Possibility of new coherently precessing spin states in superfluid ^{3}He-B

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It is argued that in the case where the magnitude of spin polarization S of ^{3}He-B is considerably different from its equilibrium value $S_0=(\chi_B/g)H_0$, new coherently precessing spin–orbit configurations are formed. Especially interesting dynamic regime is expected at $S=S_0/2$ or $S=2S_0$.

The coherent spin–orbit coupling $\mathscr{F}_D$ (of dipole–dipole origin) plays a crucial role in the properties of ordered superfluid phases of liquid ^{3}He. This interaction partially lifts the degeneracy of the ground state with respect to the Goldstone degrees of freedom of the order parameter and an equilibrium spin–orbit configuration, which corresponds to the minimum of $\mathscr{F}_D$ (for given external conditions) is established.

The spin–orbit coupling has a strong influence on the dynamic properties of superfluid A and B phases of ^{3}He. In the absence of dipole–dipole effects, spin dynamics of the system, subject to the action of a static magnetic field $H_0=H_0\hat{z}$, is reduced to purely precessional motion of the magnetization $\mathbf{M}=g\mathbf{S}$ about the z axis with the Larmor frequency $\omega_0=|g|H_0$ ($g<0$ is the gyromagnetic ratio for magnetic moments of ^{3}He nuclei) which is independent of the orientation of spins with respect to $\hat{z}$ (this orientation is characterized by $s_z=\cos\beta_s$, where β_s, is the tipping angle). This simple but fundamental property is the manifestation of the fact that in the coordinate frame rotating with the Larmor angular velocity, $\vec{\omega}_0=\omega_0\hat{z}$, the magnetic field is eliminated and the spin-system restores isotropy. The coherent spin–orbit coupling perturbs, in general, the simple Larmor state. This perturbation was observed in many experimental studies (for a comprehensive review, see Ref. 1).

Even in the presence of the dipole–dipole potential $\mathscr{F}_D$ a purely Larmor regime of the spin motion can develop, however, under special circumstances, so that the spin system precesses, with the frequency ω_0 being subject to the action of the external magnetic field only. The well-known example of such a coherent spin precessing configuration is realized in the Brinkman–Smith regime, when the orbital part of the ^{3}He-B order parameter is at equilibrium and spins are precessing at the tipping angle β_s, such that $\cos\beta_s, > -1/4$. It is clear that such a situation corresponds to the case where the spin–orbit potential is minimum, and where the uniformly precessing spin state is stable (only upon deviation from the Larmor state is the dipole–dipole torque generated and the Leggett–Takagi spin relaxation mechanism is switched on).

The spin–orbit potential $\mathscr{F}_D$ depends on the variable s_x, and the Larmor state at the minimum of $\mathscr{F}_D$ is realized for some particular stationary values of s_z; however, in special cases the coherently precessing state reveals some degree of degeneracy (discrete or continuous) with respect to s_z. Such a situation takes place for the B phase of

^{3}He due to special properties of spin–orbit coupling in the superfluid state, which gives rise to a number of peculiarities of the coherent spin dynamics of ^{3}He-B and, in particular, to the long-lived two-domain structures in the presence of the magnetic field gradient.[2–4]

It is well known that for the superfluid B phase

$$\mathscr{F}_D=\frac{2}{15}\chi_B(\Omega_B/g)^2(\mathrm{Tr}\hat{R}-1/2)^2, \tag{1}$$

where χ_B is the magnetic susceptibility, Ω_B is the frequency of the linear longitudinal NMR, which characterizes the strength of the dipole–dipole interaction between magnetic moments of ^{3}He nuclei, and the orthogonal matrix $\hat{R}$ describes the relative rotation of the spin and orbital spaces:

