

Analytic description of energy loss by a bounded inhomogeneous hot plasma due to the emission of electromagnetic waves

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(Submitted 10 September 1992; resubmitted 22 October 1992)

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Universal formulas for total power losses by bounded media due to the emission of electromagnetic (both transverse and longitudinal) waves in the regimes of nonlocal (non-diffusive) heat transport are proposed. In particular, these universal formulas generalize Trubnikov's formula for synchrotron losses to the case of arbitrary emission/absorption process and inhomogeneous nonstationary plasma. Their derivation is based on a non-diffusion method which generalizes the "escape-probability method" in the theory of radiative transfer in resonance atomic lines (RTRAL). The results suggest a qualitative model for the global heat transport in a tokamak.

1. Introduction. The heat transport by electromagnetic (EM) waves, both transverse waves (i.e., photons) and longitudinal waves (i.e., plasmons), in the wide range of parameters (which includes, in particular, magnetically confined thermonuclear plasmas) is characterized by its nonlocal (non-diffusive) nature which manifests itself in nonlocal correlation of plasma temperatures and, correspondingly, non-diffusive law of heat propagation. Mathematically, the non-diffusive nature is expressed by the fact that the original equation for heat transfer which is, in general, an integral equation in space variables, cannot always be reduced to a differential diffusion-type equation in those variables. Such a reduction leads to an infinite diffusion coefficient in the case of unbounded media or medium-size-dependent diffusion coefficient in the case of bounded media, so that the very concept of diffusion coefficient appears to be meaningless.

2. Universal formula for total energy losses. Let us consider the transfer of EM energy which is described by the intensity $J(\phi, \mathbf{r}, t)$, differential with respect to EM wave (photon or plasmon) parameters ϕ ,

$$\phi \equiv \{\omega, \mathbf{n}, \zeta\}, \quad (1)$$

where ω and \mathbf{k} are the frequency and wave vector, respectively, $\mathbf{n} \equiv \mathbf{k}/k$, and the parameter ζ describes polarization state of the wave. The space-time evolution of the intensity is guided by an equation of the well-known form:

$$\left(\frac{1}{v_g} \frac{\partial}{\partial t} + \mathbf{n}_g \frac{\partial}{\partial \mathbf{r}} \right) \left(\frac{J(\phi, \mathbf{r}, t)}{(N_\tau)^2} \right) = -\kappa \left(\frac{J(\phi, \mathbf{r}, t)}{(N_\tau)^2} \right) + \frac{Q}{(N_\tau)^2}, \quad (2)$$

where $\kappa(\phi, \mathbf{r}, t)$ is the absorption coefficient, $Q(\phi, \mathbf{r}, t)$ is the source function, N_τ is ray

refractive index, and v_g and \mathbf{n}_g are the group velocity and its direction. In a medium with dispersion, the substitution of the dependence $k = k(\omega, \mathbf{n}, \zeta)$ from the dispersion relation is implied.

In the case of radiative transfer by emission/absorption of EM waves by plasma electrons, the quantities Q and κ contain averaging over electron velocity distribution (EVD). Therefore, the source function Q may implicitly depend on the intensity J via corresponding distortions of the EVD, caused by radiative transfer. If these distortions are small, then Q does not depend on J , and Eq. (2) appears to be a closed equation for energy carriers. The multiple reflection at the medium boundary (e.g., the tokamak wall) prevents straightforward use of the analytic solution of Eq. (2) in the form of the integral over a ray path.

Another example of the reduction of the transport problem to a closed equation is the Biberman–Holstein equation for energy storage systems, specifically, storage of excited atoms, in the theory of radiative transfer in resonance atomic lines (RTRAL) in gases and plasmas in case of the so-called complete redistribution in photon frequencies in an individual event of photon scattering by the atom of the medium (approximation of complete redistribution is valid for a wide range of astrophysical and laboratory plasmas¹).

In both above-mentioned cases the energy transport appears to be characterized by the following general features.

(*) The dominant contribution to energy loss stems from those long mean-free-path energy carriers (photons or plasmons) whose path lengths are of the size of the medium, L_0 , or greater, $L_0/(1-R)$, for the case of reflection at boundaries, and R is the reflection coefficient (for the RTRAL, these are the photons in the so-called line wings).

