Nonlocal fluctuation electromagnetic response and magnetic scattering of neutrons near the superconducting transition temperature

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A general expression is derived for that frequency- and wave-vector-dependent component of the transverse dielectric constant which stems from superconducting fluctuations, $\varepsilon^f_{tr}(\Omega,Q)$, for an isotropic bulk metal in the normal state near T_c. The conditions $\Omega \ll T_c$ and $Q\xi_0 \ll 1$ are assumed. The differential cross section for magnetic scattering of neutrons at comparatively small angles near T_c is discussed.

As a normal metal approaches the temperature of its superconducting transition, its conductivity and diamagnetism increase as the result of the appearance of Cooper pairs in a fluctuation process.[1–6] This fluctuation component of the electromagnetic response is usually studied in the case in which the spatial dispersion of the conductivity and the diamagnetic susceptibility can be ignored. There are, nevertheless, problems in which the nonlocal nature of the fluctuation electromagnetic response plays an important role and must be taken into account fully. We have previously carried out a corresponding study of the fluctuation diamagnetism in the static limit.[7] In the present letter we find the fluctuation response as a function of the frequency and the wave vector for a more general case. We show that the spatial dispersion of the fluctuation response may play an important role at values of the momentum transfer which have recently been achieved in experiments on scattering by high-T_c superconductors[8] and that this dispersion may be manifested in the differential cross section for magnetic scattering.

In a normal metal near T_c, the fluctuation-related linear electromagnetic response outside the critical region is known to be described by the diagrams in Figs. 1 and 2 (a wavy line corresponds to a fluctuation propagator, and a dashed line to scattering by impurities). The diagram in Fig. 1, which corresponds to the Aslamazov–Larkin component, is usually analyzed for frequencies $\Omega \ll T_c$ and in the limit in which the momentum of the external electromagnetic field, Q, vanishes. We assume that the external momentum Q can be comparable to or even greater than the reciprocal of the correlation radius of the superconducting fluctuations, $\xi^{-1}(T)$, but that the condition $Q \ll \xi_0^{-1}$ holds, where ξ_0 is the coherence length at absolute zero. In a calculation on the diagram in Fig. 1, this circumstance leads to a strong Q dependence of the fluctuation propagators, while the electron loops can be described, as before, by the expression corresponding to the limit $Q \to 0$. The reason is that for the fluctuation propagators $K^{R,A}(p,\omega) = 1/(a+p^2/4m \mp i\gamma\omega)$ [here $a=\alpha(T-T_c)$] the characteristic momentum is $\xi^{-1}(T)$, while for the electron Green's functions it is either ξ_0^{-1} or l^{-1}, where l is the mean free path.

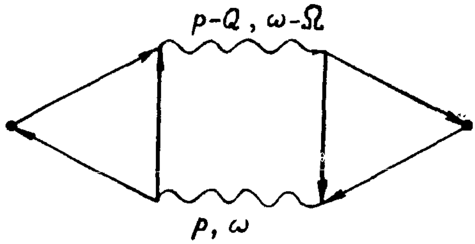

FIG. 1.

Considering the case of a transverse applied field, with div $\mathbf{E}=0$, we find $\mathbf{j}(\Omega,\mathbf{Q}) = (-i\Omega/4\pi)\varepsilon^f_{tr}(\Omega,Q)\mathbf{E}(\Omega,\mathbf{Q})$ from Fig. 1 after a corresponding analytic continuation and an integration over the frequency. Here

$$\varepsilon^f_{tr}(\Omega,Q)=\frac{64\pi e^2 T_c}{\Omega^2}\int\frac{d^3p}{(2\pi)^3}p_y^2\left\{\frac{1}{2\xi^{-2}(T)+p^2+(\mathbf{p}-\mathbf{Q})^2-4im\gamma\Omega}\right.$$
$$\left.\times\left[\frac{1}{\xi^{-2}(T)+p^2}+\frac{1}{\xi^{-2}(T)+(\mathbf{p}+\mathbf{Q})^2}\right]-\frac{1}{[\xi^{-2}(T)+p^2]^2}\right\}. \qquad (1)$$

