The behavior of deep-inelastic-scattering structure function ratio $R(x,Q^2)$ at small values of x

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(Submitted 6 December 1993)

Pis'ma Zh. Eksp. Teor. Fiz. **59**, No. 1, 3–7 (10 January 1994)

The behavior of the deep inelastic structure functions is studied at small values of x in the leading and next-to-leading orders of perturbation theory. The scheme-invariant analysis of the longitudinal and transverse structure function ratio $R(x,Q^2)$ is given. It is found that this ratio tends to zero asymptotically when $x \to 0$.

For the experimental studies of hadron–hadron processes on new, powerful LHC and SSC colliders, it is necessary to know in detail the values of parton (quarks and gluon) distributions of nucleons, especially at small values of x. The basic information on the quark structure of nucleons is extracted from the process of deep inelastic lepton–hadron scattering (DIS). Its differential cross section has the form

$$\frac{d^2\sigma}{dxdy} \propto \left[[1+(1-y)^2] - \frac{y^2/2}{1+R(x,Q^2)} \right] F_2(x,Q^2),$$

where F_2 and $F_L \equiv R/(1+R)F_2$ are the transverse and longitudinal structure functions (SF), respectively. The ratio $R(x,Q^2)$ is a good QCD characteristic because it equals to zero in the parton model. Moreover, the value of the SF F_2, whose data are usually deduced from experiment, depends essentially on the corresponding values of R. We note that the value of the ratio R is very important in the case of polarized SF which are deduced from experimentally measured asymmetry of the cross sections of polarized leptons and nucleons.

The modern DIS experimental data (see the review in Ref. 1) are not accurate enough to determine $R(x,Q^2)$. In addition, at small values of x the data for the SF F_L are not yet available. The theoretical predictions (see Ref. 2, for example) in the leading (LO) and next-to-leading (NLO) orders of perturbation theory (PT) are markedly different from each other. There is some doubt as to whether PT can be validly applied in this region.

In the present letter we are studying the behavior of $R(x,Q_2)$ at small values of x using the method[3] of replacement of the Mellin convolution by the usual integral. In addition, we will use both the usual PT and the Grunberg's method of effective charges.[4] Note that in the first two orders of PT the latter method coincides with the scheme-invariant (SI) PT.[5,6]

1. Assuming a Regge-like behavior of the gluon-parton distribution,[2)] $g(x,Q^2)=x^{-\delta}\tilde{g}(x,Q^2)$, and of the singlet quark distribution, $s(x,Q^2)=x^{-\delta}\tilde{s}(x,Q^2)$ $=g(x)/\rho(x)$[3)], we obtain the following equation for the SF F_2 and F_L:

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$$F_1(x)=s(x)+O(x) \quad (F_1=F_2-F_L), \tag{1}$$

$$F_L(x)=\alpha(Q^2)\sum_{p=s,q} B^p_L p(x)+O(x),$$

where $B^p_k(k=1, L)$ are the one-loop coefficients of the Wilson expansion of the first $(n=1)$ SF moments.

We will use below two rather strong hypotheses. We assume a similar behavior for $g(x)$ and $s(x)$ at small x, which has been confirmed by a numerical solution of the Gribov-Lipatov-Altarelli-Parisi equation given in Ref. 7, and ignore the nonsinglet part. Hence, the function ρ is independent of x. We also assume that ρ does not depend on Q^2, i.e., $\rho=$const. These hypotheses allow us to simplify the calculation fundamentally and to perform the SI analysis of $R(x,Q^2)$ at small values of x.

Note that the hypothesis about the weak dependence of ρ on x and Q^2 has been confirmed by the PD parametrizations. However, different parametrizations yield fundamentally different values of ρ. We will therefore not use the fixed value of ρ in our analysis.

