The S-channel approach to Lipatov's pomeron and hadronic cross sections

N. N. Nikolaev
L. D. Landau Institute for Theoretical Physics, GSP-1, 117940, 117334, Moscow, Russia
B. G. Zakharov
Institute for Theoretical and Experimental Physics, Bolshaya Cheremushkinskaya 25, 17259 Moscow, Russia
V. R. Zoller
IKP(Theorie), KFA Jülich, 5170 Jülich, Germany

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A generalized Balitskii-Fadin-Kuraev-Lipatov equation, which applies directly to the perturbative QCD component of the total cross section, has been derived. The gluon correlation radius $R_c \sim 0.4$ fm was used to reproduce the empirical rate of growth of the hadron–nucleon total cross sections.
The simultaneous estimate of the triple-pomeron coupling was found to be in agreement with the experiment.

Lipatov and his collaborators have shown[1-3] that at short-distance interactions the QCD pomeron has the intercept

$$\alpha_{\rm IP} = 1 + \Delta_{\rm IP} = 1 + \frac{12 \log 2}{\pi} \alpha_S, \tag{1}$$

which gives a very large $\Delta_{\rm IP} \sim 1$ even with the reasonably small strong coupling $\alpha_S = g_S^2/4\pi \sim 0.4$ appropriate for the already short distances ($r \sim 0.15$ fm) (here g_S is the color charge). On the other hand, the $\sigma_{\rm tot} \propto s^{\Delta_{\rm IP}(h,N)}$ fit of the hadronic total cross sections yields[4] $\Delta_{\rm IP}(hN) \sim 0.1$ (here s is the square of the c.m. energy).

In this paper we derive a particularly simple generalization of the Balitskii-Fadin-Kuraev-Lipatov (BFKL) equation directly for the total cross sections. We use the s-channel approach to the pomeron, which is based on the technique of light-cone multiparton wave functions introduced by two of the present authors.[5,6] This method allows us to easily introduce in the gauge-invariant manner the effect of a finite radius for gluon correlations R_c, and to evaluate the effective intercept of the pomeron for the hadronic scattering regime. We attribute the growth of the total cross section to the increase in the number of perturbative gluons in the light-cone hadrons. The light-cone wave functions of the multiparton states and their interaction cross sections were derived in our previous paper[6] and applied to an analysis of the diffractive deep inelastic scattering in the double-leading-logarithm approximation (DLLA) (see also Ref. 7). In this paper we extend the considerations of Ref. 6 to the BFKL regime, discuss the connection between the BFKL and DLLA regimes, and estimate $\Delta_{\rm IP}$ for hadronic scattering processes.

Our starting point is the lowest-order perturbative QCD cross section for the scattering of the two color dipoles **r** and **R** (here **r** and **R** are the two-dimensional vectors in the impact parameter plane)

$$\sigma_0(\mathbf{r},\mathbf{R})=\frac{32}{9}\int\frac{d^2\mathbf{k}}{(k^2+\mu_G^2)^2}\alpha_S^2\,[1-\exp(-i\mathbf{kr})][1-\exp(i\mathbf{kR})]. \qquad (2)$$

Here the effective mass of the gluon μ_G serves as a reminder that the color forces cannot propagate beyond the gluon correlation radius $R_c=1/\mu_G$, and α_S^2 must be understood as[5] $\alpha_S(\max\{k^2,1/r^2\})\alpha_S(\max\{k^2,1/R^2\})$. At $r\ll R\lesssim R_c$, Eq. (2) gives the leading term of the DLLA cross section (for a detailed discussion of the DLLA regime see Ref. 6):

$$\sigma_0(r,R)\approx Cr^2\alpha_S(r)L(R,r), \qquad (3)$$

which is independent of R_c. Here $L(R,r)\approx\log[\alpha_S(R)/\alpha_S(r)]$. In terms of the dipole–dipole cross section, (2), the perturbative part of the total cross section for the interaction of mesons A and B is

$$\sigma^{(pt)}(AB)=\langle\langle\sigma(r_A,r_B)\rangle_A\rangle_B$$

$$=\int dz_A\,d^2\mathbf{r}_A\,dz_B d^2\mathbf{r}_B|\Psi(z_A,\mathbf{r}_A)|^2|\Psi(z_B,\mathbf{r}_B)|^2\sigma_0(\mathbf{r}_A,\mathbf{r}_B). \qquad (4)$$

The advantage of the representation (4) is that it makes full use of the exact diagonalization of the scattering matrix in the dipole-size representation. Below we will discuss $\sigma(\mathbf{r},\mathbf{R})$ averaged over the relative orientation of the dipoles.

