Inverted radiative hierarchy of quark masses

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We suggest that the mass hierarchy is first generated in a sector of heavy isosinglet fermions due to radiative effects and then projected in the inverted way to the usual quarks by means of a universal seesaw. The simple left-right symmetric gauge model is presented with the *P*- and *CP*-parities and the exact isotopic symmetry, which are softly (or spontaneously) broken in the Higgs potential. This approach naturally explains the observed pattern of quark masses and mixing, providing the quantitatively correct formula for the Cabibbo angle. Top quark is predicted to be in the 90–150 GeV range.

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Recently,[1] a new approach to the fermion mass problem was suggested: The mass hierarchy is radiatively generated in a hidden sector of the hypothetical heavy fermions and then transferred to the visible quarks and leptons by means of universal seesaw mechanism.[2] Providing a qualitatively correct picture of quark masses and mixing, this approach solves many principal problems of previous models[3,4] for radiative mass generation. In particular, the correct value of the Cabibbo angle can be accommodated without trouble for the perturbative expansion, and thus for the radiative mass generation, which was the generic problem[5] of previous approaches. Moreover, within the seesaw approach the effective low energy theory, after integrating out the heavy sector, is simply the standard model with definite Yukawa couplings.[2] Thus, the dangerous flavor-changing phenomena, which are characteristic[4] of the direct models of radiative mass generation, are naturally suppressed.

The key idea of the model[1] is the assumption of the existence of weak, isosinglet, heavy fermions (Q-fermions) in a one-to-one correspondence with the light fermions. The model[1] has a field content such that only one family (namely the first) of Q-fermions becomes massive at the tree level, whereas the second family at the one-loop level and the third only at the two loops. Because of the seesaw features,[2] the mass of any usual quark or lepton appears to be inversely proportional to the mass of its heavy partner, so that the mass hierarchy between the families of light fermions is inverted with respect to the hierarchy of Q-fermion families. This pattern is attractive for the reason that we experimentally observe the small mass splitting within the lightest quark family (u and d) and then increasing splitting from family to family, with the up-quark masses growing faster: $m_u/m_d \ll m_c/m_s \ll m_t/m_b$. The latter fact can be related to the difference in the perturbian theory expansion parameters in the up and down quark sectors.

In the present letter we show that the simplest and most economical version of the model[1] provides a predictive ansatz for the quark mass matrices. We assume that the "isotopical" discrete symmetry I_{UD} between up and down quark sectors and the left-right symmetry P_{LR} and CP-invariance are violated only in the loop expansion, due to soft (or spontaneous) breaking in the Higgs potential. The appearance of both the mass splitting within the lightest family ($m_d/m_u = 1.5$–2) and the large value, compared to other mixing angles, of the Cabibbo angle ($V_{us} \simeq 0.22$) is related to the features of seesaw "projection," without trouble for the perturbation theory. The model leads to certain predictions for the quark mass and mixing pattern, which we will discuss below.

Let us consider the simple left-right symmetric model based on the gauge group $G_{LR} = SU(2)_L \otimes SU(2)_R \otimes U(1)_L \otimes U(1)_R \otimes U(1)_{\bar{B}-\bar{L}}$, suggested in Ref. 1. The left- and right-handed components of the usual quarks $q_i = (u_i, d_i)$ and their heavy partners $Q_i = U_i, D_i$ are used in the following representations:

$$q_{Li}(I_L = 1/2,\ \bar{B}-\bar{L} = 1/3), \quad q_{Ri}(I_R = 1/2,\ \bar{B}-\bar{L} = 1/3)$$

$$U_{Li}(Y_L = 1,\ \bar{B}-\bar{L} = 1/3), \quad U_{Ri}(Y_R = 1,\ \bar{B}-\bar{L} = 1/3)$$

$$D_{Li}(Y_L = -1,\ \bar{B}-\bar{L} = 1/3), \quad D_{Ri}(Y_R = -1,\ \bar{B}-\bar{L} = 1/3), \tag{1}$$