Using the above hypotheses and the exact values of Wilson's coefficients, we obtain the following relation from Eqs. (1):

$$R=\alpha(Q^2)[B^g_L\rho+B^s_L]=\tfrac{4}{3}\alpha(Q^2)[f\rho+2]. \tag{2}$$

2. In the NLO approximation Eqs. (1) are changed to the form

$$F_1(x)=[1+\alpha(Q^2)B^s_1]s(x)+\alpha(Q^2)B^g_1 g(x)+O(x)$$
$$=[1-\tfrac{8}{3}\alpha(Q^2)]s(x)-\tfrac{2}{3}f\alpha(Q^2)g(x)+O(x), \tag{3}$$

$$F_L(x)=\alpha(Q^2)\sum_{p=s,g} B^p_L[1+\alpha(Q^2)R^p_L]p(x)+O(x)$$
$$=\tfrac{4}{3}\alpha(Q^2)[(1+\alpha(Q^2)R^g_L)fg(x)+(1+\alpha(Q^2)R^s_L)2s(x)]+O(x),$$

where the products B^p_L and R^p_L are the two-loop coefficients of the Wilson expansion for the first $(n=1)$ moment of the longitudinal SF (see Ref. 8). Specifically,

$$R^g_L=-4[l(x)+\tfrac{5}{3}], \quad R^s_L=8.46-\tfrac{8f}{9}[l(x)+5.64],$$

and $l(x)=\ln(1/x)-[\Psi(\nu+1)+\gamma]$. Here $\Psi(x)$ and γ are the Eulerian Ψ-function and a constant, respectively, and ν is a coefficient which is connected with the $g(x)$ asymptotic function at large x: $g(x)\sim(1-x)^\nu$. We use $\nu=4$, in agreement with the quark count rules (see Ref. 9).

From Eq. (4) we have the following equation for $R(x)$:

$$R=\tfrac{4}{3}\alpha(Q^2)[f\rho(1+\alpha(Q^2)R^g_L)+2(1+\alpha(Q^2)R^s_L)/[1-\tfrac{2}{3}(4+f\rho)\alpha(Q^2)]. \tag{4}$$

3. The SI equation for R can be easily obtained from Eqs. (2) and (4) by introducing new SI effective coupling constant $a(x,Q^2)$, which contains a two-loop correction in its Λ parameter:

$$\Lambda = \Lambda_{\overline{\rm MS}}^{f} \exp(r/2\beta_0),$$

where

$$r = (f\rho \tilde{R}_L^g + 2\tilde{R}_L^s)/(f\rho + 2), \quad \tilde{R}_L^p = R_L^p + \tfrac{2}{3}(f\rho + 4) \quad (p = g, s).$$

We therefore have

$$R^{\rm SI} = \tfrac{4}{3} a(x,Q^2)\,[\,f\rho + 2]. \tag{5}$$

4. Let us analyze the predictions given by Eqs. (2), (4), and (5). The LO of PT predicts a fixed value for the ratio $R(x,Q^2)$. The two-loop correction is negative at small values of x and increases logarithmically in the limit $x \to 0$. This behavior agrees numerically with the predictions of Refs. 2 and 10 for the longitudinal SF (the transverse SF changes only slightly when NLO is added). Hence, the standard PT is weakly applicable at very small values of x.

In the SI approach which is based on the first two orders of PT we have no discrepancies: The variable $R(x,Q^2)$ tends to zero as $[\ln(1/x)]^{-1}$ in the limit $x \to 0$. In the doubly logarithmic approximation the effective coupling constant $a(x)$ has the form

$$a(x) = \bar{a}(x)\left[1 + \frac{\beta_1}{\beta_0}\bar{a}(x)\ln[\bar{a}(x)/\beta_0]\right],$$

where

$$\bar{a}(x) = \left[\beta_0 \ln\left(Q^2/\Lambda_{\overline{\rm MS}}^{2}\right) + 4b\ln(1/x) - \tfrac{4}{3}c\right]^{-1}$$

$$b = \frac{\rho + 4/9}{\rho + 2/f}, \quad c = \frac{17}{4} + \frac{f\rho}{2} + \frac{\rho + 4.75 - 8.19/f}{\rho + 2/f}.$$

In the SI approach we therefore do not obtain negative values of the ratio R at any values of x. In addition, the effective coupling constant decreases logarithmically at small values of x (see also Fig. 1). Hence, it has similar behavior when $x \to 0$ and $Q^2 \to \infty$. This result is obtain in the first two orders of PT only. The values of higher orders of PT are unknown. However, if we construct the SI PT following, for example, the authors of Ref. 6, where the effective coupling constant contains only NLO, we will obtain the new PT with a decreasing coupling constant when $x \to 0$. The results given by this PT therefore seem to be more reliable than in the standard case.