(**) Each of the variables of the total phase space $\{\Gamma\} \equiv \{\phi, \mathbf{r}\}$ manifests one of two limiting forms of its evolution (“redistribution”) along the trajectory of energy carrier from its birth to its death (by conversion into the heat of the medium); either no redistribution (“independent” variables) in which case for this variable the energy transport takes place independently (e.g., the variable ω for the case of absorption/emission by free electrons with fixed velocity distribution and the variable \mathbf{r} for the case of bounded electrons, see below) or complete redistribution in which case for each of those variables the transport equation may be properly averaged, according to an actual redistribution law.

We denote the whole set of first-type variables as Γ_{ind} and the second-type as Γ_{crd} . The properties (*) and (**) enable us to obtain the following general result for total power losses:

$$\frac{dE}{dt} = \int d\Gamma_{\text{ind}} (1 + \nu_{\text{que}}/\nu_{\text{csc}})^{-1} \int d\Gamma_{\text{crd}} Q(\Gamma). \quad (3)$$

Here ν_{que} is the rate of such an absorption of EM energy by the medium, which converts transported EM energy into the heat of the medium (temperature), [this is the quenching of the atomic excitation by, e.g., the impact of the particles of the medium for the RTRAL, and the absorption of the photon (plasmon) by plasma

electrons for the radiative transfer in the continuous spectrum], and ν_{esc} is the rate of free escape of EM energy from the medium. Both quantities ν_{que} and ν_{esc} are averaged over Γ_{crd} according to the corresponding redistribution laws.

In the case of the RTRAL, we have $\Gamma_{\text{ind}} = \mathbf{r}$ and $\nu_{\text{esc}} = \langle T(\mathbf{r}) \rangle / t_{\text{rad}}$, where $\langle T(\mathbf{r}) \rangle$ is the Holstein function averaged over the angles of escape of the photon, and t_{rad} is the spontaneous emission lifetime for excited atom. We thus obtain the well-known result of the RTRAL theory (for the accuracy of this result, see the review article in Ref. 1 and the references cited there).

It should be noted that the phenomenon of nonlocal, non-diffusive transport has been identified and thoroughly investigated just in the RTRAL theory in the late forties and early fifties in the pioneer works by Biberman, Holstein, and Sobolev. The derivation of an equation such as (3) and its further advances are known in the literature as the “escape-probability method” (see, e.g., Ref. 2). Therefore, Eq. (3) may be interpreted as a generalization of this method to the case of heat transport via emission/absorption of (transverse and longitudinal) EM waves by free electrons with fixed velocity distribution [for comparison of Eq. (3) with numerical calculations for the heat transport by electron cyclotron radiation see Sec. 3].

Note that Eq. (3) covers both limiting regimes of energy loss, namely, purely volumetric loss due to free escape from the whole phase space ($\nu_{\text{que}} \ll \nu_{\text{esc}}$) and purely surface loss ($\nu_{\text{que}} \gg \nu_{\text{esc}}$). In the latter case, the intensity of escaped EM field is close to the equilibrium Planck distribution with some effective, space-averaged temperature; the heat transport inside the medium is characterized in the corresponding domain of phase space by diffusion-type regime of radiative heat conduction. Nevertheless, the total losses in Eq. (3) are determined predominantly by those parts of the total phase space Γ in which the process of energy transport has essentially nonlocal character, namely, the non-diffusion regime of free escape. The latter statement constitutes the essence of a generalized escape probability (GEP) method which enables us to obtain Eq. (3) within the framework of principles (*) and (**).

3. Heat transport by plasma waves. Contrary to the RTRAL theory, nonlocal character of the transport by longitudinal plasma waves and its possible role have been realized much later, starting with the paper by Rosenbluth and Liu.³ For the case of transport by transverse waves (i.e., conventional electron cyclotron radiation) in inhomogeneous plasma, the concept of non-diffusive transport has been effectively exploited by Tamor.⁴

The straightforward analogy between the heat transport by plasma waves and the RTRAL was analyzed in Ref. 5, where the non-diffusive law of heat propagation by the Bernstein modes was obtained (approximately $t \propto r$ contrary to the standard diffusion law $t \propto r^2$).