where $i=1,2,3$ is the family index [the indices of the color $SU(3)_c$ are omitted].[1)] Only the nonzero quantum numbers are shown in the brackets: $I_{L,R}$ are the $SU(2)_{L,R}$ weak isospins and $Y_{L,R}$ are the $U(1)_{L,R}$ hypercharges. Let us introduce also one additional family of fermions with $\bar{B}-\bar{L}=1/3$ and the following hypercharges:

$$p_L(Y_L=-1/2,\ Y_R=3/2),\quad p_R(Y_L=3/2,\ Y_R=-1/2)$$

$$n_L(Y_L=1/2,\ Y_R=-3/2),\quad n_R(Y_L=-3/2,\ Y_R=1/2). \tag{2}$$

The scalar sector of the theory consists of

$$H_L(I_L=1/2,\ Y_R=1),\quad H_R(I_R=1/2,\ Y_L=1)$$

$$T_{uL}(Y_L=-2,\ \bar{B}-\bar{L}=-2/3),\quad T_{uR}(Y_R=-2,\ \bar{B}-\bar{L}=-2/3)$$

$$T_{dL}(Y_L=2,\ \bar{B}-\bar{L}=-2/3),\quad T_{dR}(Y_R=2,\ \bar{B}-\bar{L}=-2/3) \tag{3}$$

$$\Phi(Y_L=2,\ Y_R=-2),\ \varphi(Y_L=1/2,\ Y_R=-1/2),\quad \Omega(Y_L,Y_R=1/2,\ \bar{B}-\bar{L}=-1),$$

where T-scalars are assumed to be color triplets. Let us impose also CP, P_{LR}, and I_{UD} discrete symmetries. The P_{LR},[7] which is essentially the parity, and CP act in the usual way. The isotopical "up-down" symmetry I_{UD} is defined by

$$U_{L,R}\leftrightarrow D_{L,R},\quad p_{L,R}\leftrightarrow n_{L,R},\quad H_{L,R}\leftrightarrow \tilde{H}_{L,R}=i\tau H^*_{L,R},$$

$$T^u_{L,R}\leftrightarrow T^d_{L,R},\quad \Phi\leftrightarrow\Phi^*,\quad \varphi\leftrightarrow\varphi^*,\quad A^\mu_{L,R}\leftrightarrow A^\mu_{L,R}, \tag{4}$$

where $A^\mu_{L,R}$ are the gauge bosons of $U(1)_{L,R}$. Then the most general Yukawa couplings, consistent with the gauge invariance, I_{UD}, P_{LR} and CP, are

$$L_1=\Gamma_{ij}(\bar{q}_{Li}U_{Rj}\tilde{H}_L+\bar{q}_{Li}D_{Rj}H_L)+(L\leftrightarrow R)+\text{h.c.}$$

$$L_2=\lambda_{ij}(U_{Li}CU_{Lj}T_{uL}+D_{Li}CD_{Lj}T_{dL})+(L\leftrightarrow R)+\text{h.c.}$$

$$L_3=h(\bar{p}_Lp_R\Phi^*+\bar{n}_Ln_R\Phi)+h_i(\bar{U}_{Li}p_R\varphi^*+\bar{D}_{Li}n_R\varphi)+(L\leftrightarrow R)+\text{h.c.}, \tag{5}$$

where C is the charge conjugation matrix. The coupling constants h, h_i, λ_{ij}, Γ_{ij} $(i,j=1,2,3)$ are real due to CP-invariance (λ_{ij} is antisymmetric, $\tilde{\lambda}=-\lambda$, since the T-scalars are color triplets). In what follows we do not make any particular assumption on their structure. We only assume that they are typically $O(1)$, as well as the gauge coupling constants. Without loss of generality, by suitable redefinition of the fermion basis we can always take h_2, $h_3=0$, $\lambda_{13}=0$, Γ_{12}, Γ_{13}, $\Gamma_{23}=0$, which we use below.

