Effect of a finite transmission of the insulating layer on the properties of SIS tunnel junctions

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The effect of a depairing in the superconducting electrodes of an SIS tunnel junction on the steady-state properties of this junction is studied. A suppression of the superconductivity of the electrodes by a current results in substantial changes in the functional dependence $I_s(\varphi)$ and in the temperature dependence of the critical current I_c at a junction resistance $R_n \leqslant \rho\xi^*$. It is concluded that the suppression of the product I_cR_n in the Josephson junctions at the grain boundaries in high-T_c superconductors is not a consequence of a depairing in the grains themselves.

In artificial tunnel junctions based on conventional and high-T_c superconductors, the transmission of the insulating tunneling layer is generally so low that the density of the supercurrent flowing through the junction is far from the current density for the depairing of the superconducting electrodes. In granular structures in which the intergrain boundaries have a Josephson conductivity, however, the situation regarding these currents is not as obvious. In particular, the temperature dependence of the critical current seen in some high-T_c superconductors, $I_c(T) \propto (T_c - T)^{3/2}$, is approximately that of a depairing current.[1]

Our purpose in the present study is to determine the quantitative criteria on the transmission of the insulating layer, i.e., on the resistance of the junction in its normal state (R_n), at which there is a transition from a Josephson effect to a depairing effect

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in the electrodes of the SIS structure. The corresponding problem has previously been analyzed only for SNS weak links.[2,3]

I. Model of the SIS junction. We assume that the dimensions of the junction in plain view are small in comparison with the Josephson penetration depth λ_J, and we assume that the conditions of the dirty limit prevail in the electrodes of the junction. We place the origin of coordinates in the insulating layer, and we direct the x axis perpendicular to the boundaries of the structure. Under these assumptions, the properties of the SIS junction can be described by the Usadel equations[4] with the boundary conditions of Ref. 5. By virtue of the symmetry of the problem, we write these conditions in the form

$$R'(0)=0,\quad \xi^*\gamma_B I'(0)=2I(0). \tag{1}$$

Here R and I are the real and imaginary parts of the modified Usadel functions[5] Φ, $\gamma_B=R_n/\rho\xi^*$, and $\xi^*=(D/2\pi T_c)^{1/2}$ and ρ are the coherence length of the S electrode and its resistivity in the normal state. The prime means differentiation with respect to the coordinate x.

In the interior of the electrodes, the conditions on the functions Φ are determined by the requirement that they evolve into a solution which describes a uniform current-carrying state in the superconductor:[6]

$$\xi^*X'(\infty)=v_s,\quad F'(\infty)=0, \tag{2}$$

where F and X are the modulus and phase of the functions Φ, and v_s is the superfluid velocity.

Since the Matsubara frequency ω does not appear explicitly in conditions (1) and (2), we can seek a solution of the Usadel equations in the class of functions which have ω-independent values of the phase X coinciding with the phase of the order parameter, as in a calculation of the depairing current of a uniform superconductor.[6] In this case the Usadel equations and boundary conditions (1) and (2) (in the gauge with a zero vector potential) can be written

$$F=\Delta+\frac{\pi T_c}{\omega G}\left[(G^2F')'-(X')^2F^2G^2\right](\xi^*)^2, \tag{3}$$

$$X'F^2G^2=\text{const},\quad G=\omega/(\omega^2+F^2)^{1/2}, \tag{4}$$

$$\Delta\ln(T/T_c)+2\pi T\sum_{\omega\geqslant 0}\left[(\Delta/\omega)-F/(\omega^2+F^2)^{1/2}\right]=0, \tag{5}$$

$$J=\frac{2\pi T}{eR_n}\sum_{\omega\geqslant 0}\frac{F^2}{\omega^2+F^2}\sin(\varphi), \tag{6}$$

$$\xi^*\gamma_B X'(0)=\sin(\varphi),\qquad \xi^*\gamma_B F'(0)=2F(0)\sin^2(\varphi/2). \tag{7}$$

Here Δ is the modulus of the order parameter, $\varphi=2X(0)$ is the difference between the phases of the order parameter across the insulating layer, and J is the supercurrent density. The task of solving boundary-value problem (2)–(7) simplifies in several limiting cases.