$$\hat{R}=\hat{R}^{(S)}\cdot\hat{R}^{(L)-1}, \tag{2}$$

where $\hat{R}^{(S)}$ and $\hat{R}^{(L)}$ are the matrices of the three-dimensional rotations of spin and orbital spaces, respectively. Parametrizing these rotations by triples of Eulerian angles $(\alpha_s, \beta_s, \gamma_s)$ and $(\alpha_L, \beta_L, \gamma_L)$, we find

$$\mathrm{Tr}\hat{R}=\cos\beta_L\cos\beta_s+\tfrac{1}{2}(1+\cos\beta_L)(1+\cos\beta_s)\cos(\alpha+\gamma)$$
$$+\tfrac{1}{2}(1-\cos\beta_L)(1-\cos\beta_s)\cos(\alpha-\gamma)+\sin\beta_L\sin\beta_s(\cos\alpha+\cos\gamma), \tag{3}$$

where $\alpha=\alpha_s-\alpha_L$, and $\gamma=\gamma_s-\gamma_L$.

Considering what follows the case of a strong magnetic field $(\omega_0\gg\Omega_B)$, we take into account that angular variables α and γ perform fast rotations (on the time scale of order Ω_B^{-1}). Substituting (3) into (1) and averaging over the fast oscillations, we conclude that the spin–orbit potential [measured from now on in units of $(2/15)\chi_B(\Omega_B/g)^2$] is given by

$$\bar{U}_D=\{[l_zs_z+\tfrac{1}{2}(1+l_x)(1+s_z)\cos\phi-1/2]^2+\tfrac{1}{8}(1-l_z)^2(1-s_z)^2$$
$$+(1-l_z^2)(1-s_z^2)(1+\cos\phi)\}, \tag{4}$$

where $l_z=\cos\beta_L$ defines the orientation of the orbital momentum of the magnetized B phase with respect to the direction of the applied field $\vec{H}_0={}^3\mathrm{H}_0\hat{z}$, and the angular variable $\phi=\alpha+\gamma$ "survives" after averaging, since $\dot{\alpha}\simeq\dot{\gamma}$ (for the Larmor state under consideration ϕ is constant in time). Expression (4) is symmetric with respect to spin and orbital variables (as it should be for the B phase), and in somewhat different notation it was presented in Ref. 5. Recently it was used to construct the phase diagram of ^{3}He-B subject to the action of the transverse field with pump frequency $\omega_p\neq\omega_0$ and the superfluid counterflows.[6]

In order to construct the Larmor state (homogeneous spin precessing state with frequency ω_0) we must explore the manifold of variables (l_z, s_z, ϕ) realizing the minimum of (4). The equation defining this space of degeneracy is

$$\cos\phi=-\frac{(1-2l_z)(1-2s_z)}{(1+l_z)(1+s_z)}. \tag{5}$$

Excluding ϕ from (4), we see that the ground state is realized at the minimum of

$$\bar{U}_D^{(0)}=3(1-l_z)(1-s_z)[1-\tfrac{5}{8}(1-l_z)(1-s_z)] \tag{6}$$

in the domain of the (l_z, s_z) plane, where $|\cos\phi|\leqslant 1$. The two boundaries of this domain are given by equations:

$$l_z+s_z=2+5l_zs_z \quad (\cos\phi=1) \tag{7}$$

$$l_z+s_z=l_zs_z \quad (\cos\phi=-1). \tag{8}$$

On crossing curve (7) (with two branches) we merge into the domain with $\cos\phi\equiv 1$ [in the case of (8)—into domain with $\cos\phi\equiv -1$]. The minimum of (6) is attained at

$$s_z=1, \quad -1/4\leqslant l_z<1 \tag{9}$$

$$l_z=1, \quad -1/4\leqslant s_z<1, \tag{10}$$

and the residual degeneracy of the equilibrium spin state (9) with respect to l_z, and of the spin precessing state (10) with respect to s_z are lifted by external perturbations. In particular, in the presence of the transverse rf field with pump frequency ω_p slightly larger than ω_0 the coherently precessing spin state with $s_z\simeq -1/4$ is stabilized at the boundary (7). In the presence of the magnetic field gradient a two-domain state was first observed in Ref. 2, and the degeneracy of (9) with respect to l_z makes the structure of this two-domain state very sensitive to the action of superfluid counterflows in the rotating ^{3}He-B.[7–9]

