# Manifestations of a fluctuational mid-range order in superconductors with nonuniform pairing

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(Submitted 6 November 1992)
Pis'ma Zh. Eksp. Teor. Fiz. **56**, No. 11, 600–603 (10 December 1992)

Fluctuation effects in superconductors with nonuniform pairing are stronger than in the case of uniform pairing. The fluctuation component of the heat capacity is negative in a certain region of the phase diagram. A "fluctuation mechanism" is proposed for explaining the nonclassical (i.e., non-BCS behavior of the superconducting gap which has been found in tunneling and optical experiments on high-$T_c$ superconductors.

Let us consider the region in which a nonuniform superconducting state is nucleated in a ferromagnet.[1,2] Figure 1 shows a $T$, $h$ phase diagram of the system, where $T$ is the temperature, and $h$ the internal spin-exchange field.

We show below that fluctuation effects in superconductors with nonuniform pairing are stronger than those in the case of uniform pairing.

A fluctuational mid-range order in the region in which the nonuniform superconducting state is nucleated is determined by a correlation of electrons in states which are coupled by a continuous class of nonuniformity vectors $\mathbf{q}$ such that we have $|\mathbf{q}|-q_0<1/\xi$, where $\xi$ is the correlation length of the nonuniform superconducting state. The following expression can be derived for $q_0$:

$$q_0=\frac{\alpha h}{v_F}\left[1+\frac{8\pi^2}{3(\alpha^2-4)}\left(\frac{T}{h}\right)^2\right],\quad \alpha\simeq 2.4,$$

where $v_F$ is the Fermi velocity. As a result, the phase volume of the correlated states is greater than that corresponding to the uniform ($q_0\equiv 0$) case, by a factor of $(\xi' q_0)^2$, where $\xi'$ is the correlation length of the uniform superconducting state.

Formally, the system which we are discussing here is analogous to a system with a nonuniform electron-hole pairing. Effects of a fluctuational mid-range insulating order in the latter system were studied in Ref. 3. Working from the results of that paper, we conclude that the ratio of the correlation lengths of the nonuniform and uniform states is $\xi/\xi'\simeq 5$. The pseudogap in the density of electron states in the region in which the nonuniform state is nucleated is $v_F q_0$, greater than the corresponding value for the uniform case, $v_F/\xi'$.

To estimate the region in which fluctuations play an important role above the line of the phase transition to the nonuniform state, we calculate the fluctuation component of the heat capacity:

  

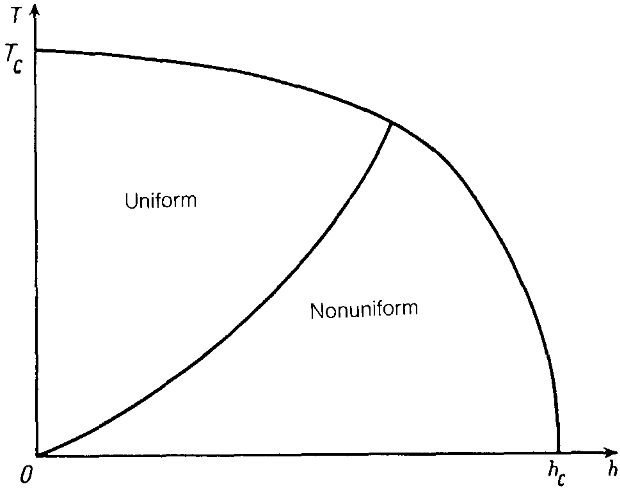


FIG. 1.

$$\Delta C=-T\frac{\partial^2[\Delta\Omega(h,T)]}{\partial T^2}. \tag{1}$$

Here $\Delta\Omega(h,T)$ is the fluctuation correction to the thermodynamic potential. It is given in the random phase approximation by

$$\begin{cases}\Delta\Omega(h,T)=TV\sum_n\int\frac{d\mathbf{q}}{(2\pi)^3}\ln(g\Phi(\Omega_n,\mathbf{q},h,T)\\ \Phi(\Omega_n,\mathbf{q},h,T)=\frac{1}{g}-\frac{1}{N(0)}T\sum_m\int\frac{d\mathbf{k}}{(2\pi)^3}G(\omega_m,\mathbf{k},h)G(-\Omega_n-\omega_m,-\mathbf{k}+\mathbf{q},-h).\end{cases} \tag{2}$$

Here $G(\omega_m,\mathbf{k},h)=[i\omega_m-\epsilon(\mathbf{k})+h]^{-1}$ is the Green's function for a ferromagnet in a nonsuperconducting state, $\omega_m=(2m+1)\pi T$, $\Omega_n=2n\pi T$, $N(0)=2\epsilon_F^2/\pi^2 v_F^3$ is the density of states at the Fermi level, $\epsilon_F$ is the Fermi energy, $V$ is the volume of the system, and $g$ is the superconducting pairing constant.

Expanding $\Phi(\Omega_n,q,h,T)$ in small parameters

$$\tilde{h}-1=h/h_c-1,\quad \tilde{T}=T/2h_c,\quad \tilde{\Omega}_n=\Omega_n/2h_c,\quad \tilde{q}-\tilde{q}_0=v_F|\mathbf{q}|/2h_c-\tilde{q}_0$$

(where $h_c\simeq 1.31T_c$, and $\tilde{q}_0=\frac{\alpha}{2}\simeq 1.2$), we find

$$\Phi(\tilde{\Omega}_n,\tilde{q},\tilde{h},\tilde{T})=\tilde{h}-1+\frac{1}{2}\frac{(\tilde{q}-\tilde{q}_0)^2}{\tilde{q}_0^2-1}+\frac{2}{3}\frac{\pi^2\tilde{T}^2}{\tilde{q}_0^2-1}+\frac{\pi}{2\tilde{q}_0}|\Omega_n|. \tag{3}$$