The perturbative $q\bar{q}g$ Fock state generated radiatively from the parent color-singlet $q\bar{q}$ state has the interaction cross section

$$\sigma_3(r,\rho_1,\rho_2)=\frac{9}{8}[\sigma_0(\rho_1)+\sigma_0(\rho_2)]-\frac{1}{8}\sigma_0(r),$$

where $\vec{\rho}_{1,2}$ are separations of the gluon from the quark and antiquark, respectively, and $\vec{\rho}_2=\vec{\rho}_1+\mathbf{r}$ (Ref. 6). The increase of the cross section for the presence of gluons is (we suppress the target variable R)

$$\Delta\sigma_g(r,\rho_1,\rho_2)=\sigma_3(r,\rho_1,\rho_2)-\sigma_0(r)=\frac{9}{8}[\sigma_0(\rho_1)+\sigma_0(\rho_2)-\sigma_0(r)]. \qquad (5)$$

The light-cone density of soft, $z_g\ll 1$, gluons in the $q\bar{q}g$ state derived in Ref. 6 is

$$|\Phi_1(\mathbf{r},\vec{\rho}_1,\vec{\rho}_2,z_g)|^2=\frac{1}{z_g}\frac{1}{3\pi^3}\mu_G^2\left|g_S(r_1^{\min})K_1(\mu_G\rho_1)\frac{\vec{\rho}_1}{\rho_1}\right.$$
$$\left.-g_S(r_2^{\min})K_1(\mu_G\rho_2)\frac{\vec{\rho}_2}{\rho_2}\right|^2. \qquad (6)$$

Here $g_S(r)$ is the running color charge, $r_{1,2}^{\min}=\min\{r,\rho_{1,2}\}$, $K_1(x)$ is a modified Bessel function, z_g is a fraction of the (light-cone) momentum of the $q\bar{q}$ pair carried by the gluon, and $\int dz_g/z_g=\log(s/s_0)=\xi$. With allowance for the $q\bar{q}g$ Fock state the dipole cross section takes the form $\sigma_{\rm tot}(\xi,r)=\sigma_0(r)+\sigma_1(r)\xi$, where[6]

$$\sigma_1(r)=\int d^2\vec{\rho}z_g\left|\Phi_1(\mathbf{r},\vec{\rho}_1,\vec{\rho}_2,z_g)\right|^2\Delta\sigma_g(r,\rho_1,\rho_2)=\mathscr{K}\otimes\sigma_0(r). \qquad (7)$$

To higher orders in ξ we have

$$\sigma(\xi, r) = \sum_{n=0} \frac{1}{n!} \sigma_n(r) \xi^n,$$

where $\sigma_{n+1} = \mathscr{K} \otimes \sigma_n$, so that

$$\frac{\partial \sigma(\xi, r)}{\partial \xi} = \mathscr{K} \otimes \sigma(\xi, r) \tag{8}$$

is the generalized BFKL equation for the dipole cross section. We emphasize that the introduction of the gluon correlation length R_c in the kernel $\mathscr{K}$ does not conflict with the gauge-invariance constraints $\sigma(r) \to 0$ and $|\Phi(\mathbf{r}_1, \vec{\rho}_1, \vec{\rho}_2, z_g)|^2 \to 0$ as $r \to 0$, and $\Delta\sigma_g(r, \rho_1, \rho_2) \to 0$ as $\rho_{1,2} \to 0$. The essential ingredient of this derivation is the subtraction of $\sigma_0(r)$ in Eq. (5), which in a simple and intuitively appealing way to take care of the virtual radiative corrections. Once the dipole–dipole scattering problem is solved, Eq. (4) gives the hadron–hadron cross section.

