Self-focusing and angular distribution of electrons moving in the field of atomic rows

M. Kh. Khokonov

Kabardino-Balkarskiĭ University, 360004, Nal'chik, Kabardino-Balkariya, Russian Federation

(Submitted 18 August 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **56**, No. 7, 349–351 (10 October 1992)

When relativistic electrons move along atomic rows as they pass through a crystal, their angular distribution as they leave the crystal is quite different from that in the case of an amorphous medium. A radiative self-focusing of electrons with energies of hundreds of GeV is predicted when the electron beam initially has a wide angular divergence.

Relativistic charged particles are extremely sensitive to the field of the atomic rows even if they are moving at angles much larger than the Lindhard critical angle. As a result, there is a "doughnut" scattering;[1] i.e., rings appear in the angular distributions due to a coherent scattering by the rows while transverse energy is conserved.[2,3] If the initial beam is azimuthally symmetric with respect to an atomic axis, the only changes which occur in the angular distribution as the beam penetrates into the crystal are those caused by incoherent collisions accompanied by a change in transverse energy. This case, the most common case in practice, captures our interest in the present letter.

Three factors make the angular distributions of electrons in oriented crystals different from those in the case of amorphous media. (a) The continuous potential of the axis (or plane) has an effect. (b) The cross section for incoherent scattering by a single atom in a channel is larger than that in an amorphous medium by a factor $\simeq S_0/S(\epsilon)$, where $S(\epsilon)$ is the transverse area accessible to an electron with an initial energy ϵ, and S_0 is the area per row. (c) The emission of photons has an effect on the motion.

Under statistical-equilibrium conditions,[4] particles with a given transverse energy ϵ are distributed in transverse phase space in accordance with the microcanonical distribution

$$dn(\mathbf{r},\mathbf{p}\epsilon)=\delta\left(\epsilon-\frac{\mathbf{p}^2}{2m}-U(\mathbf{r})\right)\frac{d\mathbf{r}d\mathbf{p}}{\Omega(\epsilon)}, \tag{1}$$

where $\mathbf{r}$ and $\mathbf{p}$ are the transverse coordinate and transverse momentum, $U(\mathbf{r})$ is the continuous potential of the atomic row (or plane), $\Omega(\epsilon)=2\pi m S(\epsilon)$ is the density of states with the given transverse energy, and m is the relativistic mass of the particle. In the planar case the density $\Omega(\epsilon)$ is equal to the period of the transverse motion.

The integration over momentum in (1) leads to known expressions for the spatial distribution of particles in a channel,[4] while the integration over coordinates yields the

 © 1992 American Institute of Physics

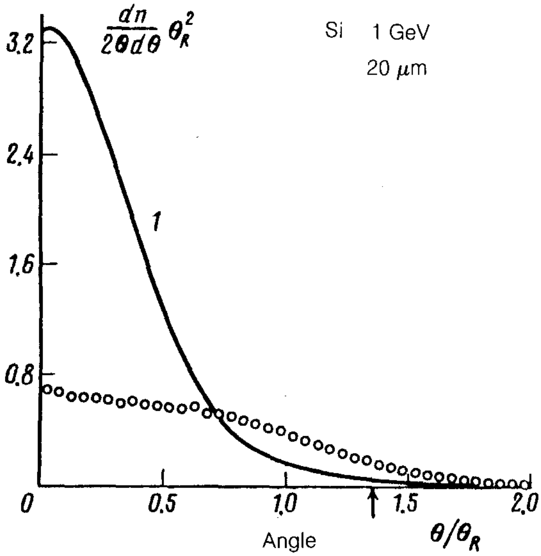

FIG. 1. Angular distribution of 1-GeV electrons which have passed through a silicon crystal 20 μm thick. *1*—Molière distribution for an amorphous target; Δ—for the $\langle 111 \rangle$ direction. The angles are given in units of the average multiple-scattering angle for an amorphous medium, θ_R ($\theta_R = 3 \times 10^{-4}$ in the case at hand). The arrow marks the critical angle for channeling, $\theta_R = 4.1 \times 10^{-4}$).

angular distribution of particles with the given transverse energy. In the axial case we have

$$dn(\mathbf{p},\epsilon) = \frac{2\pi r p dp}{m S(\epsilon) U'(r)}, \tag{2}$$

where $U'(r) = dU/dr$, and $r = r(\epsilon,p)$ is found from the condition $\epsilon(p^2/2m) + U(r)$. The transverse momentum p is related to the angle from the axis, θ, by $p = mv\theta$.