Here $\xi(T)=1/2(ma)^{1/2}$, and the vector $\mathbf{Q}$ is assumed to be directed along x. For the discussion below it is convenient to introduce a dimensionless frequency $\omega =2m\gamma\xi^2(T)\Omega$ and a dimensionless wave vector $\mathbf{q}=\mathbf{Q}\xi(T)/2$. After the integral in (1) is evaluated, the Aslamazov–Larkin component (which depends on the frequency and the wave vector) of the transverse dielectric constant of a normal metal near T_c can be written in the form

$$\varepsilon^f_{tr}(\Omega,Q)=\frac{16\gamma^2m^2e^2T_c\xi^3(T)}{\omega^2}\left\{1+\frac{i\omega}{2q^2}[1-(1+q^2-i\omega)^{1/2}]-\frac{1}{q}\left[1+q^2\left(1-\frac{i\omega}{2q^2}\right)^2\right]\right.$$
$$\left.\times\left[\arctan\left(q-\frac{i\omega}{2q}\right)+\arctan\left(\frac{i\omega}{2q(1+q^2-i\omega)^{1/2}}\right)\right]\right\}. \qquad (2)$$

Expression (2) could also be derived from the linearized time-dependent Ginzburg–Landau equation with a Langevin source.

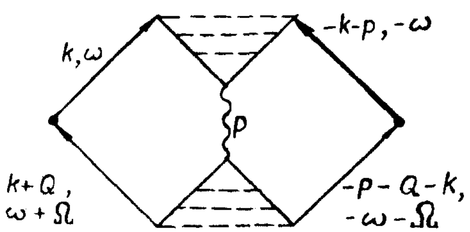

FIG. 2.

For small values of the wave vector, $q^2 \ll \omega$, we find from (2)

$$\varepsilon^f_{tr}(\Omega,Q)=\varepsilon^f(\Omega)+i\,\frac{256\gamma^2m^2e^2T_c\xi^3(T)}{15\omega^5}\,q^2$$
$$\times\left[1-\frac{5}{2}\,i\omega-(1-i\omega)^{5/2}-\frac{15}{8}\,\omega^2(1-i\omega)^{1/2}\right], \qquad (3)$$

where the expression for the fluctuation dielectric constant with exclusively a frequency dispersion,

$$\varepsilon^f(\Omega)=\frac{32\gamma^2m^2e^2T_c\xi^3(T)}{\omega^2}\left\{1+\frac{2i}{3\omega}\,[1-(1-i\omega)^{3/2}]\right\}, \qquad (4)$$

agrees with the results of Refs. 1 and 2.

At comparatively large values of the wave vector, $q^2 \gg \omega$, we find from (2)

$$\varepsilon^f_{tr}(\Omega,Q)=\frac{4\pi c^2Q^2}{\Omega^2}\,\chi^f(Q)+\frac{4\pi i}{\Omega}\,\sigma^f(Q). \qquad (5)$$

Here $\chi^f(Q)$ and $\sigma^f(Q)$ are the static diamagnetic susceptibility and the static transverse conductivity of the normal metal near T_c; both this susceptibility and this conductivity depend on the wave vector. The quantity $\chi^f(Q)$ was found in Ref. 7. The spatial dispersion of the static transverse fluctuation conductivity is

$$\sigma^f(Q)=\frac{2\sigma_{AL}}{q}\left\{\arctan\, q+\frac{1}{q}\,[1-(1+q^2)^{1/2}]\right\}, \qquad (6)$$

where $\sigma_{AL}=e^2\gamma mT_c\xi(T)/\pi$ is the fluctuation Aslamazov–Larkin conductivity.

It follows from (6) that near T_c the spatial dispersion of the fluctuation conductivity of a metal becomes important even at extremely small values of the wave vector, $Q\sim\xi^{-1}(T)\ll\xi_0^{-1}$, while the wave-vector dependence can be ignored in all other components of the conductivity.

In addition to the Aslamazov–Larkin diagram for the Maki–Thompson diagram[4,5] shown in Fig. 2, one could parametrize the momenta in such a way that the external momentum goes through only the lines of electron Green's functions and does not go through a fluctuation propagator (as shown in Fig. 2). For this reason there is a qualitative difference between the behavior of the spatial dispersion for the Aslamazov–Larkin corrections and that for the Maki–Thompson corrections. Under the condition $Q\ll\xi_0^{-1}$ we can ignore the wave-vector dependence of the Maki–Thompson correction to the conductivity of the metal, σ_{MT}. In the dirty limit, the nonlocal nature of the Maki–Thompson correction thus becomes noticeable only at $Q\sim l^{-1}\gg\xi_0^{-1}$. Incorporating the frequency dispersion of the Maki–Thompson correction leads to the following expression in the dirty limit:

$$\sigma_{MT}(\Omega)=\frac{1}{8}\,\frac{e^2}{\xi_0\left[t^{1/2}+\left(\delta-\dfrac{\pi i\Omega}{8T_c}\right)^{1/2}\right]}, \qquad (7)$$

where $t=(T-T_c)/T_c$, $\delta=\pi/8T_c\tau_\phi$, and τ_ϕ is the phase relaxation time. In the static limit, expression (7) becomes the Maki–Thompson result.