In the BFKL scaling limit of r, ρ_1, $\rho_2 \ll R_c$ and fixed α_S,

$$\mu_G^2 \left| K_1(\mu_G \rho_1) \frac{\vec{\rho}_1}{\rho_1} - K_1(\mu_G \rho_2) \frac{\vec{\rho}_2}{\rho_2} \right|^2 = \frac{r^2}{\rho_1^2 \rho_2^2}, \tag{9}$$

the kernel $\mathscr{K}$ becomes independent of R_c and with a fixed α_S it takes on the scale-invariant form. The BFKL eigenfunctions in Eq. (8) are $E(\omega, \xi, r) = (r^2)^{1/2+\omega} \exp[\xi \Delta(\omega)]$ with the eigenvalue (the intercept) [here $\mathbf{r} = r\mathbf{n}$, $\vec{\rho}_1 = r\mathbf{x}$ and $\vec{\rho}_2 = r(\mathbf{x} + \mathbf{n})$]

$$\Delta(\omega) = \frac{3\alpha_S}{2\pi^2} \int d^2\mathbf{x} \frac{2(\mathbf{x}^2)^{1/2+\omega} - 1}{\mathbf{x}^2 (\mathbf{x}+\mathbf{n})^2} = \frac{3\alpha_S}{\pi} \int_0^1 dz \frac{z^{1/2-\omega} + z^{1/2+\omega} - 2z}{z(1-z)}$$

$$= \frac{3\alpha_S}{\pi} \left[2\Psi(1) - \Psi\left(\frac{1}{2} - \omega\right) - \Psi\left(\frac{1}{2} + \omega\right) \right], \tag{10}$$

where $\Psi(x)$ is the digamma-function. The final result for $\Delta(\omega)$ coincides with the eigenvalues of the BFKL equation. In the scaling limit of $\mu_G \to 0$ Eq. (8) can be transformed to the same form as the original BFKL equation for the differential distribution of gluons.[1,2]

When ω is real and varies from $-\frac{1}{2}$ to 0 and to $\frac{1}{2}$, the intercept $\Delta(\omega)$ is also real and varies from $+\infty$ to $\Delta(0) = \Delta_{\mathbf{IP}}$ and back to $+\infty$, along the cut from $j = 1 + \Delta_{\mathbf{IP}}$ to $+\infty$ in the complex angular momentum j plane. If $\omega = i\nu$ and ν varies from $-\infty$ to 0 and to $+\infty$, then the intercept $\Delta(i\nu)$ is again real and varies from $-\infty$ up to $\Delta(0) = \Delta_{\mathbf{IP}}$ and back to $-\infty$, along the cut from $j = -\infty$ to $j = 1 + \Delta_{\mathbf{IP}}$ in the complex j plane. The choice of the latter cut is appropriate for the Regge asymptotic relations at $\xi \gg 1$, and the counterpart of the conventional Mellin representation is

$$\sigma(\xi, r) = \int_{-\infty}^{+\infty} d\nu\, f(\nu) E(i\nu, r, \xi) = r \int_{-\infty}^{+\infty} d\nu f(\nu) \exp[2i\nu \log(r)] \exp[\Delta(i\nu)\xi], \tag{11}$$

where the spectral amplitude $f(\nu)$ is determined by the boundary condition $\sigma(\xi=0, r)$:

$$f(\nu)=\frac{1}{\pi}\int dr\frac{\sigma(0, r)}{r^2}\exp[-2i\nu\log(r)]. \tag{12}$$

In the BFKL regime, the rightmost j-plane singularity corresponds to the asymptotic cross section

$$\sigma_{IP}(\xi, r)\propto r\exp(\xi\Delta_{IP}). \tag{13}$$

The solution of Eq. (8) can be written as follows:

$$\sigma(\xi, r)=r\int\frac{dr'}{(r')^2}K(\xi, r, r')\sigma(\xi=0, r'). \tag{14}$$

