Quadratic field effect in quantum-well films

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The quadratic field effect in a quantum-well film between two electrodes is analyzed. A charge double layer arises in the film near the edges of the electrodes because of the electric field. Estimates show that the height of the field-induced potential barrier in films made from several narrow-gap semiconductors may exceed the thermal potential at room temperature.

Various aspects of fields in reduced-dimensionality electron system have been the subject of a fairly large number of papers.[1] In most cases, a so-called linear field effect is seen. This is true, for example, of field-effect transistors with an isolated gate, in which case the conductance of the channel is a linear function of the voltage on the gate. In other words, when the sign of the electric field beside the gate is reversed, the increment in the source-drain current also changes sign.

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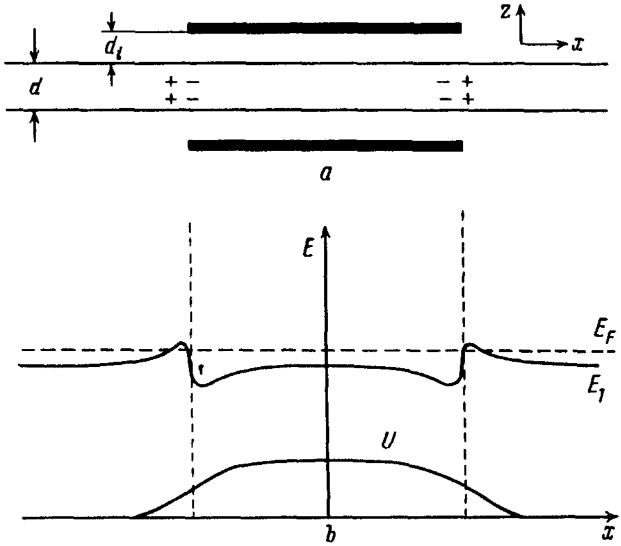

FIG. 1. *a*—Film in the electric field of symmetrically positioned electrodes; *b*—band diagram and potential relief for the electrons in the film.

In the present letter we estimate the quadratic field effect in 2D solid-state films, and we discuss the conditions under which this effect can have a substantial influence on the conductance of 2D systems.

As the 2D system we consider a degenerate electron gas in a thin film in the electric field between two electrodes (Fig. 1). If the film thickness d is smaller than the first Bohr radius a_B, the electric field can be assumed uniform over the film thickness. The electron spectrum in such a film is known to consist of 2D quantum-well subbands, whose positions are determined by the electric field. For a potential well which is symmetric (in the absence of a field), the shift of the subbands is a quadratic function of the normal component of the electric field, and there is no linear field effect.

To estimate the change in the energy position of the first subband (the "bottom" of the conduction band), we work from a solution of the one-particle Schrödinger equation

$$d^2\psi/dz^2 + (2m/\hbar)^2)(E-U)\psi = 0\,, \qquad (1)$$

with the potential energy

$$U = U_0 - qFz, \quad -d/2 < z < d/2\,;$$

$$U \to \infty, \quad |z| > d/2; \quad \Psi(-d/2) = \Psi(d/2) = 0\,, \qquad (2)$$

where U_0/q is the electric potential averaged over the film thickness, F is the normal component of the electric field, and m and q are the effective mass and the charge, respectively, of an electron.

For a triangular potential well as in (2), the solution of Eq. (1) can be expressed in terms of Airy functions of argument $s = (fz - e)/f^{3/2}$, where we have introduced

FIG. 2. The energy position of the first subband, e_1, versus the normal component of the electric field, $f(U_0=0)$.

some dimensionless variables: $f=-qFd/2E_0$, $e=(E/E_0)-u_0$, $u_0=U_0/E_0(E_0=2\hbar^2/md^2)$. Figure 2 shows the results of some numerical calculations of $e_1(f)$, the position of the bottom of the conduction band. In fields $0\leqslant f\leqslant 10$, the behavior of the change in the energy position of the bottom of the conduction band, $\delta e_1(f)$, can be approximated by

$$\delta e_1 \simeq u_0-\alpha^2 f^2, \quad \alpha=0.13\,. \tag{3}$$

To estimate the magnitude of the quadratic field effect, we restrict the discussion to the case in which the Fermi level remains in the first subband over the entire range of electric fields applied to the film. Using the known expression[1] for the surface concentration of electrons in quantum-well films at low temperatures, we find an expression for the deviation of the surface concentration N_s from its equilibrium value N_{s0}:

$$\Delta N_s=N_s-N_{s0}\simeq(4/\pi d^2)(\delta e_F-u_0+\alpha^2 f^2)\,, \tag{4}$$

where δe_F is the deviation of the Fermi level from the equilibrium position, divided by E_0.