II. The situation at temperatures $T \approx T_c$. At temperatures near the transition temperature, Eqs. (2)–(7) reduce to the Ginzburg–Landau equation. Using the first integral of these equations, and going through calculations analogous to those of Ref. 2, we easily find an algebraic equation for the coefficient α. The coefficient relates the value of the modulus of the order parameter at the boundary with the insulating layer, $\Delta(0)$, to that in the interior of the electrodes, $\Delta(\infty)$:

$$\Delta^2(0) = \alpha \Delta^2(\infty), \quad \Delta^2(\infty) = \Delta_0^2(1 - v_s^2),$$

$$v_s = [\alpha \sin(\varphi)/\Gamma_B], \quad \Delta_0^2 = [8\pi^2/7\zeta(3)] T_c(T_c - T),$$

$$8\alpha \sin^4(\varphi/2) - (1-\alpha)^2[\Gamma_B - \alpha(\alpha+2)\sin^2(\varphi)] = 0. \tag{8}$$

We also find an expression for $J(\varphi)$:

$$J(\varphi) = (J/J_0) = v_s(1 - v_s^2), \quad J_0 = (\pi\Delta_0^2)/(4eT_c\rho_n\xi_{GL}) . \tag{9}$$

Here $\Gamma_B = \gamma_B \xi^*/\xi_{GL}$, and $\xi_{GL} = (\pi/2)\xi^*(1 - T/T_c)^{-1/2}$ is the coherence length in the Ginzburg–Landau theory.

In the limit of large values $\Gamma_B \gg 1$, in the zeroth approximation in Γ_B^{-1}, it follows from (8) that we have $\alpha = 1$ and that expression (9) leads to a known result for SIS structures.[7] Deviations from that result stem from a spatially uniform suppression of the order parameter in the electrodes by a current flowing through them and also from a local decrease in Δ near the boundary of the electrodes with the insulating layer. The second of these factors is the more important; it leads to corrections to V_c which are proportional Γ_B^{-1}:

$$V_c = I_c R_n = V_0\left[1 - \frac{\sqrt{2}}{\Gamma_B}\right], \quad V_0 = \frac{\pi\Gamma_0^2}{4eT_c}. \tag{10}$$

A decrease in Γ_B is accompanied by an increase in the critical current density,

$$J_c = J_0 \frac{1}{\Gamma_B}\left[1 - \frac{\sqrt{2}}{\Gamma_B}\right], \quad J_0 = \frac{\pi\Delta_0^2}{4eT_c\rho_n\xi_{GL}}, \tag{11}$$

and by a change in the functional dependence $J(\varphi)$. The maximum of this dependence shifts toward small values, $\varphi = ((\pi/2) - \sqrt{2}/\Gamma_B)$.

Consequently, the role of a local suppression of Δ decreases with decreasing Γ_B, and in the limit of small values $\Gamma_B \ll 1$ the critical current of an SIS junction is governed by uniform depairing processes in the electrodes. In our approximation, the maximum on the $J(\varphi)$ curve shifts to values $\varphi \ll 1$, and the term proportional to $\sin^4(\varphi/2)$ in (8) can be ignored in comparison with $\sin^2(\varphi) \approx \varphi^2$. For α we then find

$$\alpha = \begin{cases} 1, & 0 \leqslant \varphi \leqslant \Gamma_B/\sqrt{3}, \qquad \beta = \varphi/\Gamma_B, \\ q/\beta, & \Gamma_B/\sqrt{3} \leqslant \varphi \ll 1, \qquad q = 1/(\beta + \sqrt{1+\beta^2}) \end{cases} . \tag{12}$$

The critical current is reached at $\varphi = \Gamma_B/\sqrt{3}$; it is equal to the depairing current: $J_c = (2/3\sqrt{3})J_0$.

