Gluon and quark contributions to the hadron mass

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Trace anomaly in QCD and low-energy theorems are used to determine the gluon and quark contributions to the hadron mass and express it in terms of the string tension and current quark masses.

1. In QCD all hadrons consist of quarks and gluons, and in the limit, where the current quark masses are much less than Λ_{QCD}, the quark contribution to the hadron mass disappears. This means that, for example, nucleons and all objects around us are massive with good accuracy due to gluons inside them. How can this conclusion be reconciled with the constituent quark model (CQM), where the nucleon mass is mostly due to the masses of constituent quarks?

In this letter we show that the dominant contribution to the mass of light flavor hadrons indeed comes from gluons in the strings that connect the quarks. The strings also create constituent quark mass, and the resulting picture agrees with CQM.

In QCD the hadron (meson or baryon) mass can be expressed in terms of the energy density of the quark and gluon fields inside the hadron, e.g., in the c.m. system:

$$M=\langle h|\Delta\Theta_{\mu\mu}|h\rangle,\quad \Delta\Theta_{\mu\mu}=\Theta_{\mu\mu}-\langle\Theta_{\mu\mu}\rangle, \tag{1}$$

where[1]

$$\Theta_{\mu\mu}=\frac{\beta(\alpha_s)}{4\alpha_s}F^a_{\mu\nu}F^a_{\mu\nu}+\sum_q m_q\bar{\Psi}_q\Psi_q(1+\gamma_q), \tag{2}$$

m_q is the current quark mass, and γ_q is anomalous dimension; the sum is over all quark species $q=u, d, s,\ldots$, while $\langle\Theta_{\mu\mu}\rangle$ is the vacuum average of $\Theta_{\mu\mu}$.

We include $1+\gamma_q$ into the definition of the quark mass which is normalized at the typical hadronic scale of 1 GeV.

The hadron wave function Ψ_h depends on the center-of-mass time y_4 and can be written in terms of the evolution operator $|h\rangle\equiv\exp(-Hy_4)\Psi_0$, where H is the complete QCD Hamiltonian. Similarly, for $\langle h|$ we write

$$\langle h|\equiv\Psi_h^+\sim\Psi_0(T)\exp\, H(T-y_4).$$

We obtain for the mass in (1)

$$M=\left\langle\int d^3y\Psi_h^+(y_4)\Delta\Theta_{\mu\mu}(y_i,y_4)\Psi_h(y_4)\right\rangle, \tag{3}$$

or

$$M=\lim_{T\to\infty}\frac{\langle\int G(T,0)\Delta\Theta_{\mu\mu}(y)d^3y\rangle}{\langle G(T,0)\rangle}=\lim_{T\to\infty}\frac{1}{T}\frac{\int d^4y\langle G(T,0)\Delta\Theta_{\mu\mu}(y)\rangle}{\langle G(T,0)\rangle}, \tag{4}$$

where G is the hadron Green's function which evolves from some initial state at the c.m. Euclidean time $\tau=0$ for a final state at the c.m. time T. The limit of large T separates the ground state hadron. Angular brackets imply averaging over all gluonic and quark fields via functional integral with the standard measure.[2] For $G(T,0)$ the Feynman–Schwinger representation[3,4] readily relates it to a path integral over Wilson loops with spin insertion $W_\Sigma(A)$

$$G(T,0)=\int D\rho(z,s)W_\Sigma(A), \tag{5}$$

where

$$W_\Sigma(A)=P\exp g\int_0^s \sigma_{\mu\nu}F_{\mu\nu}d\lambda\cdot P\exp ig\int A_\mu dz_\mu\,. \tag{6}$$

The path-integral measure $D\rho$ depends on the proper times s_i, the quark (antiquark) trajectories $z_i(\lambda)$, and contains the quark determinant $\Pi_q\det[m_q+\hat{D}(A)]$, which is properly normalized and regularized.

2. First, we consider the case in which the spin interaction [the first factor on the r.h.s. of (6)] can be assumed a perturbation. Disregarding it completely (for spinless quarks), we obtain for the hadron mass

$$M=\lim_{T\to\infty}\frac{1}{T}\frac{\int D\rho d^4y\langle W(A)\Delta\Theta_{\mu\mu}(y)\rangle}{\int D\rho\langle W(A)\rangle}\,. \tag{7}$$

Here $W(A)$ is the usual Wilson loop, i.e., Eq. (6) without the first factor.

Consider now a hadron with light valent quarks, and take the limit $m_q\to 0$. Naively, one expects that the light quark term in $\Theta_{\mu\mu}$ does not contribute to M in the limit $m_q\to 0$, and indeed one makes sure that Eq. (7), even with the determinant term, is nonsingular in this limit and $m_q\bar{\Psi}_q\Psi_q$ gives vanishing result. The heavy quark contribution from the sea is suppressed and can be disregarded in the first approximation.[5] Thus all contribution to the mass is coming from the gluon part of $\Delta\Theta_{\mu\mu}$ in the case of spinless quarks [Eq. (7)].