The specific dependence of the transverse dielectric constant on the temperature, the wave vector, and the frequency may be manifested in, for example, the behavior of the differential cross section for magnetic scattering of neutrons in a normal metal near T_c. A behavior of this sort was recently observed in experiments on polycrystalline samples of high-T_c superconductors.[8] In the absence of magnetic order, the magnetic scattering of neutrons is attributed to their interaction with the equilibrium fluctuation magnetic field. The corresponding differential cross section for the scattering of a depolarized beam into the element of solid angle dO and into the element of energy transfer $d\Omega$ at comparatively small values of the energy transfer, $\Omega \ll T_c$, is given by the following expression for an isotropic bulk sample:

$$\frac{d^2S}{d\Omega dO}=\frac{g^2e^2VT_cp'}{2\pi^2c^2}\frac{1}{p}\frac{1}{\Omega}Im\frac{1}{1-\dfrac{\Omega^2}{c^2Q^2}\varepsilon_{tr}(\Omega,Q)}. \qquad (8)$$

Here $\mathbf{Q}$ is the momentum transfer, $\mathbf{p}=m_Nv_N$ and $\mathbf{p}'=\mathbf{p}+\mathbf{Q}$ are the initial and final momenta of the electron, and $g=1.91$ is the magnetic moment of the neutron, expressed in nuclear magnetons.

Bernhoeft *et al.*[8] have measured the scattering cross section dS/dO at small angles, integrated over the energy. To calculate this cross section, we should integrate (8) over the energy Ω at a fixed scattering angle θ or, equivalently, at a fixed value of the momentum transfer corresponding to quasielastic through an angle θ: $K=2p\sin(\theta/2)$. For scattering through small angles we have $\Omega/v_N \sim K \ll p$ in the integration of (8). We can thus use the approximation $Q^2=K^2+\Omega^2/v_N^2$ in (8). For the frequencies and wave vectors of interest below, the transverse dielectric constant can be written $\varepsilon_{tr}(\Omega,Q)=4\pi i\sigma/\Omega+\varepsilon_{tr}^f(\Omega,Q)$, where σ is the static conductivity of a normal metal without superconducting fluctuations (for high-T_c superconductors, a Maki–Thompson correction can also be incorporated in σ; more on this below) and $\varepsilon_{tr}^f(\Omega,Q)$ is defined in (2). The spin component of $\varepsilon_{tr}(\Omega,Q)$ in these regions of Ω and Q usually does not have a specific temperature dependence near T_c, so it is being ignored.

Estimates based on the measurement conditions of Ref. 8 show that the inequality $c^2Q^2 \gg \Omega^2\varepsilon_{tr}(\Omega,Q)$ holds accurately. Using it, we find explicit analytic expressions for the contribution of superconducting fluctuations to the scattering cross section dS^f/dO in the angular intervals corresponding to $K \ll m\gamma v_N/\hbar^2$ and $K \gg m\gamma v_N/\hbar^2$:

$$\frac{dS^f}{dO}=\frac{16g^2e^4VT_c^2}{\pi c^4\hbar^8}(m\gamma v_N)^2\xi^3(T)\left\{\frac{2}{3\tilde{k}^3}[(1+\tilde{k})^{3/2}-1]-\frac{1}{\tilde{k}^2}\right\}, \quad K \ll m\gamma v_N/\hbar^2, \qquad (9)$$

$$\frac{dS^f}{dO}=\frac{g^2e^4VT_c^2\gamma v_N}{2\pi c^4\hbar^4 a}\left[\frac{1}{k^2}\left(1+\frac{k}{2}-(1+k^2)^{1/2}\right)+\frac{2\chi^f(K)}{3\pi\chi^f(0)}\right], \quad K \gg m\gamma v_N/\hbar^2. \qquad (10)$$

Here we have introduced the dimensionless quantities $k=K\xi(T)/2$ and $\tilde{k}=2m\gamma v_N\xi^2(T)K/\hbar^2$, and we are no longer assuming that Planck's constant is unity.