Let us assume that the discrete symmetries CP, P_{LR} and L_{UD} are softly broken only by the bilinear and trilinear terms in the Higgs potential.[2)] The latter are

$$\Lambda_uT^*_{uL}T_{uR}\Phi+\Lambda_dT^*_{dL}T_{dR}\Phi^*+\text{h.c.}, \tag{6}$$

where the coupling constants $\Lambda_{u,d}$ are generally complex, violating thereby both CP and P_{LR} invariances.

The VEVs $\langle\Phi\rangle=v_\Phi$ and $\langle\varphi\rangle=v_\varphi$, $v_\Phi\gg v_\varphi$ break down $U(1)_L\otimes U(1)_R$ to $U(1)_{L+R}$ [the VEV of Ω then breaks down $U(1)_{L+R}\otimes U(1)_{\bar{L}-\bar{L}}$ to the usual

$U(1)_{B-L}$: $B-L=Y_L+Y_R+\bar{B}-\bar{L}$]. The fermions p and n become massive, $M_p=M_n=hv_\Phi$, and the Q-fermions of the first family, U_1 and D_1 receive masses $M\simeq h_1^2v_\varphi^2/hv_\Phi$ due to their seesaw mixing with the former fermions [interactions L_3 in (5)]. At the same time, the colored scalars $T_{uL}-T_{uR}$ and $T_{dL}-T_{dR}$ are mixed due to the interaction terms (6). At this point, the radiative mass generation occurs in accordance with the chain $U_1\to U_2\to U_3$, $D_1\to D_2\to D_3$ and the Q-fermion mass matrices generated from the loop corrections due to L_2 in (5) can be presented in the form

$$M_{U,D}=M(\hat{P}_1+e^{-i\omega_{u,d}}\xi_{u,d}\tilde{\lambda}\hat{P}_1\lambda+C_{u,d}\xi_{u,d}^2\tilde{\lambda}^2\hat{P}_1\lambda^2+\cdots), \tag{7}$$

where $\hat{P}_1=\text{diag}(1,0,0)$ is a one-dimensional projector and $\omega_{u,d}=-\arg\Lambda_{u,d}$. The loop expansion factors are

$$\xi_q=\frac{1}{8\pi^2}\sin 2\alpha_q \ln R_q, \quad R_q=(M_+^q/M_-^q)^2, \tag{8}$$

where M_+^q, M_-^q are the eigenvalues of the mass matrices of the scalars $T_{qL}-T_{qR}$, $q=u$, d, and α_q are the corresponding mixing angles. In a "reasonable" range of parameters ($1<R<10$) the two-loop factor $C(R)=C(1/R)$ is nearly constant:[4] $C_{u,d}\simeq 0.65$. Equation (8) is valid in the natural regime $M<M_+^q$, $M_-^q<M_p$.

Aside from small ($\sim\varepsilon_{u,d}^2$) 1–3 mixing, the matrices $M_{U,D}$ are diagonal and the mass hierarchy between three families of Q-fermions is given by $1: x^{-1}\varepsilon_{u,d}: \varepsilon_{u,d}^2$, where we denote $x=\sqrt{C}\lambda_{23}/\lambda_{12}$, and $\varepsilon_{u,d}=\sqrt{C}\lambda_{12}\lambda_{23}\xi_{u,d}\sim 10^{-2}$–$10^{-1}$ are the effective perturbative expansion parameters for the up and down sectors, respectively.