We now use the so-called low-energy theorem[6] and reduce Eq. (7) to an indentity. Differentiating the quantity $\ln\langle W(A)\rangle$ in the bare coupling constant g_0^2, we obtain a theorem[6]

$$\int d^4y\langle W(A)\Delta\Theta_{\mu\mu}\rangle=2\sigma S\langle W(A)\rangle. \tag{8}$$

Here we invoke the area law for $\langle W(A)\rangle$ with the minimal area S and the string tension σ. With that assumption we also have $\sigma S\langle W(A)\rangle=-\sigma(\partial/\partial\sigma)\langle W(A)\rangle$. For simplicity, we ignore the perimeter term in $\langle W(A)\rangle$ and the perturbative exchanges, which produce color Coulomb interaction. This approximation is valid at large distances and therefore our results below apply to hadrons of large size R, $R\gg T_g$, where T_g is the gluon correlation length in vacuum. Recent Monte Carlo calculations[7] yield

$T_g \sim 0.2$ fm. Our consideration should therefore be reasonably good even for ground states of light hadrons and excited states of heavy quarkonia.

Hence, Eq. (7) can be rewritten as follows:

$$M = \lim \frac{1}{T} \left(-2\sigma \frac{\partial}{\partial\sigma} \right) \ln\langle G(T,0) \rangle. \tag{9}$$

The asymptotics of $\langle G(T,0)\rangle$ is $\exp(-M_0 T)$, where M_0 is the lowest hadron mass without spin interaction of quarks. As a result, we obtain a simple relation

$$M = 2\sigma \frac{\partial}{\partial\sigma} M_0(\sigma), \tag{10}$$

which is easily satisfied, since in the absence of quark masses σ is the only mass parameter and $M_0 = \text{const}\sqrt{\sigma}$. Thus, in this spinless case all the hadron mass is due to the strings that connect quarks [viz. the term $\langle W(A)\rangle$], and $\Delta\Theta_{\mu\mu}$ exactly measures how much of energy-density (and hence the mass) is contained in the strings. This picture does not exclude the notion of the constituent quarks. On the contrary, in Ref. 8 it was shown for mesons that $M_0 = 4\mu(\sigma)$, where $\mu(\sigma)$ is the constituent quark mass which depends on the hadron state. This means that a quark with an adjacent piece of the string makes a constituent quark, and the constituent quark mass here is due to the confinement. The same picture occurs for baryons.[9]

3. We now consider a heavy quark system, and still ignore the spin interaction of quarks which is small in this case, $\Delta M_s \sim (1/m_q^2)$. Equation (7) is still valid where $\Delta\Theta_{\mu\mu}$ can be replaced by

$$\Delta\Theta_{\mu\mu} \to 2\sigma S - \sum_q m_q \frac{\partial}{\partial m_q}. \tag{11}$$

Here we have used (8) and substituted $\bar{\Psi}_q \Psi_q$ by the derivative $\partial/\partial m_q$ of the action in the weight of averaging in (7).

Equation (10) is replaced by

$$M = \left(2\sigma \frac{\partial}{\partial\sigma} + \sum m_q \frac{\partial}{\partial m_q} \right) M_0(\sigma, m_q). \tag{12}$$

In the nonrelativistic approximation, M_0 can be represented as

$$M_0 = \sum m_q + \sum \left\langle \frac{p^2}{2m_q} \right\rangle + \langle V \rangle, \tag{13}$$

where the linear potential $\langle V\rangle$ is proportional to σ. From Eqs. (12) and (13) we have

$$M = 2\langle V \rangle + \sum m_q - \sum \left\langle \frac{p^2}{2m_q} \right\rangle. \tag{14}$$

Since V contains string interaction linear in distances between quarks, the virial theorem applies[10]

$$\langle V\rangle=2\sum\left\langle\frac{p^2}{2m_q}\right\rangle, \tag{15}$$

and we obtain an identity $M=M_0$.

Hence in the case of heavy quark systems, the hadron mass consists of quark masses plus gluon energy which is condensed in the form of strings between quarks.

4. We have disregarded until now the quark spin interactions. Treating it as a perturbation and taking as an example the hyperfine term for the qq system, we must add the following term[4] to M_0 in (10) or (12):

$$M_{ss}=\frac{8\alpha_s\pi}{9\mu_1\mu_2}\varphi^2(0)\vec{\sigma}_1\vec{\sigma}_2=\frac{4\alpha_s\sigma}{9(\mu_1+\mu_2)}\vec{\sigma}_1\vec{\sigma}_2 . \tag{16}$$

In this expression we have introduced the value of the eigenfunction at the origin $\varphi(0)$ for the linear interaction. For massless quarks $\mu_1=\mu_2=\mu(\sigma)$ is the constituent quark mass which depends only on $\sqrt{\sigma}$.

Therefore, the sum $M_0(\sigma)+M_{..}(\sigma)$, when substituted on the r.h.s. of (10), yields again an identity. The same line of reasoning applies to other spin-dependent terms. We thus find that the light hadron (meson or baryon) mass can be expressed in terms of constituent quark masses. It is generated by the confinement and disappears when σ tends to zero.

In conclusion we have shown in the framework of the Feynman–Schwinger representation that confinement and the notion of constituent quark mass provide a natural explanation of a hadron mass in terms of its gluonic and quark contents.

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