The conditions $K \ll m\gamma v_N/\hbar^2$ and $K \gg m\gamma v_N/\hbar^2$ correspond to the limiting cases $q^2 \ll \omega$ and $q^2 \gg \omega$ (if we have $\Omega \sim K v_N$). The dependence of the scattering cross section dS^f/dO on the momentum $\hbar K$ in the angular interval $K \ll m\gamma v_N/\hbar^2$ [see (9)] is thus related to the dependence of dielectric constant (4) on the frequency Ω. The effect of spatial dispersion is negligible. The dependence of the quantity dS^f/dO on the momentum $\hbar K$ at angles $K \gg \gamma v_N/\hbar^2$ [see (10)] is directly related to the wave-vector dependence of fluctuation conductivity (6). It follows that incorporating the spatial dispersion is important here under the conditions $K \gtrsim m\gamma v_N/\hbar^2$, $\xi^{-1}(T)$. Estimates show that values of the momentum transfer of this order of magnitude figure in the measurements of Ref. 8.

Under the condition $K \ll m\gamma v_N/\hbar^2$, either of the relations $\tilde{k} \ll 1$ and $\tilde{k} \gg 1$ can hold, generally speaking. Consequently, the inequality $K \gg m\gamma v_N/\hbar^2$ does not rule out either the limiting case $k \ll 1$ or the limiting case $k \gg 1$. In the limiting cases $\tilde{k} \ll 1$ and $k \ll 1$, expressions (9) and (10) take the same form, as they should:

$$\frac{dS^f}{dO} = \frac{g^2 e^4 V T_c^2 \gamma v_N m^{1/2}}{\pi c^4 \hbar^5 [\alpha(T-T_c)]^{1/2} K} = \frac{2 g^2 e^2 V T_c \sigma_{AL} v_N}{c^4 \hbar^2 K}. \qquad (11)$$

Under these conditions the scattering cross section increases toward T_c in proportion to $(T-T_c)^{-1/2}$ and in proportion to $1/K$.

If we instead have $\tilde{k} \gg 1$ in (9) or $k \gg 1$ in (10), the scattering cross section does not increase toward T_c, and from (9) and (10) we find

$$\frac{dS^f}{dO} = \frac{8 g^2 e^4 V T_c^2 [2m\gamma v_N]^{1/2}}{3\pi c^4 \hbar^5 K^{3/2}}, \quad \frac{\hbar^2}{2m\gamma v_N \xi^2(T)} \ll K \ll m\gamma v_N/\hbar^2, \qquad (12)$$

$$\frac{dS^f}{dO} = \frac{16}{\pi^2}\left(\frac{\pi}{2} - 1\right)\frac{g^2 e^4 V T_c^2 m\gamma v_N}{c^4 \hbar^6 K^2}, \quad K \gg m\gamma v_N/\hbar^2, \frac{1}{\xi(T)}. \qquad (13)$$

We find an expression like (11) when we consider essentially any contribution $\Delta\sigma$ to the conductivity of the metal, including one in the case of scattering angles for which we can ignore both the spatial dispersion and the frequency dispersion of $\Delta\sigma$ [under the condition $c^2K^2 \gg \Omega^2 \varepsilon_{tr}(\Omega,K)$, the contributions to the scattering from the different terms in $\varepsilon_{tr}(\Omega,K)$ are additive in a first approximation]. This statement also applied to the Maki–Thompson correction. It can be shown that the frequency dispersion of the Maki–Thompson correction can be ignored because of the short phase relaxation time $\tau_\phi \sim \hbar/T_c$ of high-T_c superconductors[10] under the condition $\hbar\Omega \ll T_c$. The contribution of the Maki–Thompson correction to the magnetic scattering is thus described by (11) with the substitution $\sigma_{AL} \to \sigma_{MT}$ over the entire range of scattering angles under consideration here. This contribution to the scattering is weaker than (for example) expression (13) to the extent that the parameter $K\xi_0$ is small, so this contribution can be ignored.