The existing analytic descriptions have the character of the fit of numerical results and pertain to (a) the specific mechanism of emission, namely, cyclotron radiation, and (b) the specific profiles of temperature and density: homogeneous, the well-known Trubnikov’s formula,⁶ or tokamak-like formula.⁷ The required analytic description may be obtained from our general result (3) for the specific case of tokamak geometry under following, usually satisfactory, conditions, namely, not too large aspect ratio,

non-circular (in particular, elongated) cross section and multiple reflection at the plasma boundaries (for transverse waves the latter is assured by highly reflecting tokamak walls, $(1-R) \lesssim 0.1$, and for longitudinal waves it depends on the edge plasma parameters). Neglecting the mixing of different modes in reflections at the boundaries, we have $\Gamma_{\text{ind}} = \{\omega, \xi\}$, and Eq. (3) reduces to the form

$$\frac{dE}{dt} = \sum_{\xi} \int d\omega \frac{\int dV \int d\Omega_n Q(\phi, \mathbf{r})}{1 + \tau_{\text{eff}}}, \quad \tau_{\text{eff}} = \frac{\int dV \int d\Omega_n Q(\phi, \mathbf{r})}{\int d\Omega_n \int (\mathbf{n}, d\mathbf{S}_s) [1 - R(\phi, S_s)]}, \quad (4)$$

where $\tau_{\text{eff}}(\omega, \xi) = \nu_{\text{que}}/\nu_{\text{esc}}$ is the effective (dimensionless) optical length which describes the trapping (or, equivalently, imprisonment, according to RTRAL theory language) of plasmons (photons). Here V and S_s are the plasma volume and surface, respectively, and $R(\phi, S_s)$ describes the dependence of the reflection coefficient on the wave parameters (first of all, the frequency). The comparison of Eq. (4) with the results of numerical calculations, in particular, for homogeneous⁶ and inhomogeneous⁴ cases, and with formula⁷ (and formula⁶ in the region of successful approximation of numerical results, see Refs. 6 and 8) shows good agreement (to an accuracy of about 30%) in the regions of applicability of these results. The GEP method allows also to obtain a universal analytic description for spatial profile of wave energy balance and for the case of arbitrary degree of mixing of different waves at plasma boundaries, both for stationary and nonstationary cases. Thus, the nonstationary counterpart of Eq. (4) has the form:

$$\frac{dE(t)}{dt} = \sum_{\xi} \int d\omega \int_0^t \frac{dt'}{t_{\text{eff}}} \int dV \int d\Omega_n Q(\phi, \mathbf{r}, t') \exp\left[-\int_{t'}^t \frac{dt''}{t_{\text{eff}}} [1 + \tau_{\text{eff}}(t'')]\right], \quad (5)$$

where t_{eff} is the characteristic time of wave trapping exclusively due to the reflection by the boundaries,

$$t_{\text{eff}}(t) = \frac{\int dV \int d\Omega_n v_g^{-1}(\phi, \mathbf{r}, t)}{\int d\Omega_n \int (\mathbf{n}, d\mathbf{S}_s) [1 - R(\phi, S_s)]}, \quad (6)$$

[Eq. (5) assumes the smallness of the retardation effects].

Equations (4) and (5) generalize Trubnikov's equation⁶ for synchrotron losses to the case of (a) arbitrary elementary process of radiation emission by high-temperature plasma with highly reflecting walls and fixed (e.g., Maxwellian) velocity distribution and (b) inhomogeneous and nonstationary plasma parameters (density, temperature, magnetic field).

4. Global heat transport in a tokamak. The application of the GEP concept [items (*) and (**) of Sec. (2)] to the energy transport by plasma waves suggests a qualitative model for the global heat transport in a tokamak, which exploits the fact of a strong coupling of essentially nonlocal and local characteristics of a plasma in Eqs. (4) and (5) for total power losses, i.e., the coupling of space-averaged emission/absorption coefficients and the coefficient for the reflection of plasma waves at a plasma boundary. In this model, the global energy transport is conducted by the mean-free-path quanta of longitudinal EM waves (plasmons), which are responsible for such a strong coupling due to their multiple reflection at plasma boundary and

appear to be the main carriers of energy, whereas the particles are invariably the main accumulators of plasma energy. Such a model suggests the possibility of an effective control of global (nonlocal) plasma parameters via a proper control of the reflection of plasma waves in edge plasma. From this viewpoint, the L–H transition may appear to be stimulated by a sharp change of the reflection coefficient due to, e.g., increased gradient of poloidal rotation of tokamak plasma or increased stability of magnetic surfaces which are responsible for the reflection of most significant energy carriers.

The stimulation of author's interest to radiative transfer problem by V. I. Kogan and to heat transport by longitudinal waves by V. S. Lisitsa with fruitful cooperation⁵ is acknowledged with gratitude.

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Translated by the author