Confinement of the hole spin in stressed Ge–$Ge_{1-x}Si_x$ superlattices

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The angular distribution of the Shubnikov–de Haas oscillations for a Ge–$Ge_{1-x}Si_x$ periodic heterosystem, which exhibits a hole quantum Hall effect, has been studied for the first time. Specific elastic deformations in this system cause the oscillations of the magnetoresistance to depend on only the projection of the magnetic field onto the axis of the superlattice.

In p-type heterostructures with quantum wells, the pronounced deformation of the lattice makes it possible to observe some specific effects which stem from the quantization of the spin and orbital angular momentum of the holes. In particular, the energy of the splitting in a magnetic field **B** of the states Ψ_{Jm} with a total angular momentum $J=3/2$, with projections $m=\pm 3/2$ onto the uniaxial-deformation direction of the superlattice (the z axis), depends on only the one component B_z. Evidently the most suitable systems for studying these effects are those in which conditions are favorable from the energy standpoint for a filling of the states with $m=\pm 3/2$ with holes. We have previously reported growing Ge–$Ge_{1-x}Si_x$, superlattices selectively doped with boron, with elastic stresses on the order of several gigapascals, in both the solid–solution layers and the germanium layers.[2] This selective doping of the solid–solution layers made it possible to produce holes with a density of $(1\text{–}10)\times 10^{11}$ cm^{-2} and a high mobility, $(1\text{–}2)\times 10^{4}$ cm^2/(V·s) at 4.2 K, in the Ge channels. It also became possible to observe Shubnikov–de Haas oscillations and a quantum Hall effect.[3] In this paper we are reporting a study of the angular distribution of the oscil-

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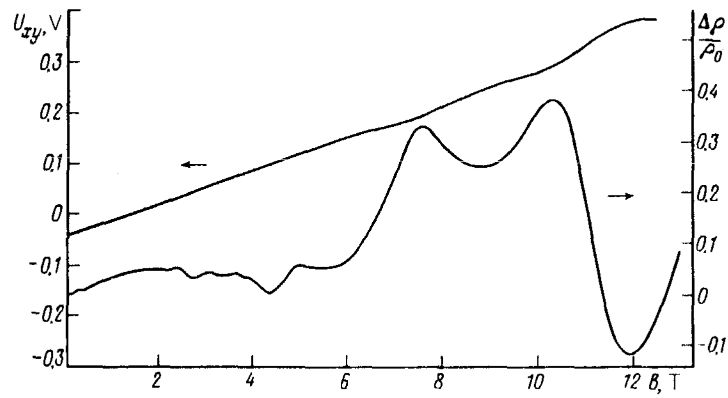

FIG. 1.

lations in the magnetoresistance in a stressed, selectively doped Ge–$Ge_{1-x}Si_x$, superlattice grown on a Ge substrate oriented along the ⟨111⟩ axis.

The test sample was grown by the hydride method. It contained alternating layers of Ge and $Ge_{0.86}Si_{0.14}$, with respective thicknesses of 11 and 18 nm. There were 90 layers of each material. The selective doping was carried out by introducing boron atoms in the middle of the solid–solution layer, in such a way that the spacers were about $(1/4)d_{GeSi}$. The Si concentration, $x=0.14$, was determined by x-ray diffraction. According to our estimates, the compressional strain in the germanium layers in the plane perpendicular to the axis of the superlattice was $\epsilon=2.4\times10^{-3}$. The Hall measurements were carried out in the double-cross geometry. From these measurements, at $T=4.2$ K, we found the hole density and mobility to be $p=5.1\times10^{11}$ cm^{-2} and $\mu=12\,000$ cm^2/(V · s). In high magnetic fields, up to 13 T, oscillations were found on the plot of the magnetoresistance ρ_{xx} versus B, and plateaus were found on the plot of the Hall resistance ρ_{xy} versus B (Fig. 1). The last ρ_{xy} plateau corresponded to a filling factor $\nu=2$ in the equation $p=\nu qB/h$, where q is the electron charge, and h is Planck's constant.

Figure 2 shows a plot of $[\rho_{xx}(B)-\rho_{xx}(0)]/\rho_{xx}(0)$ versus B for various values of the angle (ϑ) between the magnetic field and the superlattice axis (the field direction was varied in the plane perpendicular to the current direction) found for this structure at $T=4.2$ K. The shift of the maxima and minima of the magnetoresistance oscillations in the direction of increasing field is proportional to $1/\cos\vartheta$. The effect can be seen clearly in Fig. 3, which is a plot of the reciprocal of the field, $1/B$, at an oscillation maximum and the corresponding value of $\cos\vartheta$. These results conform to straight lines which pass through the origin. This behavior of the angular distribution is characteristic of a 2D system in which the spin splitting is either small or completely absent. In our case of the Ge–$Ge_{1-x}Si_x$ heterosystem, this splitting is by no means small, as we will see below. It follows from the plot of the quantum Hall effect $\rho_{xy}(B)$ in Fig. 1 that the positions of the plateaus with $\nu=2$–4 correspond to the equation $p=\nu qB/h$, with-

