^2-W^2}\ \ln\frac{Q^2}{W^2}-1\right\},$$

$$A_3=-A_6=-\frac{1}{2\pi^2}\frac{1}{Q^2-W^2}\left\{\frac{Q^2}{Q^2-W^2}\ \ln\frac{Q^2}{W^2}-1\right\},$$

$$A_4=-\frac{1}{2\pi^2}\frac{1}{Q^2-W^2}\ \ln\frac{Q^2}{W^2}, \tag{5}$$

where $0<Q^2=-E^2=-k_2^2$ and $0<W^2=-M^2=-(k_1+k_2)^2$.

In the other regions of M^2 and E^2, an analytic continuation is carried out in the following manner:

1) $0<-Q^2=E^2$: $\ln Q^2\to -i\pi+\ln E^2$.

2) $0<-W^2=M^2$: $\ln\frac{1}{W^2}\to i\pi+\ln\frac{1}{M^2}$. (6)

In the massless limit, there are thus cuts on the physical sheets of the amplitudes A_i given by expressions (5) and (6); these cuts are at $0\leqslant E^2<\infty$ and $0\leqslant M^2<\infty$ for $k_2^2\neq 0$. Only in the case $Q^2\to 0$, does a pole arise in A_3 and A_6 at $M^2=0$:

$$A_3=-A_6=\frac{2}{M^2}A_1=-\frac{2}{M^2}A_2=\frac{1}{2\pi^2}\frac{1}{M^2}. \tag{7}$$

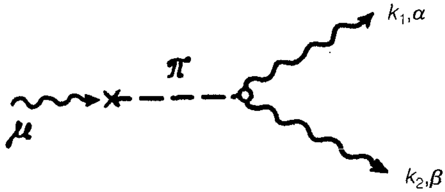

FIG. 2.

The conclusion that there is no factorization of vertices for an axial-anomaly pole seems to leap out at us. Nevertheless, we will prove it. In the axial vector channel we consider the contribution of a massless Goldstone boson (Fig. 2):

$$T^G_{\alpha\beta\mu} = \frac{f_\pi g_{\pi\gamma\gamma}}{M^2}(k_1+k_2)_\mu k_1^\delta k_2^\sigma \epsilon_{\delta\sigma\beta\alpha}\,. \tag{8}$$

The factorization of vertices in (8) is obvious. To go over to the form of (1), we use the relations

$$k_{1\mu}k_1^\delta k_2^\sigma \epsilon_{\delta\sigma\beta\alpha} = k_1^2 k_2^\sigma \epsilon_{\sigma\alpha\beta\mu} - (k_1k_2)k_1^\sigma \epsilon_{\sigma\alpha\beta\mu} - k_{1\beta}k_1^\delta k_2^\sigma \epsilon_{\delta\sigma\alpha\mu} - k_{1\alpha}k_2^\delta k_1^\sigma \epsilon_{\delta\sigma\beta\mu}\,,$$

$$k_{2\mu}k_1^\delta k_2^\sigma \epsilon_{\delta\sigma\beta\alpha} = (k_1k_2)k_2^\sigma \epsilon_{\sigma\alpha\beta\mu} - k_2^2 k_2^\sigma \epsilon_{\sigma\alpha\beta\mu} - k_{2\beta}k_1^\delta k_2^\sigma \epsilon_{\delta\sigma\alpha\mu} - k_{2\alpha}k_2^\delta k_1^\sigma \epsilon_{\delta\sigma\beta\mu}\,. \tag{9}$$

As a result, we find

$$A_1^G = -\frac{f_\pi g_{\pi\gamma\gamma}}{M^2}[(k_1k_2)+k_2^2], \quad A_2^G = \frac{f_\pi g_{\pi\gamma\gamma}}{M^2}[(k_1k_2)+k_1^2],$$

$$A_3^G = A_4^G = -A_5^G = -A_6^G = -\frac{f_\pi g_{\pi\gamma\gamma}}{M^2}. \tag{10}$$

We see from (10) that a Goldstone pole is present in the amplitudes A_i not only under the condition $k_1^2 = k_2^2 = 0$.

A massless pseudoscalar Goldstone boson is thus incapable of reproducing an axial-anomaly pole.

't Hooft has proposed an elegant principle,[7] according to which a *composite particle must reproduce an axial anomaly of its fermion constituents.*

Does it follow from the result found above that an axial anomaly pole is incompatible with a spontaneous breaking of chiral symmetry? Generally speaking, no! However, we are forced to pay a price for introducing a Goldstone boson. To see just how high this price is, we identify the Goldstone pole with the axial-anomaly pole in the case $k_1^2 = k_2^2 = 0$.

A comparison of (10) with (7) yields

$$f_\pi g_{\pi\gamma\gamma} = -\frac{1}{2\pi^2}\,. \tag{11}$$

We now consider the difference between amplitudes (5) and (10), using (11):

$$\bar{A}_1=A_1-A_1^G=\bar{A}_2=A_2-A_2^G=\frac{1}{4\pi^2}Q^2\left\{\frac{1}{Q^2+M^2}\ \ln\frac{Q^2}{-M^2}+\frac{1}{M^2}\right\},$$

$$\bar{A}_6=A_6-A_6^G=-\bar{A}_3=-A_3+A_3^G=\frac{1}{2\pi^2}\frac{Q^2}{Q^2+M^2}\left\{\frac{1}{Q^2+M^2}\ \ln\frac{Q^2}{-M^2}+\frac{1}{M^2}\right\}. \quad (12)$$

Here we have restricted the discussion to only the physically important amplitudes, and $M^2<0$.

It can be seen from (12) that there is a pole in the invariant amplitudes $\bar{A}_i$ at $M^2=0$ in the case $k_2^2\neq0$. This pole looks like a massless-boson component.

It is not difficult to verify that the amplitude

$$\bar{T}_{\alpha\beta\mu}=\sum_i \bar{A}_i t^{\,i}_{\alpha\beta\mu} \quad (13)$$

is transverse in the axial vector channel:

$$\partial^\mu \bar{T}_{\alpha\beta\mu}=0. \quad (14)$$

In other words, only (1^+) pseudovector intermediate states are possible in the axial vector channel of the amplitude $\bar{T}_{\alpha\beta\mu}$. Accordingly, in the chiral limit, with $k_2^2\neq0$, there is a pole in the amplitude $\bar{T}_{\alpha\beta\mu}$ which looks like the contribution of a massless pseudovector boson.

We thus see that in the chiral limit in the case of a spontaneous breaking of chiral symmetry (or, if you wish, of a nonlinear realization of this symmetry), a massless pseudoscalar Goldstone boson can reproduce an axial anomaly pole only as the result of a "deal" struck with a pseudovector massless boson. Consequently, in the case $k_2^2\neq0$ their resultant contribution to the invariant amplitudes A_i disappears. These amplitudes are free of kinematic singularities.

Is it possible to hold the price to this level? As far as we can see, a (1^+) massless pseudovector boson will require a (1^-) massless vector chiral partner.

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