It should be stressed that the preceeding consideration of the coherently precessing Larmor state was based on the assumption that spin dynamics is developing along the trajectories with the magnitude of spin polarization close to its equilibrium value $[S\simeq S_0=(\chi_B/g)H_0]$. In this case the angular variable $\phi=\alpha+\gamma$ is slow and we obtain Eq. (4) for the dimensionless average spin–orbit potential $\bar{U}_D$. In what follows we shall assume that when the actual value of S is considerably different from S_0 [more precisely when $|\omega_s-\omega_0|\gg\Omega_D$ with $\omega_s=\omega_0(S/S_0)$], new coherent Larmor states are possible with specific spin–orbit configurations. In this situation two cases should be distinguished. In the first nonresonant regime $[S\neq(S_0/2,2S_0)]$ there are no slow angular variables and we can safely average expression (4) over ϕ (which is now fast since $|\dot{\phi}|\gg\Omega_D$). As a result, we obtain a new effective spin–orbit potential

$$\widetilde{U}_D=\tfrac{3}{4}[1+(1-l_z^2)(1-s_z^2)+2l_z^2s_z^2], \tag{11}$$

with degenerate minima at

$$s_z^2=1, \quad l_z=0 \tag{12}$$

$$s_z=0, \quad l_z^2=1. \tag{13}$$

The profile of $\widetilde{U}_D(s_z, l_z)$ is shown in Fig. 1. An interesting property of $\widetilde{U}_D$ is that at $l_x^2=1/3$ it is independent of s_z (and vice versa). We note that the solution $(s_z=-1, l_z=0)$ belonging to (12) is similar to the reversed spin (RS) state found in Ref. 6 for the case $S\simeq S_0$ in the presence of transverse rf field with pump frequency $\omega_p>\omega_0$ which

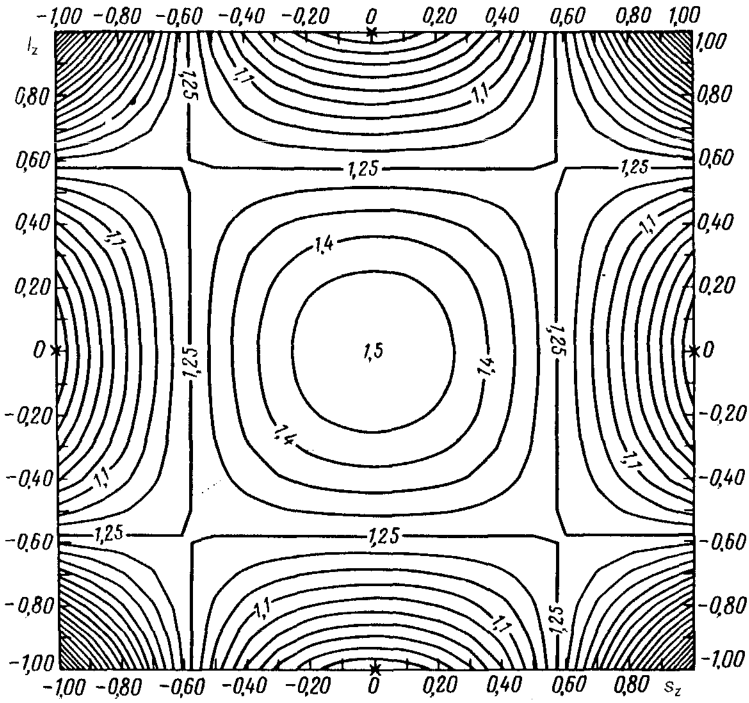

FIG. 1. The topographic profile of average spin-orbit potential $\widetilde{U}_D(s_z, l_z)$ [Eq. (11)]. Four degenerate stationary points (denoted by crosses) correspond to states (12) and (13).

stabilizes it. The RS state found by us as a stationary spin–orbit configuration of (11) at $\omega_p=\omega_0$ corresponds to the case with $|1-S/S_0|\gg\Omega_D/\omega_0$. The transverse spin (TS) state (13) belongs to the same class of solutions.