From Eqs. (2) and (3) we find the relative size of the fluctuational correction to the heat capacity $[C_n=\frac{4}{3}(\epsilon_F^2/v_F^3)TV]$:

$$\frac{\Delta C(h,T)}{C_n} = -3(\tilde{q}_0^2-1)\frac{h_c^3}{\epsilon_F^2 T}$$

$$\times \sum_{n=0}^{[1/4\pi\tilde{T}]} \frac{32\pi\tilde{q}_0^2(\tilde{h}-1)\tilde{T}^2+8\tilde{q}_0(\tilde{q}_0^2-1)(\tilde{h}-1)\tilde{\Omega}_n+3\pi(\tilde{q}_0^2-1)\tilde{\Omega}_n^2}{[2(\tilde{q}_0^2-1)(\tilde{h}-1)+\pi(\tilde{q}_o^2-1)/\tilde{q}_0\tilde{\Omega}_n+\frac{4}{3}\pi^2\tilde{T}^2]^{3/2}}, \quad (4)$$

where $[1/4\pi\tilde{T}]$ is the greatest integer in $1/4\pi\tilde{T}$.

According to the Ginzburg criterion,[4,5] the range of applicability of the random phase approximation and thus of expression (4) is bounded by the condition $\Delta C(h,T)/C_n \ll 1$.

Analyzing (4), we distinguish the following regions of characteristic behavior of $\Delta C(h,T)/C_n$:

1. $\tilde{T} \ll \frac{1}{4\pi}$; $\frac{\Delta C(h,T)}{C_n} = -\frac{3\sqrt{2}}{4\pi}\tilde{q}_0(\tilde{q}_0^2-1)\frac{h_c^4}{\epsilon_F^2 T^2} < 0$;

2. $\tilde{T} \sim \frac{1}{4\pi}$; $\frac{\Delta C(h,T)}{C_n} \sim \frac{-h_c^2}{\epsilon_F^2}\sqrt{\frac{h_c}{T}} < 0$;

3. $\tilde{T} \gg \frac{1}{4\pi}$, $\tilde{h} > 1$, $\tilde{T}^2 \ll \frac{3(\tilde{q}_0^2-1)(\tilde{h}-1)}{2\pi^2}$;

$$\frac{\Delta C(h,T)}{C_n} = -\frac{48\pi^2\tilde{q}_0^2}{\sqrt{2(\tilde{q}_0^2-1)}}\frac{Th_c}{\epsilon_F^2}\sqrt{\frac{h_c}{h-h_c}} < 0;$$

4. $\tilde{T} \gg \frac{1}{4\pi}$, $\tilde{h} > 1$, $\tilde{T}^2 \gg \frac{3(\tilde{q}_0^2-1)(\tilde{h}-1)}{2\pi^2}$;

$$\frac{\Delta C(h,T)}{C_n} = -72\sqrt{3}\tilde{q}_0^2(\tilde{q}_0^2-1)\frac{h_c^4}{\epsilon_F^2 T^2}\frac{h-h_c}{h_c} < 0;$$

5. $\tilde{T} \gg \frac{1}{4\pi}$, $\tilde{h} < 1$, $\frac{\Delta C(h,T)}{C_n}$

$$= -16\pi\tilde{q}_0^2(\tilde{q}_0^2-1)\left[2(\tilde{q}_0^2-1)\frac{h-h_c}{h_c}+\frac{1}{3}\pi^2\left(\frac{T}{h_c}\right)^2\right]^{-1}\frac{h_c T}{\epsilon_F^2}\frac{h-h_c}{h_c} > 0.$$

Ginzburg defined the fluctuation region by the condition $\Delta C(h,T)/C_n \gtrsim 1$ [$\Delta C(h,T)$ is taken in the random phase approximation]. In the case at hand, of a fluctuational mid-range order above the line of the transition to the nonuniform superconducting state, this condition yields the following estimate of the temperature range corresponding to the fluctuation region: $0 < T \lesssim h_c(h_c/\epsilon_F)$. This range is considerably broader than the corresponding temperature range of the fluctuation region in the uniform case:[4,5] $|T-T_c| \lesssim T_c(T_c/\epsilon_F)^4$.

There are two terms in the fluctuation correction to the heat capacity. The first (which is positive) stems from the contribution of a low-energy, weakly damped collective mode of Cooper pairs. The second stems from the contribution of one-particle excitations, in the density of states for which a pseudogap forms. This term is consequently negative.

The possibility of a Fulde–Ferrell–Larkin–Ovchinnikov nonuniform superconductivity is not limited to superconducting ferromagnets. For example, the existence of strong insulating correlations in high-$T_c$ superconductors in the region of superconducting compositions may induce a nonuniform superconductivity.[6] The absolute values $q_0$ of the vectors of the nonuniform superconducting state will be determined in this case by the spatial modulation of the corresponding nonuniformity of the insulating order.[3] For a similar reason, a nonuniform superconducting state is possible in superconducting superlattices. Near the transition of the system into the nonuniform superconducting state (possibly below this transition), a superconducting pseudogap with a weak temperature dependence arises. The scale of this gap is proportional to $q_0$. It may be for this reason that a pseudogap is found in optical and tunneling experiments on high-$T_c$ superconductors, simulating a nonclassical (i.e., non-BCS) temperature dependence of the corresponding long-range order of the actual superconducting gap, $\Delta(T)[\Delta(T)\to 0$ as $T\to T_c]$.

Calculating the fluctuation component of the kinetic coefficients would be an independent and interesting problem.[7,8] It follows from the discussion above that this component should increase substantially if a nonuniform pairing occurs in a system.

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Translated by D. Parsons