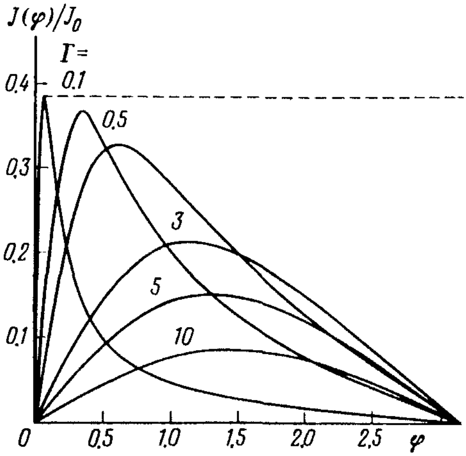

FIG. 1. The functional dependence $J(\varphi)$ for an SIS tunnel junction in which the insulating layer has a finite transmission, for various values of the parameter Γ_B. The dashed line is the numerical value of the depairing current density of a uniform superconductor.

At large values $\varphi \gg \Gamma_B/\sqrt{3}$, the order parameter near the insulating layer is greatly suppressed,

$$\alpha = \Gamma_B^2/[8\sin^2(\varphi/2)], \tag{13}$$

and the current flowing through the junction is much lower than J_c. Equations (9), (12), and (13) determine the functional dependence $J(\varphi)$:

$$J(\varphi) = J_0 \begin{cases} \beta(1-\beta^2), & 0 \leqslant \varphi \leqslant \Gamma_B/\sqrt{3}, \\ q(1-q^2), & \Gamma_B/\sqrt{3} \leq \varphi \ll 1, \\ (\Gamma_B/4)\cot(\varphi/2), & \varphi \gg \Gamma_B/\sqrt{3}. \end{cases} \tag{14}$$

Equations (8) and (9) have been solved numerically for arbitrary values of Γ_B. The results are presented as curves of $J(\varphi)$, $\alpha(\varphi)$, and $J_c(\Gamma_B)$, [or $V_c(\Gamma_B)$] in Figs. 1–3.

It can be seen from Fig. 1 that, in complete accordance with the analysis above, a decrease in Γ_B is accompanied by a transition from a sinusoidal function $J(\varphi)$ to a curve of the type in (14), which has a sharp maximum[1)] at $\varphi \approx \Gamma_B$. At the same time, there is a change in the nature of the functional dependence $\alpha(\varphi) = \Delta(0)/\Delta(\infty)$, from a smooth curve at large Γ^B (Fig. 2) to a curve with a sharp decrease in α at $\varphi \gg \varphi_m$ and $\Gamma_B \ll 1$. At $\varphi \ll \varphi_m$, for arbitrary Γ^B, the deviation of α from unity does not exceed 25% ; it reaches a maximum at $\Gamma^B \approx 0.5$.

Figure 3 shows the characteristic voltage V_c and the critical current J_c of the structure versus the parameter Γ^B (curves 1 and 3, respectively). The dashed curve is the asymptotic behavior $V_c(\Gamma_B)$ at large Γ_B, given by expression (10). At small values of Γ_B, we have $V_c \propto (T_c - T)^{3/2}$, and this parameter increases in proportion to Γ_B:

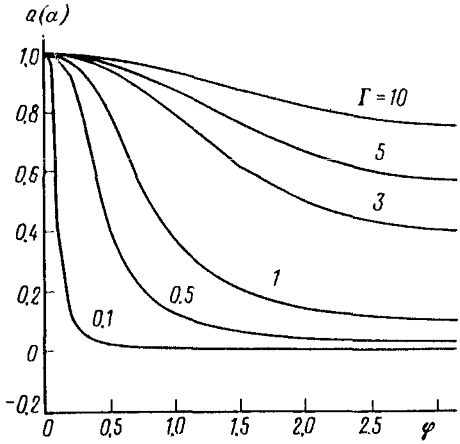

FIG. 2. Ratio of the modulus of the order parameter near the insulating layer, $\Delta(0)$, to its value in the interior of the electrodes, $\Delta(\infty)$, versus the phase difference across the junction, for various values of Γ_B.