Another regime is developed for $S=S_0/2$ (or $S=2S_0$), which is a resonant case in the sense that the angular variable $\widetilde{\phi}=\alpha+2\gamma$ (or $\widetilde{\phi}'=2\alpha+\gamma$) turns out to be slow and it "survives" the averaging procedure if one starts from general expressions (1) and (3). Under these special conditions (analogous to the case considered in Ref. 10 for the A phase) instead of (11) we obtain the following average spin–orbit potential:

$$\widetilde{U}_D^{(R)}=\tfrac{3}{4}[1+(1-l_z^2)(1-s_z^2)+2l_z^2s_z^2+\tfrac{2}{3}\sqrt{1-l_z^2}(1+l_z)\sqrt{1-s_z^2}(1+s_z)\cos\widetilde{\phi}]. \quad (14)$$

The stationary states realizing minima of (14) can be found numerically, keeping in mind that the relevant stationary point $\widetilde{\phi}_s=\pi$. It turns out that there are two degenerate stable spin–orbit configurations

$$s_z=0.75, \quad l_z=0.3 \quad (15)$$

$$s_z=0.3, \quad l_z=0.75, \quad (16)$$

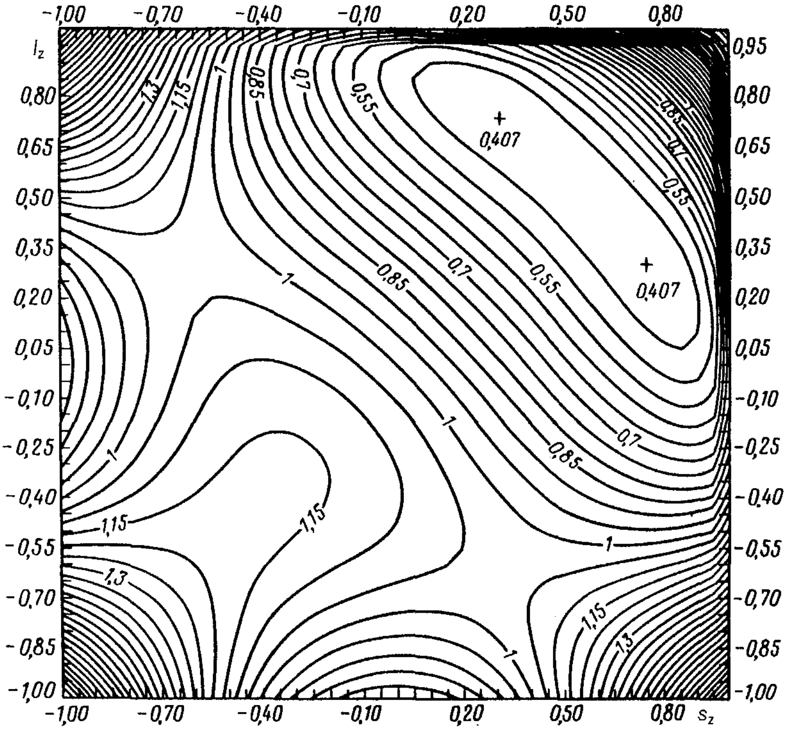

FIG. 2. The topographic profile of average spin–orbit potential $\widetilde{U}^{(R)}(s_z, l_z)$ [Eq. (14)]. Two degenerate stationary points (denoted by crosses) corresponds to states (15) and (16).

for which $\widetilde{U}_D^{(R)}$ is minimum. The profile of $\widetilde{U}_D^{(R)}$ (for $\widetilde{\phi}=\pi$) is shown in Fig. 2. We notice that two minima are located at the opposite ends of an elongated "valley" and are separated by a tiny "hill".