The *VEV*s $\langle H_L\rangle=(0,v_L)$ and $\langle H_R\rangle=(0,v_R)$, $v_R\gg v_L=(2\sqrt{2}G_F)^{-1/2}\simeq 175$ GeV break the intermediate $SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$ symmetry down to $U(1)_{em}$. Then the ordinary quarks $q=u,d$ acquire masses due to their seesaw mixing with heavy fermions $Q=U,D$ [the interactions L_1 in Eq. (5)]. The total mass matrix written in the block form is

$$(\bar{u},\bar{U})_L\begin{pmatrix}0 & \Gamma_{v_L}\\ \tilde{\Gamma}_{v_R} & M_U\end{pmatrix}\begin{pmatrix}u\\ U\end{pmatrix}_R \tag{9}$$

for up-type quarks and analogously for the down-type quarks. When $M_{U,D}\gg v_R$, v_L, the resulting mass matrix for the light states is given by the seesaw formula

$$M_{\text{light}}^{u,d}=v_Lv_R\Gamma M_{U,D}^{-1}\tilde{\Gamma}. \tag{10}$$

Substituting in it Eq. (7), we find in the explicit form

$$M_{\text{light}}=\frac{m}{\varepsilon^2}\begin{pmatrix}\varepsilon^2\gamma_{11}^2 & \varepsilon^2\gamma_{11}\gamma_{21} & \varepsilon^2\gamma_{11}\bar{\gamma}_{31}\\ \varepsilon^2\gamma_{11}\gamma_{21} & \varepsilon xe^{i\omega}\gamma_{22}^2+\varepsilon^2\gamma_{21}^2 & \varepsilon xe^{i\omega}\gamma_{22}\gamma_{32}+\varepsilon^2\gamma_{21}\bar{\gamma}_{31}\\ \varepsilon^2\gamma_{11}\bar{\gamma}_{31} & \varepsilon xe^{i\omega}\gamma_{22}\gamma_{32}+\varepsilon^2\gamma_{21}\bar{\gamma}_{31} & 1+\varepsilon xe^{i\omega}\gamma_{32}^2+\varepsilon^2\bar{\gamma}_{31}^2\end{pmatrix}, \tag{11}$$

where $m=\Gamma_{33}^2v_Lv_RM^{-1}$, $\gamma_{ij}=\Gamma_{ij}/\Gamma_{33}$, and $\bar{\gamma}_{31}=\gamma_{31}+\sqrt{C}x^{-1}$; $\varepsilon=\varepsilon_{u,d}$, $\omega=\omega_{u,d}$ for the up and down quarks, respectively.

It is obvious from (11) that $\varepsilon_u\ll\varepsilon_d\ll 1$. The up-quark mass matrix M_{light}^u is nearly diagonal. Ignoring $\sim\varepsilon_u$ corrections, we have $m_u=m\gamma_{11}^2$, $m_c=xm\gamma_{22}^2\varepsilon_u^{-1}$, and m_t

$=m\varepsilon_u^{-2}$. The quark mixing pattern is thus determined essentially by the down-quark mass matrix M^d_{light}, where $m_b \approx m\varepsilon_d^{-2}$. The contributions to the parameters of the *CKM* matrix from M^u_{light} are typically suppressed by the factor $\varepsilon_u/\varepsilon_d$, and we ignore them. After some algebra, we obtain

$$V_{us} \approx \sqrt{\frac{m_d}{m_s}} \left| 1 - \frac{m_u}{m_d} e^{i\delta} \right| \tag{12}$$

$$V_{ub} \approx \frac{\bar{\gamma}_{31}}{\gamma_{11}} \frac{m_u}{m_b}, \quad V_{cb} \approx \frac{m_d}{m_u} \left(\sqrt{\frac{m_s}{m_d}} V_{ub} + \frac{\gamma_{32}}{\gamma_{22}} \frac{m_s}{m_b} e^{i\omega_d} \right), \tag{13}$$