$$V_c=I_cR_n=V_0(2/3\sqrt{3})\Gamma_B\approx 0.385\Gamma_B V_0\propto R_n. \tag{15}$$

We see that the transition from a tunneling regime for current flow through the weak-link region, with $V_c\propto(T_c-T)$ and with V_c independent of R_n, to a depairing-current regime, with $V_c\propto(T_c-T)^{3/2}$ and with V_c directly proportional to R_n, occurs at $\Gamma^B\approx 1$, i.e., at

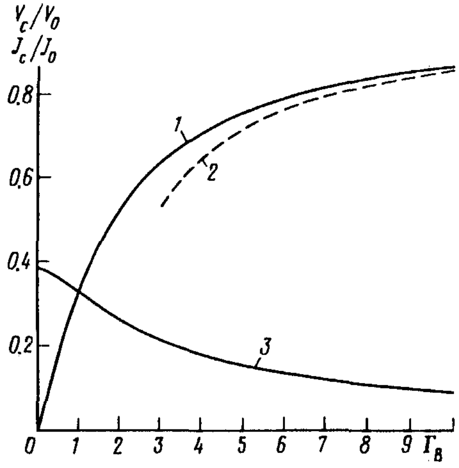

FIG. 3. Plots of two properties of an SIS tunnel junction with an insulating layer having a finite transmission versus the parameter Γ_B. *1*—The characteristic voltage V_c/V_0; *3*—the critical current J_c/J_0. Dashed curve *2* is the asymptotic dependence $V_c(\Gamma_B)$ as given by expression (11).

$$R_n \approx \rho\xi^*/(1-T/T_c)^{1/2}. \tag{16}$$

III. Approximation of high critical currents. For arbitrary temperatures, numerical methods must be used in order to solve Eqs. (2)–(7). It is nevertheless clear at a qualitative level that at small values of γ_B, at which the critical current of the junction is reached in the region $\varphi \ll 1$, by virtue of the gradients of its phase, we find the following expression for V_c:

$$\frac{eV_c}{2\pi T} = \gamma_B \frac{T}{T_c} \sum_{\omega>0} \frac{\Delta^2}{\Delta^2+\Omega^2}, \quad \Omega = \omega + \pi T_c G v_s^2. \tag{17}$$

Here the functional dependence $\Delta(\varphi)$ is found by solving the system of algebraic equations

$$\ln(T/T_c) + 2\pi T \sum_{\omega \geqslant 0} [(1/\omega) - 1/((G/\omega) + \pi T_c v_s^2)] = 0, \tag{18}$$

$$G = \Omega/(\Delta^2+\Omega^2)^{1/2}, \quad v_s = \gamma_B^{-1} \sin(\varphi).$$

These equations are the same as those which determine the temperature dependence of the critical depairing current of a uniform superconductor.[6]

It follows from (17) that at large critical currents the scaling described by expression (15)($V_c \propto \gamma_B \propto R_n$) holds at arbitrary temperatures. It is also clear at a qualitative level that at arbitrary T the transition (as γ_B increases) from this regime to the classical Josephson effect occurs under condition (16).

It can confidently be asserted on the basis of these results that the suppression of the characteristic voltage at junctions at intergrain boundaries in high-T_c superconductors with a grain size greater than $\xi^* \approx 2$ nm is not a consequence of depairing effects in the superconducting grains. In the first place, these effects lead to the scaling in (15), which is not the same as that seen experimentally[8,9] ($V_c \propto R^{-k}$, $k \approx 1$–2). Second, these effects should be seen at $R_n \approx \rho\xi^* \approx 10^{-11}\ \Omega\cdot\text{cm}^2$, i.e., at values about three orders of magnitude lower than the typical values for junctions at intergrain boundaries, $R_n \approx 10^{-8}\ \Omega\cdot\text{cm}^2$.

[1)]Incorporating the nonlinear shift in the phase difference in the electrodes, which cannot be canceled by a compensatory measurement circuit,[2] rounds off this structural feature slightly, while leaving the shape of the $J(\varphi)$ curve qualitatively the same.

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