Extrinsic geometry sensitive bosonic string theory

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We propose a modification of the string theory with the action depending on the extrinsic curvature tensor of the surface. The infrared behavior of our model is governed by a nontrivial fixed point.

Several years ago, the authors of Refs. 1–4 considered a string theory with the action containing, in addition to the usual Nambu term, a term equal to the square of the extrinsic curvature tensor of the world surface:

$$2\alpha_0 S_0 = \sqrt{g} g^{\alpha\beta} g^{\gamma\delta} \nabla_\alpha \partial_\gamma X^\mu \nabla_\beta \partial_\delta X_\mu + \sqrt{g} \lambda^{\alpha\beta} (\partial_\alpha X^\mu \partial_\beta X_\mu - g_{\alpha\beta}). \tag{1}$$

Here α_0 is a dimensionless coupling constant, and the Lagrange multiplier term keeps the metric $g_{\alpha\beta}$ identical to the induced metric $\partial_\alpha X^\mu \partial_\beta X_\mu$. The role of S_0 is, classically, to prevent (energetically) the surface from bending and to lead the theory to a "smooth" phase with the string tension scaling near the critical point. Such smooth strings are expected to provide a description of the universality classes of the three-dimensional Ising model and four-dimensional QCD,[2] thereby passing through the $c=1$ barrier of "conformal matter+Liouville"-type theories. They are also useful as the models of membranes, domain walls, etc.[1,3,4]

However, as it was shown in Refs. 1–4, the coupling α_0 turns out to be asymptotically free, which means that in the infrared the first term in S_0 is irrelevant, while in the second one the Lagrange multiplier field $\lambda_{\alpha\beta}$ develops an expectation value of the order of the cutoff, thus generating a nonzero effective string tension and driving the theory into familiar (and, from the point of view of 3D Ising model and 4D QCD, unacceptable) Liouville phase. The whole picture is analogous, apart from a numerical difference in the coefficient in the β-function, $d/2$ instead of $d-2$, to what occurs in nonlinear sigma-models, where the quantum fluctuations generate a mass gap.

This situation can be avoided by generalizing the action (1) in such a way that the β-function $\beta(\alpha_0)$ has a zero at a nonzero value of α_0. At the fixed point, the continuum theory then will remain massless, with long-range correlations between the normal vectors of the surface. Such a generalization will be described in the present letter. The details and some further results will be presented elsewhere.[8] The action to be presented below is strongly suggested by the experience gained in attempts to describe the 3D Ising model in terms of the fermionic strings[5,6] (see also Ref. 7). A basic object in our approach is a composite matrix field[5,6] $\Omega \in SO(d)$, constructed in the following way.

Let $X^\mu(\zeta)$ ($\mu=1,...,d$) describe an embedding of some (closed, orientable) surface Σ into d-dimensional Euclidean space. The tangent vectors of the surface are denoted by $X_\alpha = \partial_\alpha X$, $\alpha=1,2$, and $h_{\alpha\beta} = X_\alpha X_\beta$ is the two-dimensional metric tensor induced by the embedding. We introduce the zweibeins $e_\alpha^a (\alpha=1,2)$, such that $e_\alpha^a e_{\alpha\beta}$

$=X_\alpha X_\beta$. Then $X_\alpha = e_\alpha^a X_a$ are the orthonormal vectors tangent to the surface; let us introduce $d-2$ orthonormal normals to the surface, X_i ($i=1,\ldots, d-2$), and denote the whole set of d orthonormal d-dimensional vectors by $X_m^\mu = \{X_a^\mu, X_i^\mu\} m=1,\ldots,d$. Finally, consider a matrix field Ω representing the $SO(d)$-rotation taking the basis set X_m to some arbitrary constant orthonormal basis X_m^c.

Some comments are now in order. First, a theory with the action depending on Ω cannot, in general, be considered a string theory, because of the ambiguity in the definition of Ω: it depends on the choice of the zweibeins e_α^a and of the normals X_i. To have a string theory, one has to ensure an $SO(2)\times SO(d-2)$ gauge symmetry of the action, thus obtaining a theory with Ω taking values effectively in the Grassmanian $G_{2,d}=SO(d)/SO(2)\times SO(d-2)$. This can be done in a covariant way which we will not describe here; the net result is that after a gauge fixing one must impose the following constraints on Ω:

$$e_a^\alpha \nabla_z - e_{b\alpha} = 0 \quad X_i^\mu \nabla_z - X_{\mu j} = 0, \tag{2}$$

where the covariant derivative and the complex structure must be assumed compatible with the metric $h_{\alpha\beta}$.

The second comment is that for some embeddings the field Ω is singular. This happens when the surface has an open line of self-intersection. For $d\geqslant 4$ these singularities are not stable; however, for $d=3$ they are stable and play the most essential role in the string representation of 3D Ising model.[5,6] In the local consideration presented here they can be ignored.

The third comment is that Ω can be taken in any representation of $SO(d)$. The one relevant for the 3D Ising model is the spinor representation,[5,6] but the construction works for a general case. In the present letter we assume Ω to be a $d\times d$ matrix

$$\Omega_{mn} = X_m^c X_n. \tag{3}$$

Having all this in mind, let us write the simplest possible action which depends on Ω, i.e., that of the nonlinear sigma-model:

$$4\alpha_0 S_0 = \sqrt{h} h^{\alpha\beta} \mathrm{Tr} \partial_\alpha \Omega \partial_\beta \Omega^{-1}. \tag{4}$$

Now substituting for Ω the expression (3) and using the conditions (2), we find that when rewritten in terms of the field X, the action (4) coincides with the action (1).