Since the superconducting fluctuations are by assumption Gaussian, the description of the height of the plateau on the basis of relations (12) and (13) is valid only if the conditions for the applicability of these relations hold at $T > T_{Gi}$, where T_{Gi} is the boundary of the regions of Gaussian and strong fluctuations. In particular, a necessary condition for the validity of (13) is $[K\xi(T_{Gi})]^2 \gg 1$. For the experimental

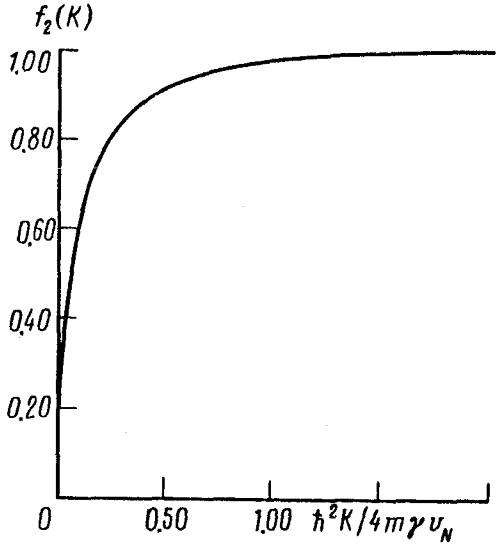

FIG. 3.

conditions in Ref. 8, the quantity $K\xi(T_{Gi})$ is apparently a few units, so (13) can be used at least for qualitative estimates.

Let us examine the scattering cross section dS^f/dO as a function of the temperature for a given value of the momentum transfer under the condition $K\xi_0 \ll 1$ (and also $\hbar K \ll p$). As the temperature is lowered, as T_c is approached, and as the Gaussian superconducting fluctuations come into play, the quantity dS^f/dO increases according to (11). Then one of conditions (12), (13) holds because of the increase in $\xi(T)$, and a plateau appears on the plot of the function dS^f/dO. This is precisely the behavior of dS^f/dO which was observed in Ref. 8.

According to results (12) and (13), the height of the plateau decreases with increasing momentum transfer and in proportion to $K^{-3/2}$ if the effects of the nonlocal nature of the fluctuation response are inconsequential ($K \ll m\gamma v_N/\hbar^2$); alternatively, it decreases in proportion to K^{-2} if spatial dispersion must be taken into account ($K \gg m\gamma v_N/\hbar^2$). The K dependence of the plateau height can also be described at intermediate values of the momentum transfer on the basis of numerical calculations. For this purpose, we write the scattering cross section in the form $dS^f/dO = f_1(K)f_2(K)$, where $f_1(K)$ represents expression (13). Figure 3 shows a plot of the function $f_2(K)$. It can be seen from the experimental results in Ref. 9 that the decrease in the plateau height with increasing K is described better by a K^{-2} dependence than a $K^{-3/2}$ dependence in the case of those results. This circumstance, along with the other estimates which we mentioned above, leads us to conclude that nonlocal effects must be taken into consideration in interpreting these measurements.

For the angular interval in (12), the plateau height is proportional to the quantity $T_c^{3/2}/\xi_0$, while for the angular interval in (13) it is proportional to T_c/ξ_0^2 [here we are assuming $\xi_0 \sim \hbar/(m\alpha T_c)^{1/2}$ and $\gamma \sim \hbar\alpha$]. It follows that for high-T_c superconductors the height of the plateau, i.e., the characteristic size of the contribution of the superconducting fluctuations to the magnetic scattering of neutrons, is larger by a factor of 10^3–10^5 than in the case of conventional, low-temperature superconductors (with their long coherence length). The results found above, however, are good for only a qualitative discussion of the experimental data of Ref. 8. A quantitative analysis of the measurements of Ref. 8 will require a generalizing of the results here to the case of anisotropic superconductors and to the polycrystalline structures of the samples of high-T_c superconductors used in Ref. 8. If we nevertheless use (13) along with the data of Ref. 8 [$K=0.035$ Å^{-1}, $v_N=6\times10^5$ cm/s] to estimate the absolute value of the change in the scattering cross section near T_c, and if we also assume $\xi_0 \approx 2$ Å, then we find a value for the cross section dS^f/dO per atom (the volume of the unit cell of $YBa_2Cu_3O_{7-\delta}$ is 173 Å^3) which is smaller than that measured in Ref. 8 by a factor of several tens. Both the anisotropy (particularly if the fluctuations are approximately two-dimensional) and the polycrystalline structure of the sample should significantly increase the scattering. Inhomogeneities in the sample may be important, even if their length scale is comparatively large. The reason is that the wave vector $K=0.035$ Å^{-1} corresponds to a wavelength of 180 Å. Under these conditions, the results found for a homogeneous system can legitimately be used only if there are no inhomogeneities with a length scale smaller than 10^3 Å.

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