5. In conclusion, we analyzed the behavior of the DIS ratio $R(x,Q^2)$ at small values of x in the first two orders of PT. We restricted the analysis to the case $g(x) \to$ const, $s(x) \to$ const, when $x \to 0$. The simple form for R was obtained. In standard PT the ratio R would be negative at a very small value of x, ($x \simeq 10^{-7}$) (see Fig. 1). In the SI approach there are no negative values of R and the effective coupling constant decreases with an increase in Q^2 and a decrease in x (see Fig. 1). This

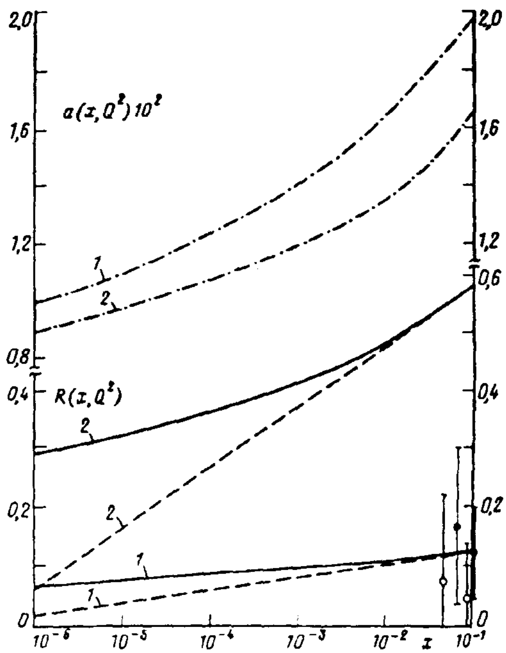

FIG. 1. The effective coupling constant $a(x,Q^2)$ (dot-dashed curve) and the ratio $R(x,Q^2)$ (the dashed and solid curves correspond to standard [i.e., Eq. (4)] and SI PT [i.e., Eq. (5)] results, respectively) are presented at $Q^2=10$ GeV^2. The symbols *1* and *2* correspond to the values *1* and *5* of ρ. The EMC and BCDMS data are indicated by white and black circles, respectively.

behavior of the SI coupling constant accounts, in part, for the weak effect of the contribution of the higher-order corrections to the results of the SI analysis obtained by us.

As is evident from Fig. 1, the values of ρ close to 1 are favored by the EMC (Ref. 11) and BCDMS (Ref. 12) data. We have used here the QCD parameter, $\Lambda(f=4)/$ MS$=200$ MeV, and the experimental EMC points for $Q^2=(12.5$ and $18)$ GeV^2 and the BCDMS points for $Q^2=(15$ and $20)$ GeV^2, respectively (notice that larger values of Q^2 correspond to larger values of x). It is expected that more information on the ratio $R=\sigma_L/\sigma_T$ at $x<10^{-2}$ and on the analysis of the Regge-like behavior of the SF in this region will be obtained from experiments on the HERA and LEP*LHC colliders.

The author is grateful to G. Grunberg, A. L. Kataev, and A. I. Savin for interest in this study and for useful remarks.

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[2)]We use the parton distributions multiplied by x and do not separate out their Q^2 dependence.
[3)]We restrict the analysis to the case $\delta=0$ which corresponds to the standard pomeron. The case $\delta\sim(1/2)$ is not very interesting, because in it the addition of the NLO leads only to a small change (see Ref. 2) of the LO predictions.

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