In the BFKL regime the evolution kernel in this expression is

$$K(\xi, r, r')=\frac{1}{\pi}\int d\nu\exp\left[2i\nu\log\frac{r}{r'}\right]\exp[\xi\Delta(i\nu)]$$
$$\propto\frac{\exp(\Delta_{IP}\xi)}{\sqrt{\xi}}\exp\left(-2\frac{(\log r-\log r')^2}{\xi\Delta''(0)}\right). \tag{15}$$

The "diffusion" kernel, (15), clearly shows that starting with $\sigma(\xi=0, r)$, which is concentrated at the small, perturbative $r\lesssim R\ll R_c$, one ends up at large ξ with $\sigma(\xi, r)$, which extends to the nonperturbative $r\sim R\exp[\sqrt{\xi\Delta''(0)}]) > R_c$. This "diffusion" toward large r is further accelerated if the running coupling is introduced. Thus, the scattering of even very small dipoles of size $r\sim R\ll R_c$ and/or deep inelastic scattering at $Q^2\gg R_c^{-2}$ will eventually be dominated by interactions of the perturbative gluons which stick out of the small dipoles at a distance $\rho\sim R_c$, and Δ_{IP} will be determined by interactions at the scale R_c and by the frozen coupling $\alpha_S^{(\mathrm{fr})}$.

Since an analytic solution of Eq. (8) at finite R_c and with the running coupling is not available, we resort to a numerical analysis. We use the running QCD coupling $\alpha_S(r)=6\pi/[(33-2N_f)\log(1/\Lambda r)]$ with $\Lambda=0.2$ GeV, and at large r we impose a simple freezing, $\alpha_S(r) = \alpha_S^{(\mathrm{fr})} = \min\{\alpha_S(R_c), 1\}$. First, we calculate $\Delta_{\mathrm{eff}}(r, R)=\sigma_1(r, R)/\sigma_0(r, R)$ for the boundary condition $\sigma(\xi=0, r)$ given by the dipole–dipole cross section, (2). The first interesting case is the scattering of the equal-size dipoles (Fig. 1). Here the kernel $\mathcal{K}$ suggests the quasiclassical estimate for the effective intercept

$$\Delta_{IP}(r)=\frac{12\log 2}{\pi}\alpha_S(r). \tag{16}$$

Indeed, we find that at small r the ratio $\beta=\Delta_{\mathrm{eff}}(r, r)/\Delta_{IP}(r)$ tends to a constant value, $\beta\approx0.57$, which is independent of the gluon correlation radius R_c. The effective intercept $\Delta_{\mathrm{eff}}(r, r)$ increases with r to $r\sim R_c$, then decreases and flattens at $r\gg R_c$, where both $\sigma_{0,1}(r)\propto R_c^2$.

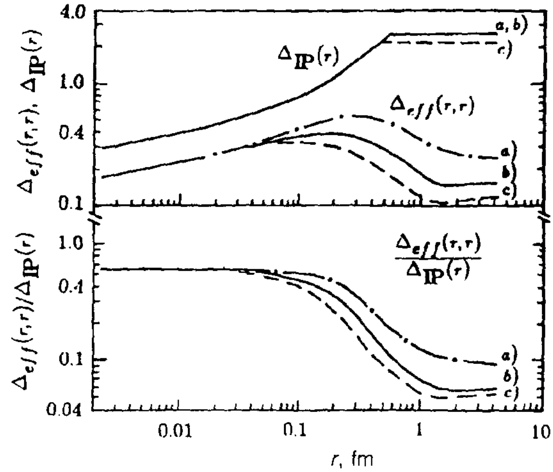

FIG. 1. The effective intercept $\Delta_{\rm eff}(r, r)$ for the scattering of two identical dipoles of size r in comparison with $\Delta_{\rm IP}(r)$, Eq. (16). The lower box shows the ratio $\Delta_{\rm eff}(r, r)/\Delta_{\rm IP}(r)$. The curves a, b, and c are for μ_G=0.3, 0.5, 0.7 GeV, respectively.

