Dicke narrowing in a plasma

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Because of elastic ion-ion scattering, the ionic spectral line narrowing is calculated for equilibrium plasma. The problem is solved for collision frequency much greater than Doppler width and in the opposite limit. It is shown that the velocity dependence of the Coulomb collision frequency decreases the narrowing.

As Dicke[1] has shown, the Doppler absorption line becomes narrower when the frequency of elastic collisions, ν, increases. The physical mechanism of the Dicke narrowing consists in the interference of wave trains emitted by an ion. A phase correlation between the trains exists if the phase changes slightly over the train's duration ν^{-1}: $kv_T/\nu \ll 2\pi$. The condition stipulates small mean free path of a probe ion compared with the light wavelength $2\pi/k$. Frequent collisions confine the region within reach of the excited ion. The ion becomes almost fixed as in the solid state; therefore, the Doppler broadening vanishes. The line narrows because the correlation function decreases not as rapidly[2] as the Doppler function $\exp(-k^2v_T^2t^2/4)$. In this paper we study the effect in an equilibrium plasma, in which scattering at small angles dominates, and the effective collision frequency depends strongly on the velocity.

Calculation of the narrowing in the Coulomb case includes the scattering to the quantum kinetic equation for the density matrix[3] $\hat{\rho}$

$$\left(\frac{\partial}{\partial t}+v\,\frac{\partial}{\partial \mathbf{r}}+\hat{\Gamma}\right)\hat{\rho}+i\,[\hat{V},\hat{\rho}]=\hat{Q}-\frac{\partial \hat{q}_\alpha}{\partial v_\alpha},\quad \hat{q}_\alpha=\frac{F_\alpha}{m}\,\hat{\rho}-D_{\alpha\beta}\frac{\partial\hat{\rho}}{\partial v_\alpha}. \tag{1}$$

Here $\hat{\Gamma}$ is the relaxation matrix. The velocity dependence of the dynamic friction force and of the diffusion tensor is

$$F_\alpha=-\nu m v_T\xi_\alpha\Phi_l(\xi),\quad D_{\alpha\beta}=\frac{\nu v_T^2}{2}\left[\Phi_l(\xi)\frac{\xi_\alpha\xi_\beta}{\xi^2}+\Phi_{\text{tr}}(\xi)\left(\delta_{\alpha\beta}-\frac{\xi_\alpha\xi_\beta}{\xi^2}\right)\right], \tag{2}$$

where ν is effective transport collision frequency, $\xi_\alpha=v_\alpha/v_T$ is the dimensionless velocity of ions, and m and v_T are their mass and thermal velocity. The functions $\Phi_l(\xi)$, $\Phi_{\text{tr}}(\xi)$ for Maxwellian distribution of the field particles are expressed in terms of the Chandrasekhar function $g(\xi)$:[4]

$$\Phi_l=\frac{3\sqrt{\pi}}{2}\frac{g(\xi)}{\xi},\quad \Phi_{\text{tr}}=\frac{3\sqrt{\pi}}{4}\frac{\Phi(\xi)-g(\xi)}{\xi}. \tag{3}$$

The function g is a combination of the error function and its derivative

$$g(\xi)=\frac{\mathrm{erf}(\xi)-\xi\mathrm{erf}'(\xi)}{2\xi^2}, \quad \mathrm{erf}(\xi)=\frac{2}{\sqrt{\pi}}\int_0^{\xi} e^{-t^2}\,dt. \tag{4}$$

The diffusion tensor is anisotropic but invariant under rotation around the velocity vector of the probe ion. At a small velocity $v \ll v_T$, the spherical symmetry is restored $\Phi_l(0)=\Phi_{tr}(0)=1$.