This observation suggests generalizing the action (1) by adding a Wess–Zumino term to it. Specifically, consider the action

$$2\alpha_0 S = \sqrt{g} g^{\alpha\beta} \nabla_\alpha \partial^\gamma X \nabla_\beta \partial_\gamma X + \sqrt{g} \lambda^{\alpha\beta} (\partial_\alpha X \partial_\beta X - g_{\alpha\beta}) + (\sigma/3) \mathrm{Tr}(\Omega^{-1} d\Omega)^3. \tag{5}$$

In the last term of this expression an integration over a three-dimensional manifold, such that its boundary coincides with the surface Σ, is understood, and Ω denotes now an arbitrary extension of the field (3) on this manifold. The couplings must be connected by Wess–Zumino quantization condition:

$$\sigma/\alpha_0 = in/4\pi, \quad n\in Z. \tag{6}$$

Let us stress again that the constraints (2), which must be added to the action, arise as a result of the gauge fixing in some $SO(2)\times SO(d-2)$-invariant theory, the explicit form of which is not important for us now.

In the rest of this letter, I will show in the one-loop approximation that at some value of the coupling constant α_0 the action (5) describes a conformally invariant theory. For simplicity, I will restrict the discussion to the case $d=3$; I will follow closely the line of calculations described by Polyakov in Ref. 2. Let us assume that the theory (5) is defined with the momentum space cutoff Λ and let us integrate over the fields with momenta which satisfy the condition $\widetilde{\Lambda}\leqslant|p|\leqslant\Lambda$. Denoting these "fast" fields by the superscript "1", we obtain the following expression for the quadratic term in the expansion of the conformal gauge ($g_{\alpha\beta}=p\delta_{\alpha\beta}$) action in powers of the "fast" fields:

$$S_2=X_1DX_1-2X_1J-\lambda_1^{\alpha\alpha}\rho_1+(\partial^2X)^2\rho_1^2, \tag{7}$$

where

$$D^{\mu\nu}=\delta^{\mu\nu}(\partial^4-\partial j_\alpha\lambda^{\alpha\beta}\partial_\beta)+2\sigma\partial_\alpha R^{\mu\nu}_{\alpha\beta\gamma}\partial_\beta\partial_\gamma,$$

$$R^{\mu\nu}_{\alpha\beta\gamma}=[\partial_\delta X^\mu_\sigma X^\nu_\delta\delta_{\alpha\beta}+\partial_\alpha X^\mu_\sigma X^\nu_\beta]\epsilon_{\sigma\gamma},$$

$$J^\mu=\partial_\alpha X^\mu_\beta\lambda_1^{\alpha\beta}+\partial^2(\partial^2X^\mu)_{\rho 1}+(\sigma/2)\partial_\alpha n^\mu[3H_{\beta\alpha}\epsilon^{\beta\gamma}+iH_{\beta\delta}\epsilon^{\beta\gamma}\epsilon_{\alpha\delta}]\partial_\gamma\rho_1.$$

Here $H_{\alpha\beta}=n^\mu\partial_\alpha X_{\beta\mu}$ is the extrinsic curvature tensor. We have set the background ("slow") value of ρ equal to 1 and n^μ is the (unique in $d=3$) unit vector normal to the surface. When deriving S_2, we must use the variation of the matrix Ω, keeping in mind the conditions (2).

One is left with a sequence of Gaussian integrations which are performed straightforwardly, except the one over λ_1, for which a decomposition suggested by Polyakov[2] is used:

$$\lambda^{\alpha\alpha}=\zeta,\quad \partial_\alpha\lambda^{\alpha\beta}=\partial^2f^\beta.$$

As a result, in the leading logarithmic approximation the effective action turns out to be

$$2\alpha_0S_{\rm eff}=[1-(\alpha_0/2\pi)\ln(\Lambda/\widetilde{\Lambda})+(3i\sigma\alpha_0/8\pi)\ln(\Lambda/\widetilde{\Lambda})](\partial^2X)^2/\rho$$
$$+(\sigma/3){\rm Tr}(\Omega^{-1}d\Omega)^3+\lambda^{\alpha\beta}\partial_\alpha X\partial_\beta X-[1-(\alpha_0/4\pi)\ln(\Lambda/\widetilde{\Lambda})]\lambda^{\alpha\alpha}\rho.$$

Performing an evident field renormalization, we finally obtain, using the quantization condition (6), the following renormalization equation for the coupling α_0:

$$\widetilde{\alpha}_0=\alpha_0-(3\alpha_0^2/4\pi)(1+n\alpha_0/8\pi)\ln(\Lambda/\widetilde{\Lambda}). \tag{8}$$

From (8) we read off the β-function:

$$\beta(\alpha_0)=-(3\alpha_0^2/4\pi)(1+n\alpha_0/8\pi). \tag{9}$$

For $n=0$ this expression coincides with the one found in Refs. 1–3 and it describes an asymptotically free coupling. For $n\neq 0$, however, the β-function (9) has a zero at $\alpha_0=-8\pi/n$. Since α_0 must be positive, we must assume $n<0$ and we obtain an infrared, stable, fixed point.

Equation (9) is the main result of the present work. Let me add some comments here. Although Eq. (9) was derived for $d=3$, an analogous result holds for arbitrary d, where the coefficient in the β-function is $d/2$, instead of $d-2$, for the usual $G_{2,d}$ Wess–Zumino–Novikov–Witten model. This difference arises, as we stressed by Polyakov,[2] as a result of restricting the field Ω by an integrability condition which states that Ω must be obtained from a surface.

Equation (9) shows that imposing such a restriction does not spoil the conformal properties of the Wess–Zumino–Novikov–Witten model, so that the model[2,5] can be regarded as some reduction of it.

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