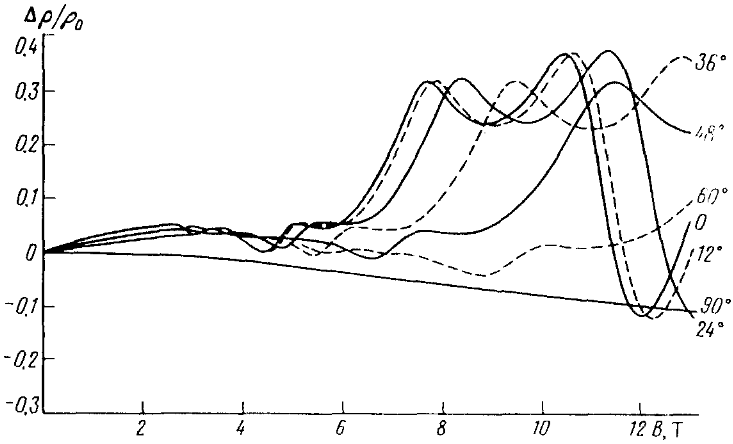

FIG. 2.

out spin degeneracy, at the value of the concentration p which is the same as that measured in weak fields. In addition, we see a well-defined plateau with $\nu=3$, which corresponds to the filling of an odd number of Landau levels. This plateau in the quantum Hall effect can be seen only if the spin splitting is significant in comparison with kT. Figure 2 shows that the minima of $\rho_{xx}(B)$ are constant at the various angles ϑ. This result corresponds to a situation in which the activation energy for the dissipative conductivity depends on only one magnetic field component, B_z.

A qualitative explanation of this effect was offered in Ref. 1, where a similar dependence was observed in an InGaAs quantum well with elastic strain. The strain combines with the quantum size effect to cause a splitting of the edge of the valence band by an amount Δ. In zeroth-order perturbation theory in the ratio e_f/Δ(e_f is the Fermi energy), the states with $m=\pm 3/2$ and $m=\pm 1/2$ do not interact, and the **B** dependence of the energy spectrum for the states with $m=\pm 3/2$ is determined by the one component B_z. For the Ge–$Ge_{1-x}Si_x$ structure which we studied, we can go through an analysis like that of Ref. 1, in a coordinate system with axes along the directions of the vectors $\mathbf{i}_x$, $\mathbf{i}_y$, $\mathbf{i}_z$, which are specified in terms of the crystallographic axes by $\mathbf{i}_x=(1/\sqrt{3})\ [-(\sqrt{3}+1)/2,\ (\sqrt{3}-1)/2,\ 1]$, $\mathbf{i}_y=(1/\sqrt{3})\ [(\sqrt{3}-1)/2,\ -(\sqrt{3}+1)/2,\ 1]$, $\mathbf{i}_z=(1/\sqrt{3})\ (\mathbf{1},\mathbf{1},\mathbf{1})$. Here we use the Luttinger Hamiltonian

$$H=\frac{1}{m}\left[\left(\frac{1}{2}\gamma_1+\frac{5}{4}\gamma_2\right)p^2I-\gamma_2(\mathbf{Jp})^2+q\hbar\left(\kappa+\frac{1}{2}\gamma_2\right)(\mathbf{JB})\right]+2\frac{1}{m}(\gamma_2-\gamma_3)$$

$$\times\left\{\frac{3}{4}\left(J_z^2-\frac{1}{3}\mathbf{J}^2\right)\left(p_z^2-\frac{1}{3}p^2\right)+\frac{1}{3}\{J_xJ_y\}[2\{p_xp_y\}-\{p_yp_z\}-\{p_zp_x\}]\right.$$

$$+\frac{1}{3}\{J_yJ_z\}[\{p_yp_z\}-\{p_xp_y\}-(p_x^2-p_y^2)]+\frac{1}{3}\{J_zJ_x\}[\{p_zp_x\}-\{p_xp_y\}$$

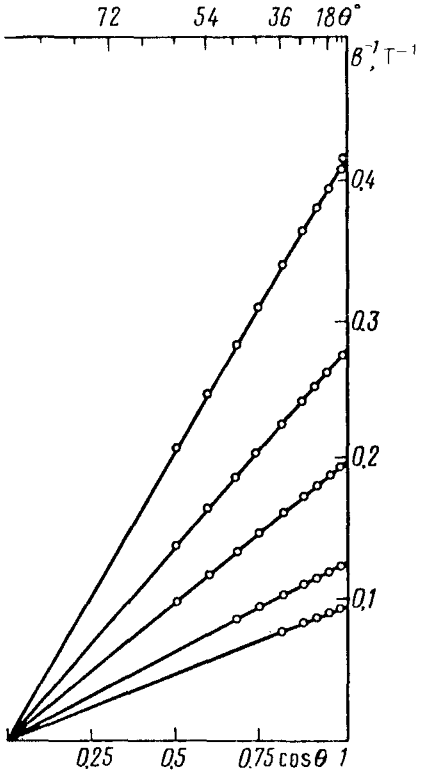

FIG. 3.