where $\delta = -\omega_d + \arg(x e^{i\omega_d} \gamma_{22}^2 + \varepsilon_d \gamma_{21}^2) \approx -\omega_d + \arg(1 + e^{i\omega_d})$ is a *CP*-violating phase. Within uncertain numerical factors (which should be ~ 1), Eqs. (13) fit the experimental values of V_{ub} and V_{cb} (notice that for $\Gamma_{32}=0$ we have $V_{ub}/V_{cb} = m_u/\sqrt{m_d m_s} = 0.11$–$0.15$). Their smallness implies that corresponding mixings cannot affect significantly the diagonal elements of M^d_{light}. As for 1-2 mixing, the situation is different. The mass splitting between u and d quarks requires some spread in Yukawa coupling constants (i.e., fluctuations near 1 within a factor of 2–3), which is perfectly acceptable: $\Gamma_{21}/\Gamma_{11} \approx \sqrt{m_d m_s}/m_u = 7$–$9$, $\Gamma_{21}/\Gamma_{22} \approx \sqrt{x/\varepsilon_d}$, etc. This in turn automatically leads to the Cabibbo angle in the needed range. Comparison of (12) with the experimental value $V_{us} \approx 0.22$ requires the large *CP*-phase, $\delta \sim 1$, in accordance with the recent data.

From the mass matrices (11) we can also derive the relations

$$\frac{\varepsilon_d}{\varepsilon_u} = \frac{m_u m_c}{m_d m_s} = \sqrt{\frac{m_t}{m_b}}, \tag{14}$$

which allow us to calculate the top quark mass. The large value of the latter shows that we must also take into account the "seesaw" corrections[9] to Eq. (10), which implies for the physical top quark mass

$$m_t^* = m_t^0 \left[1 + \left(\frac{m_t^0}{\Gamma_{33} v_L} \right)^2 \right]^{-1/2}, \tag{15}$$

where m_t^0 is the "would be" physical mass, calculated from Eq. (14). Obviously, the analogous corrections are negligible for other quark masses, since we require that all Γ's be ~ 1. On the other hand, from (11) we easily see that $\Gamma_{21}/\Gamma_{33} \approx \varepsilon_d^{-1} \sqrt{m_d m_s / m_u m_b} \geqslant 0.17 \varepsilon_d^{-1}$. In order to be consistent with perturbation theory (i.e., to avoid the appearance of Landau poles below the scale M_p), we must assume that all Yukawa coupling constants, including Γ_{21} and λ's, are less than 2, which implies that $\Gamma_{33} \leqslant 1$. Consequently, using the known values of u, d, s, c, and b quarks from (14) and (15), we obtain $m_t^* = 50$–150 GeV. The large spread is due mainly to the uncertainties in the light quark masses. Obviously, the lower limit is not relevant because of the recent *CDF* limit $m_t' > 90$ GeV. One can even turn the logic around and say that the experimental lower bound on the top quark mass favors the lower values of m_d/m_u and m_s from those allowed in Ref. 8.

Finally, we wish to stress that in our approach the strong *CP*-problem can be automatically solved without the axion. Because of the *P*- and/or *CP*-invariances, the initial $\Theta_{QCD}=0$ and the so-called $\Theta_{QFD}=\mathrm{ArgDet}\hat{M}$, which arises from the total mass matrix $\hat{M}$ of all fermions q, Q and p, n, is also vanishing at the tree level due to the seesaw pattern.[10] The loop corrections can provide, however, $\bar{\Theta}=10^{-9}$–10^{-10}, which makes this scenario in principle accessible to the search of the *DEMON*—dipole electric moment of neutron.

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[1)] The inclusion of leptons in this model is straightforward and will be presented elsewhere. In fact, $U(1)_{\bar{B}-\bar{L}}$ can be unified with $SU(3)_c$ within Pati-Salam[6] type $SU(4)$. The $U(1)_L\otimes U(1)_R\otimes I_{UD}$ part can also be enlarged to $SU(2)'_L\otimes SU(2)'_R$. In this case the isotopic symmetry is obviously continuous.

[2)] Actually, these symmetries can be spontaneously broken as a result of the introduction of P_{LR}—and I_{UD}—odd real scalars.[1]

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