The kinetic equation (1) for the off-diagonal element can be solved in the limiting case of rare collisions as an expansion in the parameter $\nu/kv_T \ll 1$, while the excitation term $\hat{Q}$ of the levels has a Maxwellian distribution in velocities. The work done by the field of the traveling wave is written as an expansion

$$\mathscr{P}=\mathscr{P}_0\,\mathrm{Re}\left[i\sum_{n=0}^{\infty}\left(\frac{i\nu}{kv_T}\right)^n I_n(z)\right],$$

$$I_n=\frac{1}{\pi^2}\int d^3\xi\left(\frac{\hat{L}}{z-\xi\cos\vartheta}\right)^n\frac{\exp(-\xi^2)}{z-\xi\cos\vartheta};$$

$$\mathscr{P}_0=-2\hbar\omega|G|^2\Delta N\frac{\sqrt{\pi}}{kv_T}, \quad \hat{\mathscr{L}}=\frac{\partial}{\partial\xi_\alpha}\left(-\frac{F_\alpha}{m}+D_{\alpha\beta}\frac{\partial}{\partial\xi_\beta}\right), \tag{5}$$

where $z+(\Omega+i\Gamma_{mn})/kv_T$, ΔN is the population difference, Γ_{mn} is the uniform width, Ω is the detuning between the wave frequency ω and the Bohr frequency of the transition $\omega_{mn}=(E_m-E_n)/\hbar$, and $G=V_{mn}\exp(-i\mathbf{kr}+i\Omega t)/2\hbar$ is the matrix element of the interaction in frequency units.

While collisions are rare, consider only terms with $n=0,1$. In zero order we obtain the Voigt contour $I_0=-iw(z)$ (w is the probability integral of the complex argument). For remote wings of the line $|z|\gg 1$ integral I_1 gives the coefficient at z^{-4}:

$$\mathscr{P}=\mathscr{P}_0\frac{kv_T}{\sqrt{\pi}}\left[\frac{\Gamma_{mn}}{\Omega^2}+\frac{k^2v_T^2}{2}\frac{3\Gamma_{mn}+\nu/\sqrt{2}}{\Omega^4}+\cdots\right]. \tag{6}$$

Comparing (6) with the results of the weak collision approximation, we calculate the change in the velocity of a probe ion: $\dot{\mathbf{u}}=-2\nu\Phi_l(u)\mathbf{u}$; i.e., at $\xi \ll 1$ the collision frequency $\bar{\nu}$ of the model is replaced by $2\nu\Phi_l$ in the Coulomb case. Averaging over velocities decreases the value $\bar{\nu}$ by $2\sqrt{2}$ times. At the line center ($\Omega=0$) we substitute $z=iy$ in (5) and calculate the integral in the Doppler limit $y=\Gamma_{mn}/kv_T\to 0$. The work done by the field increases slightly:

$$\mathscr{P}=\mathscr{P}_0\left(1+b\frac{\nu}{kv_T}\right), \quad b=\frac{2}{\sqrt{\pi}}(\sqrt{2}-\ln(\sqrt{2}+1))\cong 0.601. \tag{7}$$

The coefficient b differs from that in the model ($4/3\sqrt{\pi}\cong 0.752$) by 25% because of the velocity dependence. A gain at the line maximum is proportional to the frequency ν. The line therefore narrows, since the area under the contour is independent of ν.

In order to analyze the opposite limiting case $\nu/kv_T \gg 1$, we build up the perturbation theory, using a weak collisional model as the zero-order approximation. Equation (1) for the off-diagonal elements of the density matrix $\rho_{mn}(\mathbf{k},\mathbf{v})=\exp(-i\Omega t+i\mathbf{kr})\int_0^\infty \sigma(\mathbf{k},\mathbf{v};t)dt$ is convenient to rewrite in the integral form for the auxiliary function σ

$$\sigma(\mathbf{k},\mathbf{v};t)=\sigma_0(\mathbf{k},\mathbf{v};t)+\int d^3v'\int_0^t dt' f_0(\mathbf{k},\mathbf{v},\mathbf{v}';t-t')\hat{\delta}\sigma(\mathbf{k},\mathbf{v}';t'), \tag{8}$$

where

$$\sigma_0(\mathbf{k},\mathbf{v};t)=iG\Delta N\int d^3\mathbf{v}' f_0(\mathbf{k},\mathbf{v},\mathbf{v}';t)\,\frac{\exp(-v'^2/v_T^2)}{(\pi v_T^2)^{3/2}} \tag{9}$$