$$-(p_x^2-p_y^2)]+\frac{1}{6}(J_x^2-J_y^2)[(p_x^2-p_y^2)+\{p_zp_x\}-\{p_yp_z\}]\Big\}$$

$$+a(\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz})I+\frac{d}{3\sqrt{3}}(\mathbf{J}^2-3\mathbf{J}_z^2)(\epsilon_{zz}-\epsilon_{xx})+V(z). \qquad (1)$$

Here I is the unit matrix: $\mathbf{J}=(J_x,J_y,J_z)$ is the angular-momentum operator for an angular momentum of 3/2; $\mathbf{p}=(p_x,p_y,p_z)$ is the momentum operator in the magnetic field; $V(z)$ is the self-consistent electric potential; ϵ_{ij} are the components of the strain tensor; a and d are constants of the strain energy; $\gamma_1,\gamma_2,\gamma_3$, and κ are Luttinger parameters; and m is the mass of a free electron. In addition, for the symmetrized product of operators we introduce the notation $\{AB\}\equiv\frac{1}{2}(AB+BA)$, and for the scalar product we introduce $(\mathbf{CD}=C_xD_x+C_yD_y+C_zD_z$. Ignoring the interaction of the heavy- and light-hole bands in (1), we find the heavy-hole Hamiltonian

$$H_{hh}=\frac{1}{2m}[(\gamma_1+\gamma_3)(p_x^2+p_y^2)+(\gamma_1-2\gamma_2)p_z^2+6q\hbar k\sigma_zB_z]+a(\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz})$$

$$-\frac{d}{\sqrt{3}}(\epsilon_{zz}-\epsilon_{xx})+V(z), \quad (2)$$

where σ_z is the spin-1/2 Pauli matrix. We see immediately from (2) that the splitting of the Landau levels is determined by the component B_z alone and is equal to $6\kappa\mu_B$B $\cos\vartheta$ (at $B\simeq 5$T and $\vartheta=0$, the numerical value is 5 meV; here μ_B is the Bohr magneton, and the values of the Luttinger parameters correspond to Ge).

The condition for observing a ρ_{xx} maximum at small values of n can be written

$$e_f=e_0+(n+1/2)\hbar\Omega\cos\vartheta\pm 3\kappa\mu_B B\cos\vartheta, \quad (3)$$

where e_0 is the energy of the first quantum-well subband, $\Omega=(\gamma_1+\gamma_3)qB/m$ is the cyclotron frequency, expressed in terms of the Luttinger parameters, and n is the index of the Landau level. From (3) we find the family of $B^{-1}(\cos\vartheta)$ curves (which we mentioned earlier) corresponding to the maxima of ρ_{xx} for the various Landau levels. In principle, the Luttinger model can be used to calculate the energy spectrum of the valence band in a magnetic field. Several points must be taken into consideration here. First, the self-consistent potential is determined by the splitting of the valence band, and the magnitude of this splitting has not been established reliably for the case at hand. Second, according to Ref. 4, the effective masses of the top of the valence band of an elastically deformed Ge–$Ge_{1-x}Si_x$ superlattice are determined by interactions with not only the split-off valence bands but also the remote conduction bands. It is thus necessary to calculate the band structure and simultaneously fit the Luttinger parameters to the experimental data. Putting aside an exact calculation for a future study, we approximate the straight lines in Fig. 3 with the help of a model which uses states with $m=\pm 3/2$ alone and which uses the effective hole mass found[5] for motion in the plane of the layer from cyclotron-resonance measurements in an undoped superlattice with parameter values similar to those of the structure which we are discussing here. To determine the effective mass of the holes for motion in the direction transverse with respect to the layer, we fit the energy spectrum and the Fermi level to the observed picture of magnetoresistance oscillations. In this manner we estimate the difference e_f-e_0 to be 10 meV. On the other hand, the splitting of the edge of the valence band due to the elastic strain and the quantum-well effects in the layer generate a shift of about 50 meV for the band with $m=\pm 1/2$. It follows from these estimates that the hole gas in this structure fills only the band of hole states with $m=\pm 3/2$. The complex shape of the oscillations in weak fields is determined by the presence of the second quantum-well subband with the same value of the angular-momentum projection J_z.

In summary, we have observed, for the first time, a quantization of the spin and orbital angular momentum in a stressed Ge–$Ge_{1-x}Si_x$ superlattice. This quantization is manifested in the circumstance that ρ_{xx} and ρ_{xy} depend on only one magnetic-field component, B_z. This phenomenon is a consequence of the combined effects of the elastic strain and the size effects in the layers of the superlattice. These effects lead to a splitting of the edge of the valence band, and they greatly weaken the interaction between states with different projections of J_z onto the superlattice axis.

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