Another interesting regime is the DLLA of unequal dipoles, $r \ll R$. In this limit Eq. (7) takes the form[6]

$$\sigma_{n+1}(r)+\mathscr{K}\otimes\sigma_n(r)=\frac{3r^2\alpha_S(r)}{\pi^2}\int_{r^2}^{R_c^2}\frac{d^2\vec{\rho}}{\rho^4}\sigma_n(\rho), \tag{17}$$

which is equivalent to the GLDAP evolution equation[8] and to a first order in ξ gives $\Delta_{\rm DLLA}(r, R)=\frac{2}{3}\log[\alpha_S(R)/\alpha_S(r)]$. Like any logarithmic estimate, this formula works up to a constant term ~ 1. In Fig. 2 we show our results for $\Delta_{\rm eff}(r, R)$ for $R=1$ fm. The difference $\delta=\Delta_{\rm eff}(r, R)-\Delta_{\rm DLLA}(r, R)$ flattens at $r/R \lesssim 0.2$, which is a signal of the onset of DLLA.

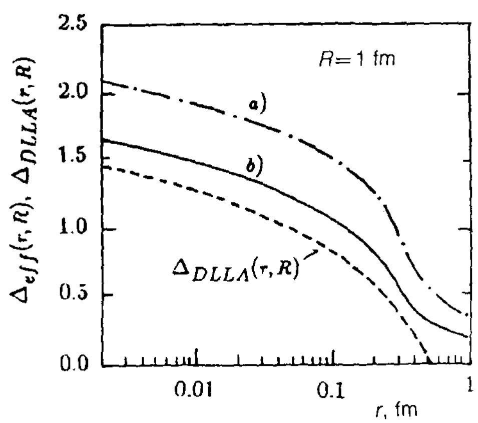

FIG. 2. The effective intercept $\Delta_{\rm eff}(r, R)$ for the scattering of unequal dipoles of sizes r and R in comparison with DLLA, formula $\Delta_{\rm DLLA}(r, R)$. The curves a and b are for μ_G=0.3 and 0.5 GeV, respectively.

The position of the rightmost singularity in the j plane can easily be estimated from the asymptotic behavior of the numerical solution of Eq. (7). At R_c=0.4, 0.28, and 0.22 fm, i.e., at the frozen coupling $\alpha_S^{(\mathrm{fr})} = 1.0$, 82, and 0.63, we find the estimates $\Delta_{\mathrm{IP}} \approx 0.52$, 0.41, and 0.36, respectively. A detailed discussion of the convergence to, and properties of, the limiting cross section, $\sigma_{\mathrm{IP}}(\xi, r)$, will be presented elsewhere. We note here that these estimates are significantly below the Collins–Kwiecinski lower bound, $\Delta_{\mathrm{IP}} > 3.6\, \alpha_S^{(\mathrm{fr})}/\pi$ (Ref. 9). (The derivation of this bound in Ref. 9 is flawed by the infrared cutoff which breaks the initial symmetry of the BFKL kernel in the momentum space.)

The two cases of certain theoretical interest, although of little practical interest, are worth mentioning. The first is the case of finite R_c at fixed α_S, the second is the case in which $\mu_G \to 0$ in the wave function (6) while keeping R_c finite in the strong coupling. Both share the property of restoration of the scaling invariance of the kernel $\mathcal{K}$ on the infinite semiaxis $\log r < \log R_c$ or $\log r > \log R_c$, where $\alpha_S(r)$ freezes, respectively. On the corresponding semiaxis, the eigenfunctions are essentially identical to the BFKL set, the spectrum of eigenvalues is evidently continuous, and the j-plane partial waves have the cut in the j plane. Here we differ from Lipatov,[3] who concluded that the running coupling leads to the discrete spectrum of eigenvalues and to a sequence of poles in the j plane. The numerical analysis shows that the tip of the cut in the j plane is very close to Δ_{IP} as given by Eq. (1) with $\alpha_S = \alpha_S^{(\mathrm{fr})}$; a more detailed analysis is needed to verify the possibility of a finite departure from Eq. (1) which depends on the value of $\alpha_S^{(\mathrm{fr})}$.