is the unperturbed function σ, and $\hat{\delta}=\hat{\mathscr{L}}-\hat{\bar{\mathscr{L}}}$ is the perturbation. Here $\hat{\bar{\mathscr{L}}}$ is the Fokker–Plank operator and $f_0(\mathbf{k},\mathbf{v},\mathbf{v}';t)$ is the Green's function of the kinetic equation for the weak collision model. In this model the coefficients of the Fokker–Plank operator are independent of the velocity $\Phi_l=\Phi_{tr}=1$. We denote the collision frequency in this model as $\bar{\nu}$. The function f_0 is the Fourier transform of the standard Green's function[4] of the weak collision model along the relative coordinates $\mathbf{r}-\mathbf{r}'$.

$$f_0(\mathbf{k},\mathbf{v},\mathbf{v}';t)=\frac{1}{[\pi v_T^2(1-\exp(-2\bar{\nu}t))]^{3/2}}$$
$$\times\exp\left\{-\frac{[\mathbf{v}-i\mathbf{k}v_T^2/\bar{\nu}-(\mathbf{v}'-i\mathbf{k}v_T^2/\bar{\nu})\exp(-\bar{\nu}t)]^2}{v_T^2[1-\exp(-2\bar{\nu}t)]}-\left(\frac{kv_T}{\bar{\nu}}\right)^2\frac{\bar{\nu}t}{2}\right.$$
$$\left.-(\Gamma_{mn}-i\Omega)t-i\mathbf{k}(\mathbf{v}-\mathbf{v}')/\bar{\nu}\right\}. \tag{10}$$

We express the work done by the field in terms of the function σ

$$\mathscr{P}=-2\hbar\omega\,\mathrm{Re}\left(iG^*\int d^3v\int_0^\infty dt\sigma(\mathbf{k},\mathbf{v};t)\right). \tag{11}$$

Let us solve the integral Fredholm equation as an expansion in the small parameter kv_T/ν, using the fact that $\sigma_0(\mathbf{k},\mathbf{v},t)$ is the eigenfunction of the operator $\hat{\delta}$ at $\mathbf{k}=0$ and arbitrary $\bar{\nu}$. Therefore, $\sigma_0(\mathbf{k},\mathbf{v},t)$ satisfies Eq. (8) in the highest order in kv_T/ν. We obtain formally the correction σ_1 of the first order and replace σ by σ_0 under the integral sign in (8). We see that at $t<\nu/k^2v_T^2$ the correction obtained increases linearly in time. So the correction increases by a factor of $\nu^2/k^2v_T^2\gg 1$ for a long time limit $t\gg\nu/k^2v_T^2$. The usual procedure to exclude growing secular terms in perturbation theory involves renormalization of the frequency[5] ν.

After the effective collision time ν^{-1} the probe particle forgets its previous history. As a result, the Green's function can be factorized at $t\gg\nu^{-1}$

$$f_0(\mathbf{k},\mathbf{v},\mathbf{v}';t)\approx\frac{1}{[\pi v_T^2]^{3/2}}\exp\left\{-\frac{[\mathbf{v}-i\mathbf{k}v_T^2/2\bar{\nu}]^2}{v_T^2}\right\}\exp\left[-\left(\frac{kv_T}{2\bar{\nu}}\right)^2(2\bar{\nu}t-3)\right.$$
$$\left.-(\Gamma_{mn}-i\Omega)t-i\frac{\mathbf{kv}'}{\bar{\nu}}\right]. \tag{12}$$