In the central part of the film the electron density evidently remains at its equilibrium value (by virtue of the symmetry of the system), while the potential energy of the electrons increases by an amount $\alpha^2 f^2 E_0$.

Let us assume that the external electric field is uniform beside the electrodes and vanishes rapidly with distance away from them. Since the Fermi level is uniform along the film ($\delta e_F=0$) in the absence of an electron current, a charge double layer arises near the edges of the electrodes. Such layer is characteristic of a heterojunction formed by putting two materials differing in work function in contact (Fig. 1). The jump in the bottom of the conduction band in this case is $\delta E_1(0)=\alpha^2 f^2 E_0$, and the width of the charge layer is comparable to the first Bohr radius a_B. Outside this layer the electron density remains at its equilibrium value. The latter circumstance is an important distinction between the quadratic field effect and the linear effect.

The qualitative arguments are supported by a direct solution of the Poisson equation with the charge in the film as in (4). It can be shown that a necessary condition for the formation of an effective barrier in the film is

$$d_i \ll r_f , \tag{5}$$

where d_i is the width of the insulating gap between the electrode and the film, and $r_f^2 = (d a_B)/8$. The maximum deviation of the electron density in the double layer from its equilibrium value is

$$\Delta N_{sm} \simeq (2a^2 f^2)/(\pi d^2) . \tag{6}$$

If the field varies smoothly over the film ($d_i \gg r_f$), there is essentially no buildup of charge in the film.

A potential barrier in the film gives rise to a modulation of the conductivity of the film. As a current flows along the film, the conductivity at low temperatures is dominated by the tunneling component of the current, which is proportional to the transmission coefficient T_f. Assuming a triangular barrier of height $U_b = \delta E_1(0)/2$ and width $w = r_f$, we can write T_f as

$$T_f(f) = \exp[-(2\alpha/3)(d/a_B)^{-1/2}|f|] . \tag{7}$$

Let us estimate the parameters of films from the standpoint of practical applications. It follows from expressions (3), (6), and (7) that the quadratic field effect should be seen most clearly in films with small effective electron masses and large dielectric constants, at thicknesses $d \simeq a_B$. Specifically, we have $\delta E_1(0) \propto d^4$, $\Delta N_{sm} \propto d^4$, $\ln T_f \propto d^{3/2}$ and $\delta E_1(0) \propto \epsilon^2/m^3$, $\Delta N_{sm} \propto \epsilon^2/m^2$, and $\ln T_f \propto \epsilon^2/m^2$ at $d = a_B$. Narrow-gap n-type semiconductors satisfy these conditions. According to our estimates, the height of the potential barriers in films based on II–VI and III–V compounds (HgSe, HgTe, GaSb, InAs, and InSb) would be $U_b = 0.04$–0.13 eV, depending on the film material, if the field in the insulating gap between the electrode and the film is $F_i = 5 \times 10^6$ V/cm. (We assumed the film thickness to be $d = 6$ nm and the dielectric constant in the gap to be $\epsilon_i = 2$. We took values of the effective masses and the dielectric constants of the materials from a handbook.[2]) We also have $T_f = 3 \times 10^{-3}$–7×10^{-2}. In this case the value of ΔN_{sm} is $\Delta N_{sm} \simeq 3 \times 10^{12}$ cm^{-2}. The semimetal bismuth (Bi) could apparently also be used as a film material. It has a small effective electron mass and a large dielectric constant.

The regions of charge accumulation which form in films by virtue of the quadratic effect can be utilized in order to achieve 1D or 2D quantum effects: dots, wires, rings, etc.

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[1] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[2] I. K. Kikoin (editor), *Handbook of Tables of Physical Quantities*, Gosatomizdat, Moscow, 1976.

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