The coherently precessing states of ^{3}He-B with different spin-orbit configurations are shown in Fig. 3, in the plane $(S_z, S_\perp)$, where the longitudinal component of the spin $S_z = Ss_z$ and its transverse component $S_\perp = S\sqrt{1-s_z^2}$. We recall that for a given spin–orbit state the orientation of the relative rotation axis $\hat{n}$ can be determined using the relation $\hat{l}_i = \hat{s}_\mu R_{\mu i}(\hat{n}, \theta_0)$, where θ_0 is the Leggett angle ($\cos\theta_0 = -1/4$). Among other coherently precessing states we have shown the possibility of realizing new stable spin–orbit configurations appropriate to $S=S_0/2$ [half spin (HS) mode] and to $S=2S_0$ [double spin (DS) mode].

From the experimental point of view, it is important to determine the proper route to HS and DS modes. A brute-force way is to increase (decrease) abruptly the strength of the applied magnetic field to the value $H=2H_0$ ($H=H_0/2$), but, as was shown by Volovik[11] by means of a computer simulation (using the LESTER package elaborated by Golo and Leman), the efficient way to reach HS state is to tip first the magnetization by an angle π, to let the spin system relax along the trajectory with

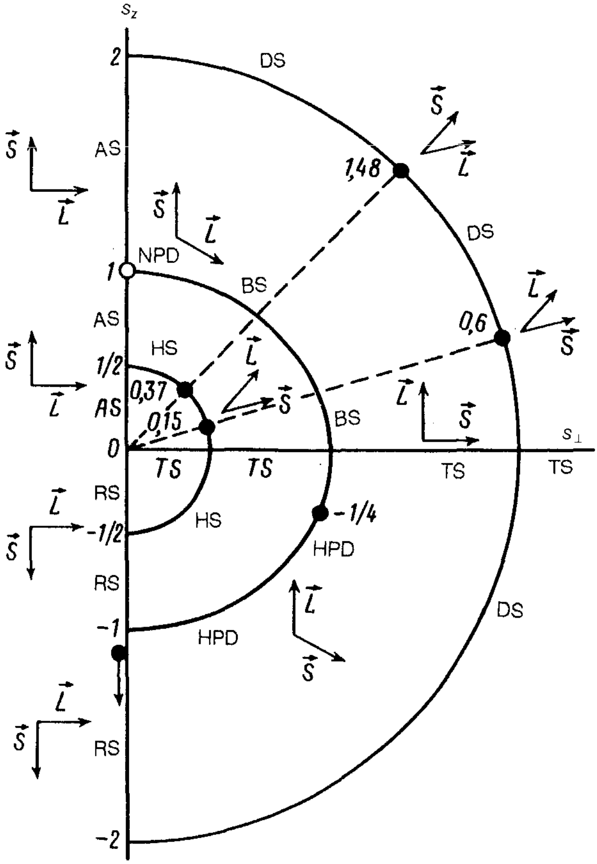

FIG. 3. Coherently precessing states of [3]He-*B*. Two pairs of HS and DS states with $S_z=(0.15;\ 0.6)$ and $S_z=(0.37;\ 1.48)$ correspond to degenerate minima of spin–orbit potential $\tilde{U}_D^{(R)}$, shown in Fig. 2. AS—Aligned spin mode, RS—reversed spin mode, TS—transverse spin mode, HPD—uniformly precessing domain, HS—half spin mode, DS—double spin mode, BS—Brinkman-Smith mode, NPD—nonprecessing (equilibrium) state, ●—Stationary states at ω close to Larmor frequency.

$S_\perp=0$ to the value $S_z=-1/2$ and then to follow the route at $S=1/2$ and increasing s_z.

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