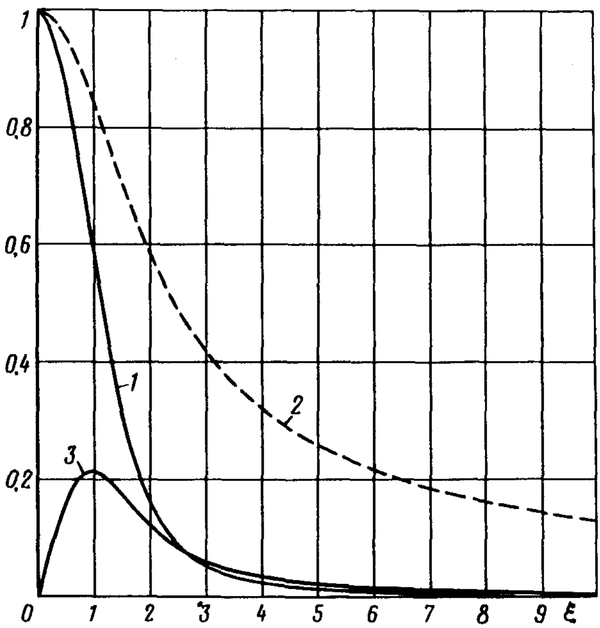

FIG. 1. Normalized diffusion coefficients along the direction (curve *1*) and transverse (curve *2*) to the velocity of the probe ion, and the Chandrasekhar function (curve *3*) vs dimensionless velocity.

Requiring at long time limit $t,t' \gg \nu^{-1}$

$$\int d^3v' f_0 \ (\mathbf{k},\mathbf{v},\mathbf{v}';t) \frac{\hat{\delta}}{\nu} f_0 \ (\mathbf{k},\mathbf{v}',\mathbf{v}'';t') = O\left(\frac{k^4 v_T^4}{\nu^4}\right), \tag{13}$$

we exclude the secular terms and obtain expression for the effective collision frequency

$$\bar{\nu} = \frac{\nu}{\sqrt{2}} + O\left(\frac{kv_T}{\nu}\right). \tag{14}$$

As a result, within the accuracy up to $k^2 v_T^2/\nu^2$ we have

$$\mathscr{P}(\Omega) = \mathscr{P}_0 \frac{kv_T}{\sqrt{\pi}} \operatorname{Re} \int_0^\infty dt \exp\left\{-\frac{1}{2}\left(\frac{kv_T}{\bar{\nu}}\right)^2 [\bar{\nu}t - 1 + \exp(-\bar{\nu}t)] - (\Gamma_{mn} - i\Omega)t\right\}. \tag{15}$$

Hence, near the line center $\Omega \ll \bar{\nu}$ we have

$$\mathscr{P}(\Omega) = \mathscr{P}_0 \frac{kv_T/\bar{\nu}}{\sqrt{\pi}} \frac{\Gamma_{mn}/\bar{\nu} + \frac{1}{2}(kv_T/\bar{\nu})^2}{(\Gamma_{mn}/\bar{\nu} + \frac{1}{2}(kv_T/\bar{\nu})^2)^2 + (\Omega/\bar{\nu})^2} \left[1 + O\left(\frac{k^2 v_T^2}{\nu^2}\right)\right]. \tag{16}$$

At the line wings $\Omega \ll \nu$, omitting terms less than ν^4/Ω^4, we obtain Eq. (6), which is valid at $|z| \ll \nu/kv_T$.

Note that we can hold terms of the order of $(kv_T/\nu)^2$ in the limit $\Gamma_{mn} \ll k^2 v_T^2/\nu$. In the opposite case, they can be omitted. The work at the line center $P(0)$ is shown in Fig. 2 as a function of collision frequency. The ratio between the uniform width and

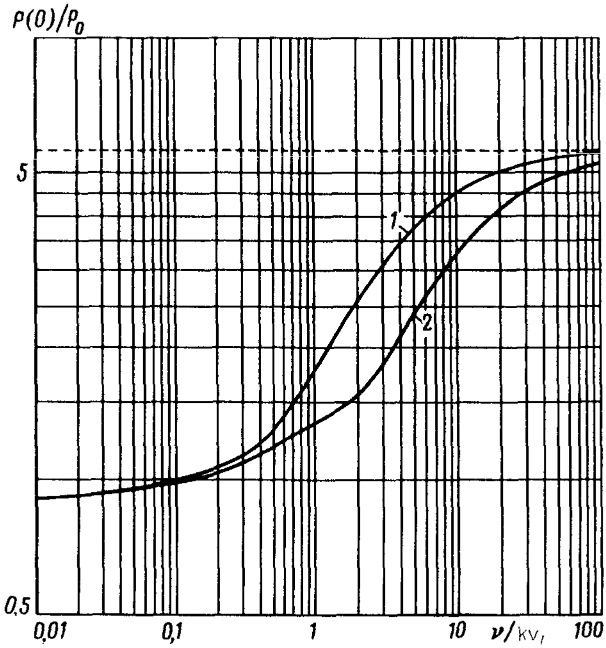

FIG. 2. The work done by field at the line center as a function of collision frequency. Curve *1* corresponds to the weak collision model with $\bar{\nu}=2\nu\Phi_l(0)$; curve *2* describes the case of Coulomb scattering (1).