Finally, let us consider the πN interaction as the typical hadronic scattering. The plausible assumption is that the growth of the hadronic cross section is dominated by the perturbative gluons. In this case we have $\sigma_1^{(\mathrm{pt})}(\pi N) = \langle\langle \sigma_1(r_\pi, r_N)\rangle_N\rangle_\pi$ and $\Delta_{\mathrm{IP}}(\pi N) \approx \sigma_1^{(\mathrm{pt})}(\pi N)/\sigma_{\mathrm{tot}}(\pi N)$. In Fig. 3 we show our prediction of the perturbative QCD contribution to the total cross section $\sigma_0(\pi N)$ and the plot of $\Delta_{\mathrm{eff}}(\pi N)$ versus the gluon correlation radius R_c. We reproduce the empirical value $\Delta_{\mathrm{IP}}(hN) \sim 0.1$ at $R_c \sim 0.4$ fm when $\sim 40\%$ of $\sigma_{\mathrm{tot}}(\pi N)$ is of the perturbative origin.

In addition to the increase in the total cross section, the variations of the dipole cross section for the presence of gluons also contribute to the triple-pomeron coupling $A_{3\mathrm{IP}}$. The method for calculating $A_{3\mathrm{IP}}$ was presented in Refs. 6 and 7. With a correlation length $R_c \sim 0.4$ fm, we find $A_{3\mathrm{IP}}(\pi N) \sim 0.04\ \mathrm{GeV}^{-2}$, consistent with the experimental determinations.[10]

In conclusion, we have derived a generalized BFKL equation for the total cross sections, with allowance for the finite gluon correlation radius R_c. We presented the first estimates of the perturbative QCD contribution to the rate of the growth of $\sigma_{\mathrm{tot}}(pN)$ and to the triple-pomeron coupling, which are consistent with experiment if $R_c \sim 0.4$ fm. [Incidentally, the instanton model of the QCD vacuum and the lattice QCD calculations give a very close value of R_c (Ref. 11).] The irrefutable advantage of having the equation for the total cross section and of using the dipole-size representation, which diagonalizes the scattering matrix, is that it allows an easy incorporation of the unitarity constraints. To this end, we recall that allowance for the unitarity corrections in the DLLA limit has already led to an important conclusion[6] that the unitarity correction to the structure functions in the diffractive deep inelastic

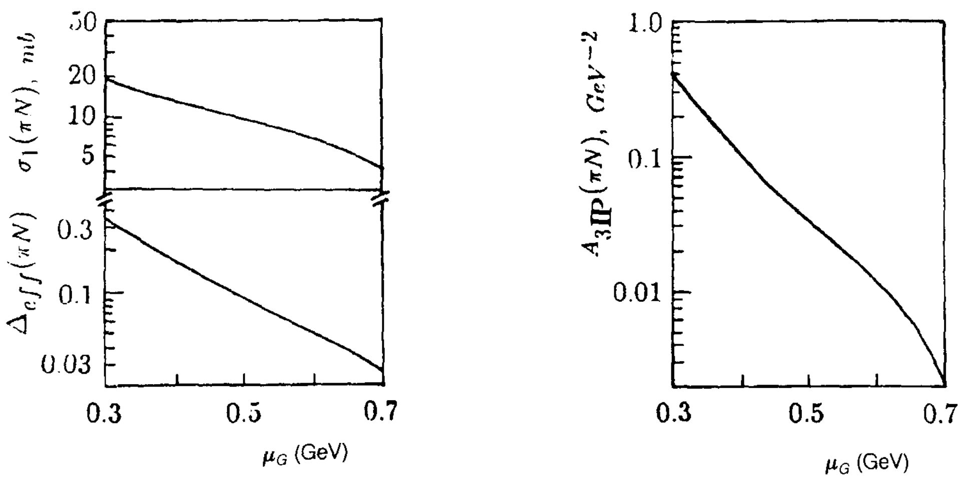

FIG. 3. The effective intercept $\Delta_{IP}(\pi N)$, the perturbative QCD contribution to the total cross section $\sigma_0(\pi N)$, and the effective triple-pomeron coupling for the plot of the pion-nucleon scattering versus μ_G.

scattering at small values of x satisfies the linear GLDAP evolution equations.

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