According to (2), the effect of the field of the row on the angular distribution is substantial for not only channeled particles but also above-barrier particles.

Figure 1 compares the angular distribution of 1-GeV electrons which have passed through a silicon crystal 20 μm thick, along the $\langle 111 \rangle$ axis, with a Molière distribution for an amorphous medium. In the first case, the data were found through a numerical simulation by a method like that of Ref. 5. In contrast with Ref. 5, however, we used a Molière potential with a Debye–Waller factor for calculating the cross sections for incoherent scattering, and we also considered all events of scattering by the individual atoms, without any restriction on scattering angles.

At electron energies of hundreds of GeV, radiation becomes important. Here the analysis is simplified because the synchrotron approximation can be used to calculate the radiation cross sections. If we assume that the photons are emitted along the velocity vector, the change in the transverse energy of an electron upon the emission of a photon with an energy ω is

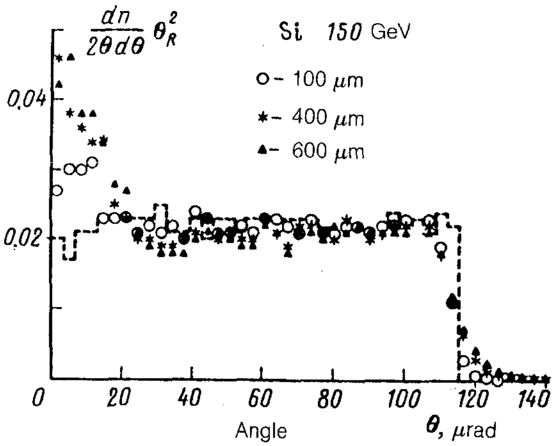

FIG. 2. Angular distribution of electrons with an initial energy of 150 GeV after they have passed along the ⟨110⟩ axis through silicon crystals of various thicknesses. The dashed line corresponds to an angular divergence $3\theta_L$ of the incident beam. The intensity is given in units of θ_R^2, where $\theta_R = 1.7\times10^{-5}$.

$$\delta\epsilon = -\frac{\omega}{E}[\epsilon - U(r)], \tag{3}$$

where E is the total energy of the electron before the emission.

Figure 2 shows angular distributions of 150-GeV electrons which have passed through ⟨110⟩ silicon crystals of various thicknesses. We see that a self-focusing occurs at the exit from the crystal: The number of electrons making small angles with the axis, $\theta < 0.5\theta_L$ (θ_L is the Lindhard critical angle), as they leave the crystal is larger than the corresponding number as the electrons enter the crystal—despite the fact that (as a calculation shows) the total mean square angle increases slowly as the electrons enter the crystal.

That a radiative self-focusing would occur is not obvious at the outset; according to (3), the angle between the direction of the motion and the channel axis does not change during the emission. After the emission, however, the electron goes into a state with a smaller transverse energy and thus a new transverse-momentum distribution, (2). This change is ultimately responsible for the effect. A self-focusing due to a different mechanism has been proposed previously by Kumakhov[6] for nonrelativistic positive particles.

I wish to thank my colleagues J.-U. Anderson, V. A. Ryabov, and E. Uggerhøj at the University of Århus for many useful discussions. I am particularly indebted to J. Lindhard for his interest and critical comments.

[1] S. K. Andersen, O. Fich, H. Nielsen *et al.*, Nucl. Phys. B **167**, 1 (1980).
[2] V. V. Beloshitskiĭ and M. A. Kumakhov, Fiz. Tverd. Tela (Leningrad) **15**, 1588 (1973) [Sov. Phys. Solid State **15**, 1060 (1973)].
[3] I. A. Golovchenko, Phys. Rev.. B **13**, 4672 (1976).
[4] J. Lindhard, Kgl. Dan. Vidensk. Selsk. Mat. Fys. Medd. **34** (1965).

[5]Yu. V. Kononets and V. A. Ryabov, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 303 (1988) [JETP Lett. **48**, 333 (1988)].
[6]M. A. Kumakhov, Pis'ma Zh. Tekh. Fiz. **14**, 250 (1988) [Sov. Tech. Phys. Lett. **14**, 112 (1988)].

Translated by D. Parsons