the Doppler width is $\Gamma_{mn}/kv_T=0.1$. The general solution based on the exact properties of the collision integral, obtained by Alekseyev and Malyugin,[6] is expressed in terms of the eigenvalues. It is difficult to find explicit spectrum of collision operator (1); nevertheless, the qualitative behavior of solution (16) is Lorentzian.

Estimation for high-current discharge [3] ($N_i=3\times10^{14}$ cm^{-3}, $T_i=1$ eV, $\lambda=488$ nm ArII) gives $\nu\sim3\times10^7$ s^{-1}, whereas the Doppler width is $kv_T\sim3\times10^{10}$ s^{-1}, so the effect is small $\nu/kv_T\sim10^{-3}$. It seems that to make the narrowing more pronounced, we should have a higher charged particle density. However, the uniform width also increases with N_e because of level deactivation and Stark broadening. For example, for dense plasma of *z*-pinch[7] ($N_i=2\times10^{18}$ cm^{-3}, $T_i=3$ eV, $\lambda=164$ nm HeII) $\nu/kv_T\sim1$; however, $kv_T\ll\Gamma_{mn}$. It is difficult to observe the narrowing for hydrogen-like lines because of strong natural broadening ($\propto Z^4$) and linear Stark effect. In recent studies of neon-like spectral line profiles of an x-ray laser (the charge of an ion $Z=24$)[8] the importance of ion–ion collisions is shown. Taking the experimental values from Ref. 8 ($T_i=400$ eV, $\lambda=20.6$ nm SeXXV) and assuming solid state density ($N_i\sim10^{20}$ cm^{-3}), we have $\nu\ll kv_T\sim10^{13}$ s^{-1}. The Dicke effect might be significant for transitions with a small uniform width ($\Gamma_{mn}\ll kv_T$).

Thus, the narrowing is strongest provided that $\nu\ll kv_T\ll\Gamma_{mn}$. In order to emphasize the effect, we must use the lines of the multielectron spectrum of multiple ions, since $\nu\propto Z^2$, and long wave transitions diminish Γ_{mn} and increase ν/kv_T. The plasma should be comparatively dense and cold. High ion density and low temperature lead to an increase in $\nu\propto N_iv_T^{-3}$. However, at excessively low ionic temperature or high charged particle density the Doppler width can become less than a uniform width.

[1] R. H. Dicke, Phys. Rev. **89**, 472 (1953).
[2] S. G. Rautian and I. I. Sobel'man, Usp. Fiz. Nauk **90**, 118 (1966) [Sov. Phys. Usp., **9**, 701 (1967)].
[3] Gi. I. Smirnov and D. A. Shapiro, Zh. Eksp. Teor. Fiz. **76**, 2084 (1979) [Sov. Phys. JETP **49**, 1054 (1979)]; S. A. Babin and D. A. Shapiro, in: "Spectral Line Shapes," **6**, Eds. L. Fommhold and J. W. Keto, New York, AIP, 1990.
[4] S. Chandrasekhar, Rev. Mod. Phys. **15**, 1 (1943).
[5] A. H. Nayfeh, *Perturbation Methods,* New York: John Wiley & Sons, 1973.
[6] V. A. Alekseyev and A. V. Malyugin, Zh. Eksp. Teor. Fiz. **80**, 897 (1981).
[7] J. P. Marangos, D. D. Burgess, and K. G. H. Baldwin, J. Phys. B: At. Mol. Opt. Phys. **21**, 3357 1988.
[8] J. A. Kock, Abstracts of 11th ICSLS (Carry le Rouet, June 8–12, 1992), Inv21.

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